RIGA TECHNICAL UNIVERSITY

Faculty of Transport and Mechanical Engineering

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"Transport"

OPTIMIZATION METHODS AND MODELS CREATION FOR LOGISTIC COMPANY SUCCESSFUL DEVELOPMENT

Doctoral Thesis Summary

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DOCTORAL THESIS

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CONFIRMATION

Hereby I confirm that I have worked out the present doctoral thesis submitted for the doctoral degree at Riga Technical University. This thesis has not been submitted for the doctoral degree at any other university.

Olga Girvica …………………………….(Signature)

Date: 06.06.2013.

The doctoral thesis is written in English. The doctoral thesis consists of introduction, 6 chapters and conclusions. List of references include 87 sources. There are 4 appendixes, 9 figures and 19 tables. The thesis is printed on 137 pages.

ABSTRACT

The promotional work "Optimization Methods and Models Creation for Logistic Company Successful Development" has been worked out by Olga Girvica to obtain the scientific degree of Doctor of Science in Engineering. Scientific supervisor of the work is Dr.hab.sc.ing., professor Alexander Andronov.

Due to specific geographic location of Latvia transport area plays one of the main roles in the national economy and also important for the economic development of the whole European Union. High attention is paid by the government for development of Logistics in Baltic area.

A broad area of my research is the lack of practical methods for optimization of the costs and activities for the logistics centre in the market conditions of World recession in Latvia.

There are some specific tasks for Logistic centre successful development and design together the facility and delivery process to better reveal and support of customer purposes. One of the main objectives for any logistic company is to simplify the supply chain, minimize storage and maximize speed and efficiency for delivery of goods. Warehousing and distribution is an important part of any supply chain. The essential part is the information technology and efficient flow of information between the members of the chain as well as understanding the business and needs of clients.

The goal of research is to develop the new optimization models and methods for solving the tasks of logistic company in the market conditions of World recession. Development of the new logistic models enables planning of the optimal use of any business resource (vehicles, finance and others), also minimization of the business costs and maximization of the received profit. This helps to model of a complete logistics system (chain) from the raw materials producer till the recipient of the end product. The models can be used both within creation of the overall transport system development strategy and within solution of specific related tasks.

Using the methods of mathematical programming three new models for work optimization of logistics companies were presented.

1. The design of optimal circular routing for goods delivery. Using the method of stochastic optimization a new Mathematical model for Optimal Planning of Circular Delivery Routes was suggested. Using this model the optimal route which provides the minimal delivery destination for each group of the delivery points was made. Differ from well known "task of commercial traveller" our problem is more complex because additionally it takes into account many cycles, weight of goods and a weight restriction for each cycle and valuable by its singularity (originality). Considered decisions and right strategy for creating of circular delivery route for goods distribution from Logistic centre to final customers allow to decrease the costs and optimize the work of the Logistic centre by more effective use of transport and minimization of goods lean time.
2. The model of hierarchic optimization of the resources distribution among the units of the Logistic centre. The problem of resources distribution in optimal way

among the units of the Logistic centre which has the treelike structure was solved with help of this model. There are some various ways of development for each unit of the Logistic centre. The ways differ from each other by necessary resource and profit receiving.

3. The Logistic centre Supply Chain optimization model. The model allows solve the task of the decision-making process for Logistics centre Supply Chain optimization through the creation of the new supply and sales channel in optimal way. There are some various decisions could be taken at each stage of process development. Two aspects, such as maximum probability of the best effect achievement and average profit maximization are taking as the criterions for effectiveness of making decisions.

Acquisition and development of the optimization models and methods of the vehicles use and logistics center activities is a significant step for securing optimal planning and further successful development of the transport system in Latvia and EU.

Contents

1. INTRODUCTION

1.1.ACTUALITY OF THE PROBLEM

In the short and long term perspective of the European transport system development the emphasis is made on modernization and renovation of the transport infrastructure that will enable securing the quality and volume of the freight traffic. A particular attention is devoted to such aspects as efficiency, profitability, environment protection and safety.

Latvian Ministry of Transport has declared that the priority of the transport industry is the construction of the industrial and distribution park, and has offered Latvia as a Logistics center in the region, which can implement a 24-hour concept, what means that from Latvian logistics and distribution warehouses goods can be transported to the stores in Helsinki, Stockholm, Minsk, Warsaw or Moscow [4].

A broad area of my research is the lack of practical methods for optimization of the costs and activities for the logistics centre in the market conditions of World recession in Latvia.

For this reason development of the new optimization models and methods in the logistics system is important which would take into account the rapidly changing situation and new influencing factors.

1.2.THE OBJECTIVES AND TASKS OF RESEARCH.

The main objective of the promotional work is development of the mathematical models and methods for the successful operations of logistic company. This includes the following aspects:

1. The design of optimal circular routing for goods delivery.
2. Developing of the model of hierarchic optimization of the resources distribution among the units of the Logistic centre.
3. Creation of the Logistic centre Supply Chain optimization model. Two aspects, such as maximum probability of the best effect achievement and average profit maximization are taking as the criterions for effectiveness of making decisions.

In this respect the following tasks were considered:

1. Considering problems of logistic system successful optimization.
2. Creation research information base through the collecting and analysing of statistical data of transport system in Latvia.
3. Investigation of the information technology and information flow between the members of the chain from the business and needs of client’s point of view.
4. Investigation nowadays used models and methods for logistics activity optimization.
5. Observation of the typical tasks and factors influencing the success in the work of the Logistic Centre.
6. Creation of the computer programs solving the research problems.

7. Using the new developed models and methods for the real actual problems solving for several Latvian companies.

1.3.METHODOLOGY AND METHODS OF THE RESEARCH.

1. In the methodology basis of this research there are real methods, which are used in real logistic business activities and based on real statistical data. The first ones received from the practical working experience of researcher in different fields of transport area during 17 years. Others (statistical data) used in this research are collected from the working history of several Logistic companies. For macro level of research statistical data was received from central statistical bureau in Latvia and EU.
2. In the basis of offered approaches main tasks and factors were delineated that undoubtedly influencing the research problems.
3. As research methods were used modern mathematical tools: graph theory (oriented graphs, routes and the shortest routes, trees), probability theory (discrete random variables and their probability distribution functions), random search method (Monte Carlo method), dynamic programming (one-dimensional resources distribution problem), controlled Markov chain (discrete time Markov chain, many steps decision making method)
4. All the created methods and algorithms are realized by Computer programming MathCad 14 environment that gives possibility for easy using of necessary results in practice.

1.4.SCIENTIFIC NOVELTY.

Due to specific geographic location of Latvia transport area plays one of the main roles in the national Latvian economy and also important for the economic development of the whole European Union. High attention is paid by the government for development of Logistics in Baltic area.

A broad area of my research is the lack of practical methods for optimization of the costs and activities for the logistics centre in the market conditions of World recession in Latvia.

The main question of my research: Is it feasible to develop effective strategic models for optimization of the costs and activities for the logistic centre in the market conditions of World recession in Latvia?

Every business has a competitive strategy. However many strategies are implicit, having evolved over time, rather than explicitly formulated from the thinking and planning process. Implicit strategies frequently lack focus. Produce inconsistent decisions, and unknowingly become obsolete. My work provides a process to develop a logistic strategy and make it explicit so it can be examined for focus, consistency, comprehensiveness and currency.

Logistics strategy can be a black art – understanding the dynamics of the industry, knowing what drives the competition and what they are likely to do next, planning and timing own moves, positioning products and services, allocating scarce resources in the most effective way [33]. The researcher has to demystify these areas and know what is the best use of a logistic simulation tool that allows to test and evaluate strategy alternatives in a risk-free environment and in conditions of the world recession now. “Strategy is not a one-time search for a sustainable competitive advantage, but a continuous monitoring of the environment, consumers, and competitors with the object of making the right moves in a dynamically changing competitive landscape." - Philip Kotler [38].

The paper is devoted to development of the new strategic optimization models for the work of a logistics company in the market conditions of World recession. Development of the new logistic models enables planning of the optimal use of any business resource (vehicles, finance and others), also minimization of the business costs and maximization of the received profit. This helps for modelling of a complete logistics system (chain) from the raw materials producer till the recipient of the end product. The models can be used both within creation of the overall transport system development strategy and within solution of specific related tasks.

1.5. READINESS OF THE THEME

The survey of literature were made and used as the base for theoretical framework. During research the following methods were investigated: Mass Service System models (learned by many authors such as G.L.Brodeckij [69], G.G.Levkin [75], U.M.Nerush [78]), Mixed-integer programming (learned by D.Bienstock [10], G.Neinhauser, R.Fukasawa [23], J.Lysgaard [23], M.Poggi de Aragao [23], M.Reis [23], R.F.Werneck [23]), heuristic algorithms (presented by works of J.L. Bentley [7], R. Battiti [10]), different algorithms for the Capacitated Vehicle Routing Problem (learned by G.Dantzig [17], R.Ramser [17], N.Christofides [13], [14], A.Mingozzi [14], P.Toth [56], [57], D.Vigo [56], [57], M. Fisher [20], C.Martinhon [42], A.Lucena [42], N.Maculan [42]). Most of the existing models offer to optimize the process through optimization steps for separate units solving the certain single tasks, but not the system as whole. However on practice only the way for the whole system improvement is workable. Also many models do not take into account all needed restrictions and external factors for achieving the best results. In this research are offered the models which besides of the local optimization give possibility to create the optimization strategy for logistic company.

1.6. PRACTICAL VALUE, REALIZATION AND APPLICATION OF THE WORK

1. On the basis of the obtained results a number of guest lectures on the subject “Transport aspects in logistics” for the second year students of bachelors’ studies programme of the Riga Technical University Mechanical Engineering faculty has been prepared.
2. It is important that all the results of my research were used in practical area that improves the work of several commercial companies and allow them to achieve better

business results. During research time all used business statistic data were given from the real Latvian companies. Using the real data gives an opportunity to check and analyse the gotten results in practice. There are three companies took part in research: SIA “66North Ice-Balt”, SIA “66North Delivery and Logistics” and SIA “EURECO”. After the new models based on dynamic programming method were implemented into the work of logistic companies the financial indices significantly increased.

3. The new models can be used both within creation of the overall logistic system development strategy and within solution of specific related tasks.
4. The main results of this research are published in 5 articles and presented at 9 scientific conferences, at which the author presented 6 reports on the subject of the promotion work. The list of articles and reports at conferences are given bellow.

1.7.DEFENDABLE THESIS

1. Mathematic models and a number of methods, which allow effectively solve important logistic tasks.
2. Transport routing optimization method for logistic company. Our model generalizes classical “commercial traveller” (salesman) problem taking into account: many “travellers” (transport vehicles), different weight capacities, different goods demands in destination points.
3. Hierarchic model for logistic company optimal development. The model allows distribute existing financial resources among the own units of the company.
4. Optimization method for Supply Chain planning. There are two aspects: average profit maximization and maximum probability of the best effect achievement have been taken as the criterions for effectiveness of making decisions. In the last case it could be talked about risk management.

1.8.STRUCTURE OF THE DOCTORAL THESIS

The thesis consists of introduction, 6 chapters, conclusions, and list of references, 3appendixes, 9 figures and 19 tables, total 137 pages (appendixes included)

Chapter 1. Evaluation of Latvian Transport System and Its Development Perspectives in Global European Logistic Network. This chapter devote to the Latvian Transport Industry. Transport and logistics is one of the priority industries in Latvia. Here presented the perspectives and the main goals for Latvian transport development set by Latvian Ministry of Transport.

Chapter 2. Logistics System Sentence and Problems. This chapter is dedicated to main important questions solving by Logistics. First of all, Logistics operations and functions are observed. Systemic approach to Logistics including purchasing, production, sales and return logistics are given as well. Also the financial risks and supporting information technology are learned as the important indicators in Logistics.

Chapter 3. Tasks for the Logistic Centre Successful Development and Methods for Solving. This chapter shows the typical strategic tasks, which are met by logistic company during developing time of the Logistic centre. Considered decisions and right strategy allows decreasing the costs of the Logistic centre and minimizing lean time for goods delivery from Logistic centre to its final customers in Latvia. Most of investigated methods: Mass Service System models, Mixed-integer programming, heuristic algorithms, different algorithms for the Capacitated Vehicle Routing Problem offer to optimize the process through optimization of separate units solving the certain single tasks, but not the system as whole. Besides, many of them do not take into account all needed restrictions and external factors for achieving the best results.

Chapter 4. Mathematical Method for Optimal Planning of Circular Delivery Routes. In this chapter a routing problem is solved. Here presented the optimization method for circular delivery routes organization. The described problem is a generalization of a classical problem of Hamilton’s cycle [66]. It is well known that the last problem have not an efficient algorithm for solving. Our problem is more complex because additionally it takes into account many cycles, weight of goods and a weight restriction for each cycle. So here a method of stochastic optimization is used.

Chapter 5. Hierarchic Optimization of the Resources Allocation Among the Units of the Logistic Centre. This chapter shows how using the dynamic programming method the real formulated task for distribution of necessary resource among the units of the Logistic centre in Latvia is solved and the solution for getting of optimal profit were found for each unit.

Chapter 6. Optimization of the Supply Chain for the Logistic Centre. This chapter present that the dynamic programming method could be used for solving the task of decision-making process for the new supply and sales channel development for getting of optimal profit that is proved by observed real sample of the Logistics Centre in Latvia.

1.9. APPROBATION OF THE DOCTORAL THESIS

The main results of the doctoral thesis have been presented on 11 international scientific conferences with reports on 6 of them:

1. International Conference on Reliability and Statistics in Transportation and Communication (RelStat`08) Riga, Latvia, October 21 – 24, 2008. Report: Main Tasks for the Logistic Center Successful Development, author: Girvica O.
2. International Conference “Breaking Bureaucracy Barriers in Latvia” 2008, Riga, Latvia
3. International Conference “Russian Market. Practice. Perspectives.” 2008, Riga, Latvia
4. The 50th Scientific Conference of Riga Technical University. Riga, Latvia, October 12 – 16, 2009. Report: „Resursu sadalījuma starp loģistikas centra vienībām hierarhiska optimizācija”, author: Girvica O.
5. International Conference on Reliability and Statistics in Transportation and Communication (RealStat’09). Riga, Latvia, October 21 – 24, 2009.
6. International Science Conference on Business Competitiveness in Local and Foreign Markets: Challenges, Riga, Latvia, April 29 – 30, 2010.
7. The 51th Scientific Conference of Riga Technical University. Riga, Latvia, October 11 –

15, 2010. Report: „Lēmumu pieņemšanas process loģistikas centra piegādes ķēžu optimizācijai", author: Girvica O.

8. International Conference on Reliability and Statistics in Transportation and Communication (RelStat`10) Riga, Latvia October 21 – 24, 2010. Report: „Optimization of the Supply Chain Process for the Logistic Centre", author: Girvica O.
9. 9th International Science Conference on Information Technologies and Management'2011. Riga, Latvia, April 14 – 15, 2011. Report: "Financial Risks in Logistics", author: Girvica O.
10. International Science Conference on Changes in Global Economic Landscape – in Search for New Business Philosophy. Riga, Latvia, April 28 – 29, 2011 Report: "New Supply Chain Creation for Logistics Centre Work Optimization", author: Girvica O.
11. Stakeholders debate "The Integrated Transport System in the Baltic Sea Region – the Transit Role of Latvia", Riga, Latvia, September 14, 2011.

The author of the doctoral thesis is the author of 5 scientific research publications:

1. Girvica O. MAIN TASKS FOR THE LOGISTIC CENTRE SUCESSFUL DEVELOPMENT (GALVENIE UZDEVUMI LOĢISTIKAS CENTRA SEKMĪGAI DARBĪBAI). Proceedings of the 8th International Conference „Reliability and Statistics in Transportation and Communication. RelStat'2008".October 15-18, 2008, Rīga, Latvija, 60-63 lpp. SCOPUS.
2. Girvica O. MODERN STRATEGIES FOR THE COSTS OPTIMIZATION OF THE LOGISTIC CENTRE (MŪSDIENĪGAS STRATĒĢIJAS LOĢISTIKAS CENTRA IZDEVUMU OPTIMIZĀCIJAI). "Mašīnzinātne un transports" RTU Zinātniskie raksti. 2010 ISSN 1407-8015, Rīga, 123.-128. lpp. EBSCO.
3. Girvica O. HIERARCHIC OPTIMIZATION OF THE RESOURCES ALLOCATION AMONG THE UNITS OF THE LOGISTIC CENTRE (RESURSU SADALĪJUMA STARP LOĢISTIKAS CENTRA STRUKTŪRVIENĪBĀM HIERARHISKĀ OPTIMIZĀCIJA). "Mašīnzinātne un transports" RTU Zinātniskie raksti. 2010 ISSN 1407-8015, Rīga, 106.-110. lpp. EBSCO.
4. Girvica O. OPTIMIZATION OF THE SUPPLY CHAIN PROCESS FOR THE LOGISTIC CENTRE. "Transport and Telecommunication" TSI journal, 2010, Vol.11, N2 ISSN 1407-6160 Riga pp.12-17. SCOPUS.
5. Girvica O. NEW SUPPLY CHAIN CREATION FOR LOGISTICS CENTRE WORK OPTIMIZATION "Journal of Business Management" 2011, Nr 4, ISSN 1691-5348, Riga. pp.170 – 177. EBSCO.

2. DESCRIPTION OF THE MAIN RESULTS OF THE WORK

2.1. LOGISTICS AND ITS IMPORTANCE IN LATVIAN ECONOMY

Since the ancient times the advantages for development of the transport sector in Latvia are its favorable geographic location in Europe and at the Baltic Sea. Ourdays transport industry is one of the significant in Latvian economy. Total transport volume takes more than 12% in average of Latvian GDP during the period from 1990. Advanced transport network creates a perfect base for formation and successful development of logistics in Latvia. The Ministry of Transport [4] has declared that the priority is the construction of the industrial and distribution park, and the ministre has offered Latvia as a Logistics center in the region, which can implement a 24-hour concept, what means that from Latvian logistics and distribution warehouses goods can be transported to the stores in Helsinki, Stockholm, Minsk, Warsaw or Moscow.

In Logistics similarly as in other business areas companies meet several type of financial risks in their daily operations. In this work were observed just a few most common risks and risk management recommendations for company in international logistics. Financial risks differ by its nature and methods of management in order to reduce the risks. Many risks are closely connected to each other and all of them in the end appeared in the liquidity of the business.

One of modern strategies in logistic is to change the traditional supply chain to a new one. The main difference and a key point for the new view of Supply Chain is creation of the system with general hub of logistic service provider (Logistic Centre) which provide and manage all processes of Supply Chain and provide all logistic functions under common strategy.

The objective is to simplify the supply chain, minimize storage time and volume and maximize speed and efficiency of deliveries. This will allow producers to specialize in production. The essential part is the information technology and efficient flow of information between the members of the chain as well as understanding the business and needs of clients. The main disadvantage of such approach is its long-run effect. It is real paradigm shift not only for the company but as well for all its clients and suppliers. This approach is actually based on trust and common values of the partnering companies.

Based on the own practical experience in logistics area and results of theoretical observation author considers that working strategy for Logistic Centre successful development is defined by the solving process of following specific tasks:

1. Creating of the system for goods receiving into warehouse.
2. Developing of the system for goods storage in the warehouse.
3. Control and monitoring of goods movements in/out warehouse.
4. Optimization of goods distribution till final consumer.
5. Goods lean time decreasing.

Most of the logistic tasks mentioned above could be solved with the help of variety of Economic and Mathematic methods and models, although they do not use all necessary restrictions and factors for receiving the best results. Also there is no suitable common strategy of optimization for the whole logistic system like Logistic Centre. Most of the learned methods and models solve certain single tasks, but do not optimize the system in general. In this research author offers the models which besides of the local optimization give possibility to create the optimization strategy for logistic company (Logistic Centre) by solving the main tasks as a complex. Among the tasks were chosen three - the most significant and actual ones for optimization of work of Logistic Centre.

2.2. MATHEMATICAL METHOD FOR OPTIMAL PLANNING OF CIRCULAR DELIVERY ROUTES

First of all let's determine what the "optimization" means. According to Merriam-Webster Dictionary: "Optimization is an act, process, or methodology of making something (as a design, system, or decision) as fully perfect, functional, or effective as possible; specifically the mathematical procedures (as finding the maximum of a function) involved in this" [24]. According to another definition - "In mathematics, computational science, or management science, mathematical optimization (alternatively, optimization or mathematical programming) refers to the selection of the best element from some set of available alternatives. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain, including a variety of different types of objective functions and different types of domains."[39]

The new optimization models for logistic company successful development were created during this research. The first of them is optimal planning of circular delivery routes.

Company sells the products and household goods. The considerable part of the goods (about 60%) is imported from the East Europe, Russia and Ukraine. In the same time about 40 % of goods are of Latvian origin. The trade activity of the company could not be without a good work of the Logistic centre of the company. The Logistic centre has five own trucks with the cargo capacity of 22 tons each. Three of the trucks deliver goods from Europe, Russia and Ukraine, the rest two distribute the goods from warehouse in Riga to clients in Latvia. The analysis of the activity and cost evaluation of the Logistic centre showed that goods are often arrived with delays and expenses were constantly growing during last few months. The expanding of the trade activity and appearing of the new clients demanded additional transport services from the Logistic centre. Often all five trucks were using on the local Latvian market. Therefore some goods from Europe were coming late to the stock in Riga. That brought the additional costs to the company because of the shortage of the import goods, additional payments to the drivers, working extra hours, and expenses related to the empty race. The main reason for that was the wrong planning of the distribution routes, which demand additional time and fuel. The real task was put for the Logistic centre.

Problem Setting

Let us view the task for creation of circular delivery routes. The Logistic centre has *k* own trucks with the cargo capacity of C_i tons for the *i*-th truck, $i = 1,\ldots,k$.

The company has a net of the clients (and corresponding towns or point). Let *n* be client number. The destinations between ones (in km) are setting by a *distance matrix D* (Table 2.1), were matrix element $D_{i,j}$ is a distance between *i*–th and *j*–th clients (points, towns). If points (*i- j)* are unconnected then $D_{i,j} = \infty$. Additionally it is known goods (cargo) in kg Go_i for the *i*-th client. It is necessary planning *k* circular routes for the goods delivery, where each rout has initial and final point (town) with number 0. It is supposed that the *r*-th rout is performed by *r*-th truck.

Let us set a mathematical formulation of the problem. At beginning, we restrict ourselves by case $k = 2$. We introduce the following vectors: $R^{\langle r\rangle} = \left(i(0,r), i(1,r),\ldots, i(r^*+1,r)\right)^T$ sets the *r*-th route, where r^* is a number of rout nonzero points (towns), *i* – number of the town in chain *r*, $t_0 = t_{r^*+1} = 0$. Now we can formulate the problem by such a way:

Minimize total loss of kilometres

$$L(R) = \sum_{r=1}^{2}\sum_{j=0}^{r^*} D_{i(j,r),i(j+1,r)} \tag{2.1}$$

subjected by restrictions:

- Both routs are summarized including all the points (towns)

$$R^{\langle 1\rangle} \vee R^{\langle 2\rangle} = \{0\ ,1,\ldots,n\}, \tag{2.2}$$

- Not any of routs cross the other one

$$R^{\langle 1\rangle} \wedge R^{\langle 2\rangle} = \{0\}, \tag{2.3}$$

- The volume of weight should not to exceed the capacity of each truck

$$\sum_{j=1}^{r^*} Go_{i(j,r)} \le C_r,\ \ r = 1,\ 2 \tag{2.4}$$

Where C_r is a capacity of truck, *r* is the number of cycle.

The described problem is a generalization of a classical problem of Hamilton's cycle [64], [66].

As it is well known that the last problem haven't an efficient algorithm of a solving. Our problem is more complex because additionally it takes into account many cycles, weight of goods and a weight restriction for each cycle. So we use a method of stochastic optimization.

Optimization Method

We use the graph theory [13], [16], [65], [74]. Namely, our undirected graph contains vertices (towns) and edges (ways between towns). Each edge has a length, it is a length of corresponding way. Each town (node) has certain weight *Ci* for *i*-th town.

Firstly we fix a number m of nonzero points in the first rout, so $(n - m)$ is a number of nonzero points in the second rout. The main steps of one iteration are the following.

1) At random generate m elements from set $\{1, 2, \ldots, n\}$. Let $G^{\langle 1 \rangle}$ be corresponding sample, $G^{\langle 2 \rangle}$ be its complement. There are 2 routes created from total number of towns.

2) Verify that for each sample restriction (2.4) is satisfied. If not then return to the step 1.

3) Verify that for each sample all points are attainable from initial point 0. If not then return to the step 1.

4) Enumerate all permutations of set $G^{\langle 1 \rangle}$. Calculate loss value (2.1) for each permutation and determine a permutation with minimum loss value. If it equals infinity, go to step 1.

5) Repeat the step 4 for the second sample $G^{\langle 2 \rangle}$.

6) Calculate a total loss by formula (2.1).

7) If new result is better than previous one then remember it as record.

Described iteration is repeated necessary times *RN*. The best result is accepted as final one.

Main Calculation Procedures and Computer Programmes

We give a shot description on that using above mentioned notation the list of steps.

1. A generation of m-sample $G^{\langle 1 \rangle}$ from n element $\{1, 2, \ldots, n\}$ is performed by programme *Sample(n. m).*
2. To make a verification of the step 3, at first we need forming an adjacency matrix for point sets $G^{\langle 1 \rangle}$ and $G^{\langle 2 \rangle}$. It performs a programme *MF(group)* where group = $G^{\langle 1 \rangle}$ or

$G^{\langle 2 \rangle}$. An attainable property for point *j* and point set *M* is verified by programmes *AccStep(M, j)* and *Acc(M, j).* Full verification is performed by means of programmes *Gr(fic)* and *TwoGr(fic),* where *(fic)* is a fictive variable.

3. Programme *PG(group)* gives all permutations of set group (the step 4).

4. Partial and total loss of the step 6 is calculated by programmes: *LP(rout)* – for one rout, and *LAll(ROUTS)* for two routs *ROUTS = (*$ROUTS^{<1>}$, $ROUTS^{<2>}$*).* Optimization procedures are performed by means of programmes; *Sopt1(group)* - for one set group, *Sopt(GROUPS)* - for two set group *GROUPS = (*$GROUPS^{<1>}$, $GROUPS^{<2>}$*).*

5. Programme *Opt(RN, GROUPS)* performs all procedure *RN* times and give the best result. RN is the number of successful creation (generation) of groups.

Listings for all programmes and printed results are presented in Attachment 1.

Numerical Example.

Firstly we describe an initial data. There are 41 tons of cargo should be distributed between clients in the following points of Latvia: 1. Daugavpils – 6100 kg, 2. Rezekne – 4700 kg, 3.Jelgava – 3200 kg, 4. Saldus – 5000 kg, 5. Jekabpils – 5700 kg, 6. Liepaja - 2400 kg, 7. Tukums – 5700 kg, 8. Madona – 5100 kg, 9. Ventspils – 4500 kg.

The geographical location of clients is shown in the Fig. 2.1.

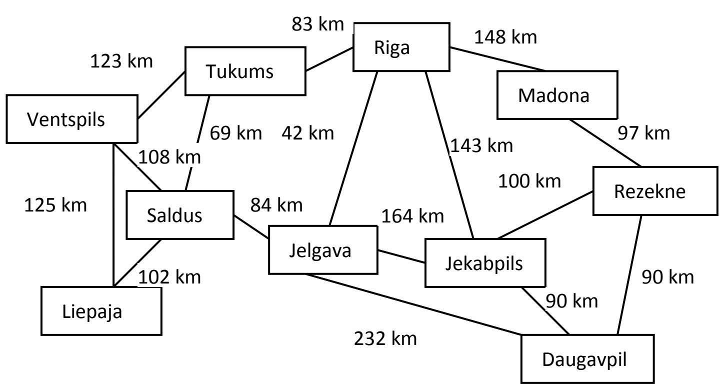

Fig.2.1. Points Location and Destination between Them

Corresponding *distance matrix D* is the following as mentioned in Table 2.1 (here Riga has a number 0 and *q* means infinity):

Table 2.1

$$D = \begin{pmatrix} 0 & q & q & 42 & q & 143 & q & 83 & 148 & q \\ q & 0 & 90 & 232 & q & 90 & q & q & q & q \\ q & 90 & 0 & q & q & 100 & q & q & 97 & q \\ 42 & 232 & q & 0 & 84 & 164 & q & q & q & q \\ q & q & q & 84 & 0 & q & 102 & 69 & q & 108 \\ 143 & 90 & 100 & 164 & q & 0 & q & q & q & q \\ q & q & q & q & 102 & q & 0 & q & q & 125 \\ 83 & q & q & q & 69 & q & q & 0 & q & 123 \\ 148 & q & 97 & q & q & q & q & q & 0 & q \\ q & q & q & q & 108 & q & 125 & 123 & q & 0 \end{pmatrix}$$

The Logistic centre has two trucks with the capacity of 22 *t* each. The task is to create 2 circular routes with the minimal summary extension for delivery of goods. In the same time each route should be connected to the truck in such way that the initial cargo volume does not exceed 22 *t*.

Now we consider a routing problem: how to organize circular deliver routes? Each rout must contain the specific town (Riga by number zero), number of other towns equals *n*. Further we use the above described algorithm.

First of all *m* number of points is chosen from the total quantity of *n* towns. So 4 points are determined from total amount of 8, besides the point number 0 (Riga) which obligatory takes place in each rout. The second group of routs is created by points which are not included into first group. In other words, the second group equals the total amount of points minus the first group. Finally, there are two groups which are tested and stand up to both restrictions volume capacity (2.4) and loss of kilometres (2.1).

The next step is using the programme *PG(group)* to get all possible permutations of created group. Using programmes *LP(rout)* for one rout and *LAll(ROUTS)* for two routs total loss in kilometres is calculated. As a result the number of km for each rout is following:

1st rout:

$$\mathrm{LP}\left(\begin{pmatrix} 0 \\ 8 \\ 2 \\ 1 \\ 5 \end{pmatrix}\right) = 568$$

2nd rout:

$$\mathrm{LP}\left(\begin{pmatrix} 0 \\ 3 \\ 4 \\ 6 \\ 9 \\ 7 \end{pmatrix}\right) = 559$$

Process of optimization provided by programmes *Sopt1 (group)* for one group, *(Sopt GROUPS)* for two groups and finally programme *Opt(RN, GROUPS)* which runs procedure necessary *RN* times including the testing for kilometres losses by both routes.

As the result of iteration of all permutations (described above in p. Optimization method) the best combination is determined.

This task was solved by the following way as presented bellow.

The optimal routes are shown in the Figure 2.2. From this could be seen that one truck has a circular route starting from Riga with delivery to 8. Madona, then 2. Rezekne, 1.Daugavpils, 5.Jekabpils and back to Riga. Another truck also starts from Riga with the first delivery to 3.Jelgava, then 4. Saldus, 6. Liepaja, 9. Ventspils, 7. Tukums and returning back to Riga. Total distance is equal to 1.127×10^{3}.

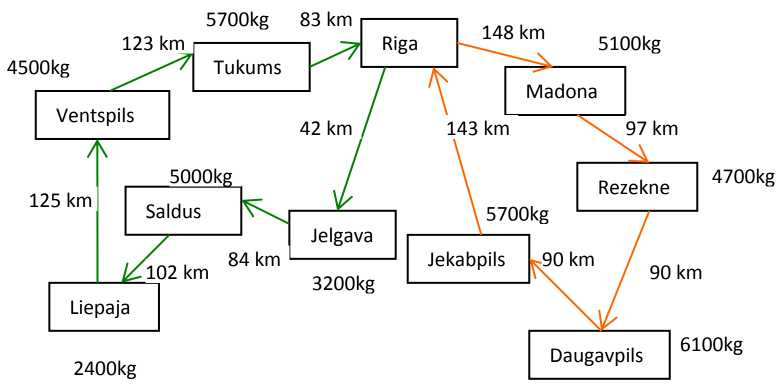

Fig. 2.2. The final optimal routes

As it is seen from Fig.2.2 both routes are formed based on the transport unit capacity. as it shown in Table 2.2.

Table 2.2

Grouping of Delivery Routes Based on the Transport Unit Capacity

Route 1		**Route 2**	
Delivery Point	**Delivery Volume, kg**	**Delivery Point**	**Delivery Volume, kg**
Tukums	5700	Madona	5100
Ventspils	4500	Rezekne	4700
Liepaja	2400	Jekabpils	5700
Saldus	5000	Daugavpils	6100
Jelgava	3200	-	-
Total	20800	Total	21600

Now we wish to show how the presented results have been got. We have three experiments with various initial content of client groups. These initial groups $G^{\langle 1\rangle}$ and $G^{\langle 2\rangle}$ are presented in two first columns of bellow tables. We call by *iteration* a sequential of steps 1 – 7 that improves previous result. The tables 2.3. – 2.5. contain results of the three experiments: gotten results for each iteration and gotten loss value (the last row). We see that suggested method works well.

Table 2.3

Experiment N1

0^{th} iteration		1^{th} iteration		2^{th} iteration		3^{th} iteration	
$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$
0	0	0	0	0	0	0	0
1	5	5	8	3	7	5	7
2	6	1	2	1	4	1	9
3	7	4	6	2	6	2	6
4	8	3	9	8	9	8	4
	9		7		5		3
8.032×10^5		2.094×10^4		1.113×10^4		1.127×10^3	

As it seen from the Table 2.3 each next of iterations improves the previous result.

Bellow Table 2.4 presents results of the quickest experiment for optimal decision. The needed result (1.127×10^3) was gotten on the third try of procedure repeating.

Table 2.4

Experiment N2

0^{th} iteration		1^{th} iteration		2^{th} iteration	
$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$
0	0	0	0	0	0
4	9	7	3	5	7
2	5	9	4	1	9
6	7	6	1	2	6
8	1	5	2	8	4
	3		8		3
1.804×10^5		2.094×10^4		1.127×10^3	

Gotten results during the 3rd experiment are shown in bellow Table 2.5. In all experiments the each next iteration results improve the results of previous iteration.

Table 2.5

Experiment N3

0^{th} iteration		1^{th} iteration		2^{th} iteration		3^{th} iteration	
$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$	$routs^{\langle 1\rangle}$	$routs^{\langle 2\rangle}$
0	0	0	0	0	0	0	0
6	1	8	3	9	3	8	7
2	7	6	4	6	5	2	9
5	4	9	2	4	1	1	6
8	3	7	1	7	2	5	4
	9		5		8		3
7.040×10^4		2.093×10^4		1.100×10^4		1.127×10^3	

Details of all experiments with various initial content of client groups are presented in Attachment 1.

Based on the experiments results we confirmed that offered method well suit for solving of delivery routes creation problems.

2.3. HIERARCHIC OPTIMIZATION OF THE RESOURCES ALLOCATION AMONG THE UNITS OF THE LOGISTIC CENTER

Problem Setting

Now turn to another task solving process. Here presented the model of hierarchic optimization of resources distribution between the different business units. One Latvian company is the producer and trader of some products. The significant part of the goods that is about 40% of total amount is imported from abroad. In the same time 60% of products has Latvian origin and is produced on the company's own production and with subcontractors. In order to minimize storage time and volume of produced goods and maximize speed and efficiency of goods deliveries to clients the supply chain was created by logistic service provider that was presented by company own Logistic Centre. One of ideas of Logistic chain is to design together the facility and delivery process to better reveal and support of customer purposes. Using the information technology and efficient flow of information between the members of the chain as well as understanding the business and needs of clients Logistic Centre provides just in time deliveries of raw materials and ready-made goods. Activity of the company would not be successful without a good work of Logistics Centre, which has the following hierarchic structure shown by the Fig.2.3.

The Logistic Centre consists of two departments: Logistic department and Production plant. The logistic department of the company rents the warehouse of 600 square meters. There are 7 employees work in logistics: 3 persons are in the office and 4 are workers on the warehouse. The operational costs of the logistic department is about 15 000 LVL per month. The tasks of the logistics department are to provide the delivery of raw materials for production units, to manage the warehouse of the ready made goods, to provide the delivery of ordered goods to clients in time and to control the quantity of goods on the store without it overloading. Own transport vehicles assist in distribution of goods and quick connection with production units.

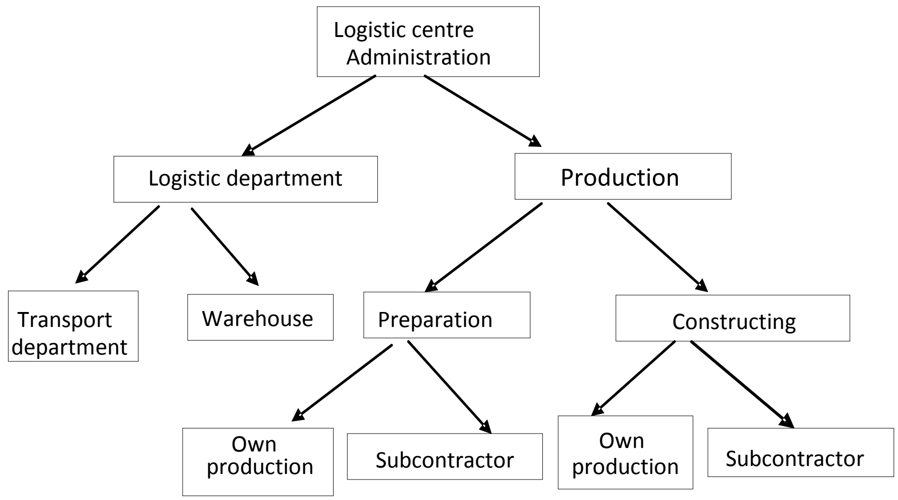

Fig. 2.3. The Hierarchic Structure of Logistic Centre.

There are 70 employees are working in production process. Company owes 1000 square meters building and has own processing equipment. The current costs for production is about 45 000 LVL.

The important task of the Logistics Centre is to combine together two processes, each with own specific and demands: the production process and the goods realization (sales) process. In our case the logistic department is the part of the both processes: raw materials supply for production and providing the delivery of goods to the final user for sales.

One of the tasks for logistic department is to make the constant monitoring of open (not delivered) sales orders and make analysis of goods production volume. Based on the results of warehouse monitoring the production order for needed goods quantity is created for the processing plant and its subcontractors and estimation time of arrival on stock for each product is calculated.

In conditions of economic depression the volume of client orders decreased, that causes the stock overloading of goods and production volume decrease. Management of the company has to decrease the increasing costs without destroying the working process in the company. To find the right solution of this problem management decides to make the profit and loss analysis for various scenarios of company development in order to distribute available financial resource between the departments of the Logistic Centre on the most efficient way.

Logistic Centre consists of separate units. We consider the task of optimal distribution of limited resource between these units. The company has hierarchic structure, that other words means that each unit (besides of terminals) has two other units under his control, so in the end we get the treelike structure presented in the Fig.2.4.

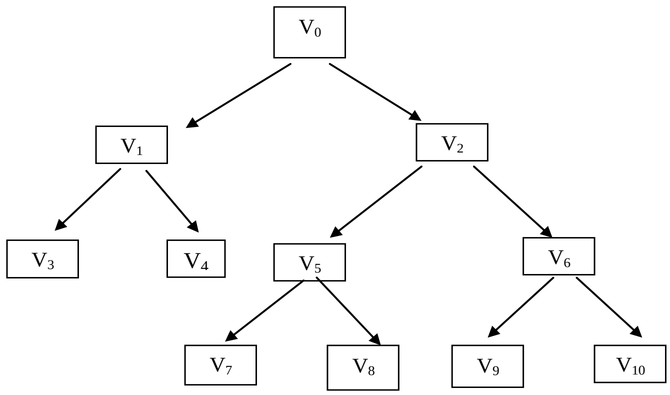

Fig.2.4. The Treelike Structure

This structure is shown by the relation matrix *T*:

$$T := \begin{pmatrix} 1 & 3 & 5 & -1 & -1 & 7 & 9 & -1 & -1 & -1 & -1 \\ 2 & 4 & 6 & -1 & -1 & 8 & 10 & -1 & -1 & -1 & -1 \end{pmatrix}$$

To make the matrix understandable for the Mathcad 14 program it has to be transposed into one shown by Table 2.5.

Table 2.5

The Relation Matrix T

	0	1
0	1	2
1	3	4
2	5	6
3	-1	-1
4	-1	-1
5	7	8
6	9	10
7	-1	-1
8	-1	-1
9	-1	-1
10	-1	-1

Rows of the matrix mean company's units, but the columns – number of the subordinate units. Column under number "0" shows number of unit, which is the left son, but column under number "1" presents the right son. Meaning of value -1 shows that there is no subordinate unit. So Table 2.5. corresponds to the treelike structure from Fig. 2.4. There are eleven numbers of concerned nodes, $n = 11$.

There are various variants of the development from *1* to n_i for each *i* unit. Each *j* ($j = 1, 2, \ldots, n_i$) variant assigns the following parameters: 1) concerned the *i-th* unit's profit $Mi_{j,0}$ from realization of the given variant *j*. It is significant, that profit depends on not the resources of the present unit only, but on the resources given to the subordinate units, too; 2) resource $Mi_{j,1}$ needed for the first subordinate unit (left son); 3) resource $Mi_{j,2}$ for the present unit; 4) resource $Mi_{j,3}$ needed for the second subordinate unit (right son).

Data for each *i* unit given by matrix *Mi* in the same way as it is shown for matrix *M0* bellow in the Table 2.6. Matrix rows correspond to offered variants for present unit. Each row has above mentioned parameters for present variant.

Table 2.6

0-NODE INDICES

$$M0 := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 6 & 19 & 3 & 48 \\ 2.5 & 9.1 & 0 & 37 \\ 4.3 & 26 & 2 & 36 \end{pmatrix}$$

The task is to distribute the resource between units in such way that the total profit would be maximal.

Method of Dynamic Programming and Its Application

This task is solving by method of *dynamic programming*, created by Richard Bellman [6], [18], [68]. The concept of dynamic programming means that the large problem is broken into incremental sub-problems, where each sub-problem can be solved and the solution is optimal. As a result, by using a formula the final solution can be generated without being required to alter the previously solved sub-problems or re-calculating some parts of the algorithm.

The dynamic programming supposes to make a decision step by step. In our case it means to take a decision for each separate unit consequently. Let's take the moment of time when the decision should be made for item *v,* if available resource is equal to *r*. This resource must be distributed among the item *v* (father) and all his offspring. Let we have *k* various solutions for unit *v*.

Then the Bellman function *F(v,r)* gives a maximal profit in this case. The value of *F(v,r)* means a maximum profit which could be got from unit *v* and all his offspring for the resource *r*.

These calculations should be used starting from the end units and come to the tree root.

Every time Bellman functions are calculated for the units which have sons (subordinate units) with already calculated Bellman function values. The end units have no sons (the condition $T_{v,0} + T_{v,1} = -2$ takes place for the item number v), and the whole resource is given to this unit. Therefore to apply the j–th project for such v item, it is necessary to satisfy condition (2.5)

$$Mv_{j,2} \leq r\ . \tag{2.5}$$

Among projects for the v-th item, the project j^* is chosen for which

$$j^* = arg\ \{\ j=1,...,n_i:\ max\ \{\ Mv_{j,0} : Mv_{j,2} \leq\ r\ \}\}\ . \tag{2.6}$$

For the rest units v=1,2,...,10 the condition

$$Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r \tag{2.7}$$

should be satisfied.

Now among all projects for the v-th item, the project j^* is chosen for which

$$j^* = arg\ \{\ j=1,...,n_i :\ max\ \{\ Mv_{j,0} + F\ (v0, Mv_{j,1}\ ) + F\ (v1, Mv_{j,3}) : Mv_{j,1} + Mv_{j,2} + Mv_{j,3} \leq r\}\} \tag{2.8}$$

where
$v0=T_{v,0}$ - is the number of the left son of v ;
$v1=T_{v,1}$ - is the right son of v.

As the results Bellman functions $F\ (v,\ r)$ will be calculated for all units and amounts of given resources r = 0, 1, ..., R. Above mentioned procedure is named as *inverse algorithm (bottom-up) of dynamic programming.*

Direct algorithm (top-down) gives the optimal quantities of assignment resources for each unit v. It is realised in opposite direction of the above mentioned – from the root to the end units, each time moving from farther to one of his sons.

Optimal solution for root (unit with number 0) corresponds to the project j^* that satisfies the equality

$$F(0,R) = M0_{j^*,0} + F(1,M0_{j^*,1}) + F(2,M0_{j^*,3}),$$

$$M0_{j^*,1} + M0_{j^*,2} + M0_{j^*,3}\ \leq\ R, \tag{2.9}$$

because for unit number 0 we have all resource: $r^*(0)=R$
In addition we fix the resource assigned for the sons

$$r(T_{0,0}) = M0_{j^*,0}\ , r\ (T_{0,1}) = M0_{j^*,2} \tag{2.10}$$

The next step is to go to one of the sons v, knowing the resources given to them. In case the unit v has resource r and he has the son $v0 = T_{v,0}$ (left son) and the son $v1 = T_{v,1}$ (right son), then the optimal solution r^* for unit v will be found as the project j^* that satisfies the equality

$$F(v,r) = Mv_{j*,0} + F(v0,Mv_{j*,1}) + F(v1,Mv_{j*,3}),$$

$$Mv_{j*,1} + Mv_{j*,2} + Mv_{j*,3} \le R . \quad (2.11)$$

The resources assigned for the sons are following:

$$r\,(T_{v,0}) = Mv_{j*,0}\,,\ r\,(T_{v,1}) = Mv_{j*,2} \quad (2.12)$$

Direct algorithm is finished when all the points (units) are calculated till that ones who have no sons.

Computer Realization

There are following information about the computer programs using for realization of above mentioned Bellman algorithm.

Programming language we use is the wide known mathematical package Mathcad 14.

We create the united matrix *M* for using it in Mathcad programme:

$$M^T = [\ M0^T\ M1^T\ \dots\ M10^T\ ]^T \quad (2.13)$$

Now we have the problem to find information concerning the concrete unit. To solve it we create the special Mathcad-program *Locman*. Then for an integer *i* Locman gives a number of matrix *M* row, that is the first row with information about the matrix *Mi*.

The programme *Father(v)* gives a vector with 2 components, one of which is -1. The second component is an integer one shows the number of unit, which is the father for unit *v*. The component location shows which son is the present unit: if it is on the top position then given unit is the left son, but in case it is located bellow given unit is the right son.

Let us consider an example:

$$Father(9) = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$$

Here the father of the unit number 9 is the unit number 6 and the unit number 9 is the left son of the unit number 6.

$$Father(10) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

The father of the unit number 10 is the unit number 6 and the unit number 10 is the right son of the unit number 6.

The inverse algorithm of dynamic programming is realised by program *F(v,r),* which gives Bellman function value for unit *v*, which has got the certain resource *r*. As the result we get the value of maximum profit and the number of the optimal solution from matrix *M*.

The direct algorithm is realized by program *GlobD*. In other words *GlobD* gives the matrix of optimal decisions for all units with resource equal *R*. Matrix rows related to the units and contain the optimal value for this unit. Here, r_i is the given for *i* unit, d_i – optimal decision for *i* unit.

Numerical Results

For our example we have the following numeric data mentioned in Table 2.7.

Table 2.7

Nodes Indices

$$M1 := \begin{pmatrix} 0 & 11 & 3 & 5 \\ 0.2 & 3.5 & 2 & 4 \\ 1.6 & 16 & 4 & 6 \end{pmatrix} \quad M2 := \begin{pmatrix} 6 & 14 & 2 & 32 \\ 2.3 & 6 & 0 & 31 \\ 2.7 & 9 & 2 & 24 \end{pmatrix} \quad M3 := \begin{pmatrix} 4.6 & 0 & 11 & 0 \\ 4 & 0 & 4 & 0 \\ 7.3 & 0 & 16 & 0 \\ 40 & 0 & 71 & 0 \end{pmatrix}$$

$$M4 := \begin{pmatrix} -4.6 & 0 & 5 & 0 \\ -3.8 & 0 & 4 & 0 \\ -5.7 & 0 & 0.57 & 0 \\ 8 & 0 & 90 & 0 \end{pmatrix} \quad M5 := \begin{pmatrix} 3 & 6.3 & 0.5 & 7.2 \\ -1 & 3.8 & 1 & 1.28 \\ 1.9 & 4.3 & 1.5 & 3.6 \end{pmatrix} \quad M6 := \begin{pmatrix} 3 & 18.2 & 1.8 & 12.3 \\ 3.3 & 19.3 & 2.5 & 9.6 \\ 0.8 & 17.6 & 0.56 & 6 \end{pmatrix}$$

$$M9 := \begin{pmatrix} 1.74 & 0 & 18.25 & 0 \\ 2.32 & 0 & 19.3 & 0 \\ 0.8 & 0 & 17.6 & 0 \end{pmatrix} \quad M10 := \begin{pmatrix} 1.23 & 0 & 12.3 & 0 \\ 0.98 & 0 & 9.57 & 0 \\ 0 & 0 & 5.6 & 0 \end{pmatrix}$$

The hierarchic structure was presented before in Table 2.5. The information about nodes indices is presented by Matrix *M0* (Table 2.6) for 0-unit (root) and matrixes *M1-M10* for other units from the Table 2.7.

Let us put the global available resource as *R:*=64. The created program gives the following results:

$$F(0,R) = 15.9.$$

One corresponds to the third solution for the item 0 (for the root)*:* $j^* = 3$.

Gotten results for each unit are presented by matrix *OptD* (see Table 2.8).

Table 2.8

The Optimal Solutions for Resource of 64 units (Matrix OptD)

V	0	1	2	3	4	5	6	7	8	9	10
*r**	64	26	36	16	6	9	24	0	0	0	0
F(*v*,r)	15.9	8.9	2.7	7.3	0	0	0	0	0	0	0
*j**	3	3	3	3	0	0	0	0	0	0	0

The matrix columns correspond to the item. The matrix has four rows. The zero row contains the number *(v)* of the items. The first row contains the value of the optimal resource *(r*)* for the corresponding item, the second row contains the value of the Bellman function *(F),* the third row contains the number of optimal solution *(j*)* from matrix *M.*

Analysing the results of calculations presented by above Table 2.8 we conclude that available resource of 64 thousand lats could bring to the company maximum profit of 15,9 thousand lats in case the resources will be distributed in following order (optimal solution number 3 will be taken). So the majority of resources 36 thousand lats should be given to the 2^{nd} unit Production and the smaller part of 26 thousand – to Logistic department (the 1^{st} unit), where Transport (unit 3) demands the most of support – 16 thousand lats. Own processing (unit 6) as always takes the most efforts and recourses; in this case it should be 24 thousand lats. Although Production is the usual cost centre, it will give profit of 2,7 thousand lats. But the biggest part of profit 8,9 thousand lats would come from Logistic department as this department collect money from clients and provide the sales for the whole company.

The problem of resources distribution in optimal way among the units of the Logistic centre was observed. The Logistic centre has the treelike structure. There are some various ways of development for each unit of the Logistic centre. The ways differ from each other by necessary resource and profit receiving. The task is solving by using the method of *dynamic programming*, created by Richard Bellman [6]. During the work using the Mathcad 14 package the special program, which helps to make the calculations, was created.

Using the dynamic programming method the real formulated task for distribution of necessary resource of 64 thousands lat among the units of the Logistic centre in Latvia is solved and the solution for getting of optimal profit were found for each unit.

2.4. OPTIMIZATION OF THE SUPPLY CHAIN FOR THE LOGISTIC CENTER

Considered Supply Chain

In order to optimize the supply chain process through the Logistics Centre the management of the textile company makes a decision to create the new supply channel for materials and sales of ready-made goods.

It is necessary to make certain decisions regarding relating parts of supply chain process such as purchase and delivery of raw materials, production and sales of ready products. Therefore the model with different possible scenarios of development has been created.

The first stage is choosing the producer of basic material (clothing). There are offers from three main production companies. The first of them offers the high technological and specialised materials of the best quality according to existing market prices. The second one is ready to supply wide use material of good quality with discount of 10% from market price. The third producer makes middle class materials of lower quality according to confirmed standards and gives discount of 25% from market price. It is necessary to foresee three scenarios of development for each producer that are 1) materials will be produced in time; 2) materials will not be produced by any reason; 3) materials will be produced with delays. Two months are planned to spend for material production.

The second stage is delivery of materials to the final goods production place. Choosing the transport way it is important to pay attention to delivery time, safety and transport rates. Transportation could be provided by shipping line using combined cargos sea container, by air, by road and using the express delivery by courier mail. If materials are produced in time, the low cost transportation ways are preferable, for example, delivery by sea as the cheapest but certain time demanded. Aircraft delivery usually is chosen for quick deliveries. Delivery by road is effective for rather short destinations. In case of time shortage priority of delivery belongs to express service of courier mail as the quickest possible, however the most expensive as a rule.

After delivery cargo could appear in three conditions:

1) materials are delivered till final destination and come to production process;

2) cargo is delivered till transit terminal (cargo warehouse, sea port airport) and further delivery till production place is necessary. In same time perhaps part of materials could be stocked temporarily in terminal by any reasons;

3) there is probability of cargo damages and shortage.

The third stage is sales of the ready-made goods. There are three current channel of goods realization:

1) through company's own shops net;

2) through the wholesalers;

3) through the foreign distributors.

Each products realization scenario has one of the three uncertain results:

1) goods have high demand that create successful sales and high profit,

2) goods take middle market position and taking into account the costs for goods creation they are not profitable but sales cover the losses,

3) in spite of all efforts, products are not interested the customers, sales figures are very low, and company has losses.

As the criterions for effectiveness of making decisions let's look at

1) maximum probability of the best effect achievement

2) average profit maximization.

In the best way, with high demand and successful sales the goods collection should be realized for the minimum time.

Consequently, the task consists in the taking decision regarding the basic materials suppliers, transport way selection and sales channel. In other words it is necessary to choose the way from selecting the raw materials till final products creation that allows getting the maximum profits to the company.

Mathematic Model and Method for the Decision Taking Process of Supply Chain Creation

Concerned decision-making process could be presented as a net structure *T*, as it shown on the Fig.2.5. "Decision-making net structure" images the immediate and future decisions regarding materials supply and goods realisation channel. The net includes arcs and vertices of two kinds.

Circle points describe state of the system after decision taking moment. All circle points have the ordinal numbers from 0 to 10. Then state without entering arcs is named source point and corresponds to the initial moment of decision-making process. Further vertices 1, 2, 3 describe the condition related to the suppliers. For instance, state 1 – the most successful status, to be exact materials produced in time.

At least one or more arcs enter all states except of number 0. Arcs correspond to transitions from one state to others. Terminals are the states without running out arcs. They correspond to the final moments of decision- making. In Table 2.9 is described sense of all 10 states.

Diamond vertex corresponds to making decision. Diamonds 0, 1, 2 correspond to producer's choosing. For example, the vertex 0 is the 1st above described producer. The equivalence between vertex number and above mentioned decision is presented in Table 2.10.

Diamond entering arc shows the concrete making decision. Running out arc shows the system possible state after this decision-making.

Table 2.9

Status Description of Possible Results

State number	Condition
0	Initial moment of decision-making process
1	Materials are produced in time
2	Materials are not produced by any reason
3	Materials are produced but with delay
4	Materials are delivered to destination point and given to further production
5	Materials are delivered to transit terminal (cargo warehouse, sea port, airport) and demand further delivery till production place
6	Destroying or missing of materials
7	Part of materials is put on stock for future production
8	Goods have high demand that create successful sales and high profit
9	Goods take middle market position and taking into account the costs for goods creation they are not profitable but sales cover the losses
10	In spite of all efforts, products are not interested the customers, sales figures are very low, and company has losses

Making of concrete decision does not mean getting of single result. On the vertex running out arcs probabilities of possible further state are mentioned. In total probabilities sum is equal to 1. For instance, as shows bellow Fig. 2.5., after choosing the producer of basic materials the system could be found in state 1 with probability 0,1 if the first producer is chosen. In case the second producer is chosen it will be there with probability 0,5. The system could be in the same 1 state with probability 0,8 if the third producer is chosen. State 2 is the final as the activities come to unsuccessful result and have no further development. So there are two states with future development left.

The next row of diamonds (2nd stage) is choosing of the transport company and delivery way from basic material production till the place of further processing. There diamonds numbers show the choice of the early mentioned transporters.

The last third row of diamonds (3rd stage) present the choice of ready-made goods realisation way.

Table 2.10

Vertexes Allocation of Decision-making Net Structure (Problem Setting)

State number / ***Development stages***	**0**	**1**	**2**	**3**
1-st stage: choosing of producer	Producer of the high technological and specialised materials of the best quality according to existing market prices	Producer of wide use material of good quality with discount of 10% from market price	Producer of middle class materials of lower quality according to confirmed standards and gives discount of 25% from market price	-
2-nd stage: choosing of transport	Delivery by sea as the cheapest but time-consuming	Air delivery as more expensive but rather quick	Express service of courier mail as the quickest possible, however the most expensive	Delivery by road is effective for rather short destinations
3-rd stage: choosing of realisation way	Through own shops net	Through the wholesalers	Through the foreign distributors	-

Let us describe the mathematic view (conception) of initial data.

States are known for each position following for the previous ones. For example, they are presented by matrix T, shown on the Table 2.11. Rows of the matrix correspond to current states, columns corresponds to different decisions. Matrix elements show numbers of the future states. Here the symbol –1 means the absence of future state.

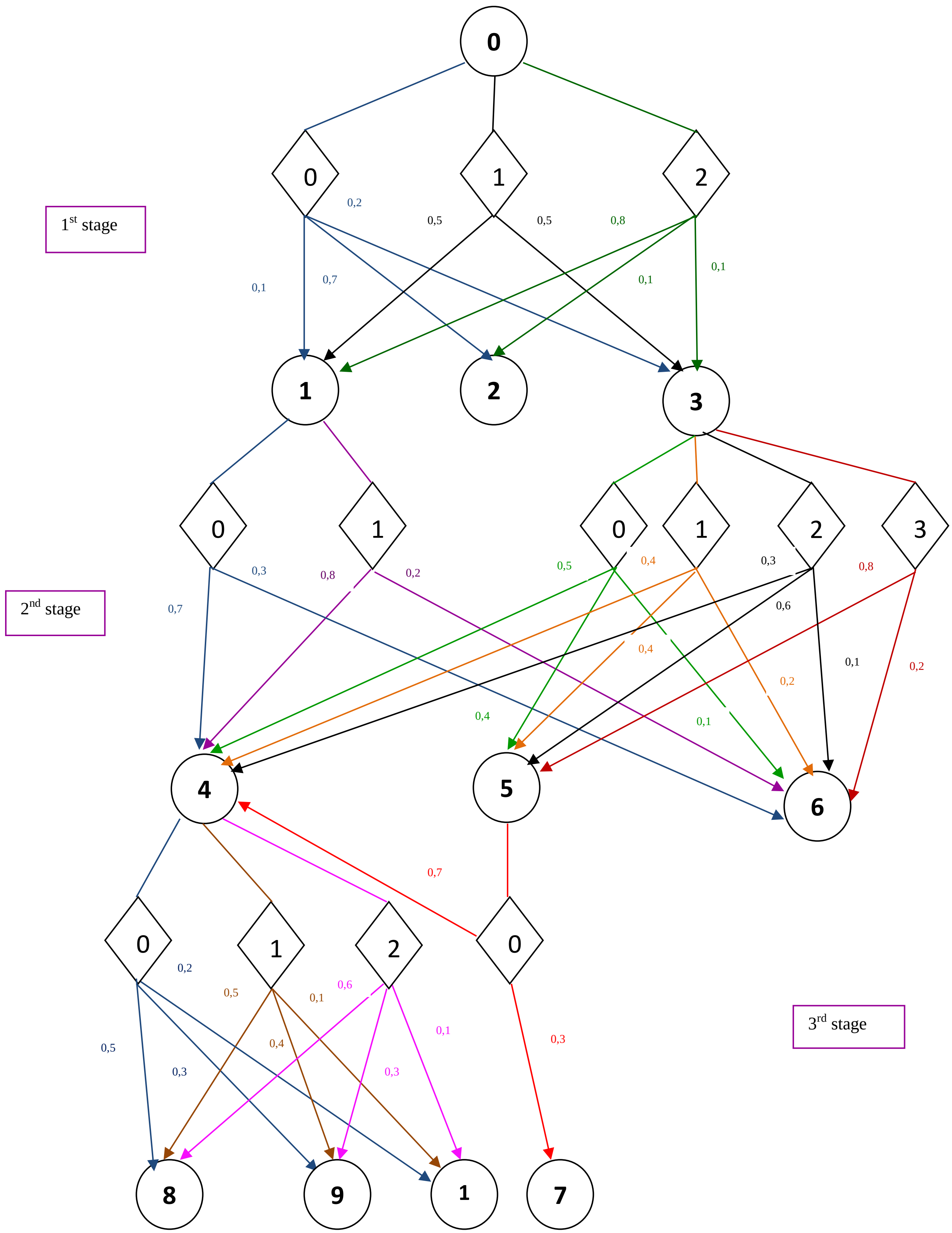

Figure 2.5. Decision – making Net Structure Controlled Markov Chain

Table 2.11

Matrix *T*

$T^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	1	4	-1	4	8	4	-1	-1	-1	-1	-1
1	2	6	-1	5	9	7	-1	-1	-1	-1	-1
2	3	-1	-1	6	10	-1	-1	-1	-1	-1	-1

One single decision should be made from several ones in each state except of terminal. The probability of transition to the next state is changing depending on taken decision. Let $Prj_{i,k}$ be a probability of a transition from state *i* to state *k* if decision *j* has been chosen. A corresponding matrix is named as *Prj*. For instance matrix *Pr0* in bellow table 2.12. Rows of the matrix *Prj* correspond to different vertexes, columns corresponds to different states. Matrix *Prj* elements show corresponding probabilities. For that numbers of states are determined by matrix *T*.

Table 2.12

Matrix Pr0

$$\mathrm{Pr0} := \begin{pmatrix} 0.1 & 0.7 & 0.2 \\ 0.5 & 0 & 0.5 \\ 0.8 & 0.1 & 0.1 \end{pmatrix}$$

We have expected revenue amount for achieving of all states. They are presented by vector *c*. Table 2.13. contents corresponding example.

Table 2.13

Expected Revenues for Each State *c*.

State, j	**Profit, cj**	**Probability Coefficients, ĉj**
0	0	0
1	-10000	0
2	-500	0
3	-12000	0
4	-5000	0
5	-500	0
6	-5000	0
7	-2000	0
8	30000	0
9	40000	0
10	60000	1

There are several criteria for effectiveness of making decisions could be offered:

1) maximum probability of the best effect achievement
2) average profit maximization.

Reward of different states sum up together. Total value is a random variable as result of random transition. The task is to choose the decision for each state in such way that the average amount of total profits becomes maximum.

Method of Dynamic Programming will be used for this aim.

Dynamic programming supposes the decision-making step by step. In our case it means decision-making for each state individually. Look at the moment of time when it is need to make a decision for state *j*. Here it is important to mention that if state *j* is not terminal, till that moment should be checked states with bigger then *j* numbers.

Let's enter Bellman function $F(j)$, this is the maximum average profit, which could be get starting from state *j* till the end moment of decision-making process. To calculate these functions we have the following Bellman equations:

$$F(j) = \underbrace{\max}_{k \in D(j)} \left\{ c_j + \sum_{i \in \mathrm{Sjk}} \Pr j_{i,k} F(i) \right\}, \qquad (2.14)$$

where

Sjk –set of state numbers, following the state *j* if decision *k* is taken

D(j) – a set of possible decisions in state *j*.

These equations should be used starting from the terminal states and going to the root (initial state). Terminal states are final, so the first item leaves in brackets in the formula (2.14). In the same time decision *k** is fixed for each state as the optimal one. For this decision the value in brackets coincides with *F(j)* in formula (2.14). So this procedure is named as *the inverse running* of dynamic programming.

Direct running gives the quantities (order) of optimal decisions for all states. It is realised in opposite direction of the above mentioned inverse running – from the root to the end units, each time moving from state *j* to one of the following states that corresponds to optimal decision *k** in the state *j*. Direct algorithm is finished when all the states are calculated till that ones who have no future states.

Computer Realization

The described algorithm is realized by program *OptValue*. This programme gives the matrix that has two columns: the first one corresponds to maximum profits *F(j)*, the second one corresponds to optimal decisions *k** for each position *j*. This program was created by using the mathematical package MathCAD 14

Primary data for program is the following:

- Matrix *T*, describing the examined net. Rows of the matrix correspond to net states, rows elements show numbers of the further states. Value –1 means the absence of the next states.
- Vector *c* describes profit, that comes for achieving each state.
- Matrix *Prj* of transit probabilities for state *j*. Rows of the matrix correspond to different decisions *k*, but columns correspond to the next state (with respect to the matrix *T)*.

main program *OptValue* uses the auxiliary program *Pr(j)* that gives the matrix *Prj* according to number *j*.

Numerical Results and Analysis

For our example we have the following numeric data mentioned in Tables 2.12, 2.13, 2.14.

Table 2.14

Matrixes of the probabilities.

$$\mathrm{Pr0} := \begin{pmatrix} 0.1 & 0.7 & 0.2 \\ 0.5 & 0 & 0.5 \\ 0.8 & 0.1 & 0.1 \end{pmatrix} \qquad \mathrm{Pr1} := \begin{pmatrix} 0.7 & 0.3 \\ 0.8 & 0.2 \end{pmatrix} \qquad \mathrm{Pr2} := (1)$$

$$\mathrm{Pr3} := \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.4 & 0.4 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0.8 & 0.2 & 0 \end{pmatrix} \qquad \mathrm{Pr4} := \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.3 & 0.1 \end{pmatrix} \qquad \mathrm{Pr5} := (0.7 \;\; 0.3)$$

As the criteria we choose optimisation of achieving the state 10, as the profit vector $\hat{c} = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1)^T$.

The next step is using the program *OptValue*, to make the calculation for optimal decisions and maximum average profit, done by known rules for Markov chains. Listing for the program *OptValue* shows Table 2.15 as follows:

```
OptValue:= | n ← rows(T)
           | j ← n
           | while j > 0
           |   j ← j − 1
           |   k ← 0
           |   for i ∈ 0.. K − 1
           |     k ← k + if(T[j,i] > −1, 1, 0)
           |   Opt[j,0] ← c[j]  if k = 0
           |   otherwise
           |     PrT ← Pr(j)
           |     m ← rows(PrT)
           |     Opt[j,0] ← −1000
           |     Opt[j,1] ← −1
           |     for i ∈ 0.. m − 1
           |       R[i] ← c[j]
           |       for v ∈ 0.. k − 1
           |         S ← T[j,v]
           |         R[i] ← R[i] + PrT[i,v]·Opt[S,0]
           |       if R[i] > Opt[j,0]
           |         Opt[j,0] ← R[i]
           |         Opt[j,1] ← i
           | Opt
```

Table 2.15

k -real number of suns

m – real number of decisions

OptValue =

	0	1
0	0.16	1
1	0.16	1
2	0	0
3	0.16	3
4	0.2	0
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	1	0

Table 2.16 presents the results of calculations.

Table 2.16

Maximum Probability of The Best Effect Achievement

State number	0	1	2	3	4	5	6	7	8	9	10
Profit	0.16	0.16	0	0.16	0.2	0	0	0	0	0	1
Making decision	1	1	0	3	0	0	0	0	0	0	0

Using the data from Table 2.13. we calculate the average profit maximization in terms of money. Here the profit vector is

$c = (0\ -10000\ \ -500\ \ -12000\ \ -5000\ \ -500\ \ -5000\ \ -2000\ \ 30000\ \ 40000\ \ 60000)^T$

Table 2.17

Maximum Average Profit for Vector c

State number	0	1	2	3	4	5	6	7	8	9	10
Profit in money term	15600	16200	-500	15000	34000	-2500	-5000	-2000	30000	40000	60000
Making decision	1	1	0	3	2	0	0	0	0	0	0

The task of the decision-making process for Logistics centre Supply Chain optimisation through the creation of the new supply and sales channel in optimal way was observed. There are some various decisions could be taken at each stage of process development. The ways differ from each other by necessary resource and profit receiving. Two aspects, such as maximum probability of the best effect achievement and average profit maximization are taking as the criterions for effectiveness of making decisions.

The task is solving by using the method of *dynamic programming*, created by Richard Bellman. During the work using the MathCAD 14 package the special program which helps to make the calculations is created.

Using the dynamic programming method the real formulated task of decision-making process for the new supply and sales channel development for the Logistics centre in Latvia is solved and the solution for getting of optimal profit is found.

3. RESULTS AND CONCLUSIONS

1. The promotional work "Optimization Methods and Models Creation for Logistic Company Successful Development" is devoted to development and application of the new optimization models for the work of a logistics company in the market conditions of World recession. Because of the lack of practical methods for optimization of the costs and activities for the logistics centres in Latvia the presented work is actual and has good perspectives for the further progress.
2. The number of important tasks concerning to the carried out research is studied: World recession influence on the Latvian transport system and development perspectives, logistics system sentence and problems in Latvia, typical strategic tasks for the Logistic centre successful development.
3. Using the methods of mathematical programming three new models for work optimization of logistic companies were created. The first is a new model for optimal planning of circular delivery routes. In difference with the classical problem of Hamilton's cycle our problem is more complex because additionally it takes into account many cycles, weight of goods and weight restriction for each cycle.
4. The next new model of hierarchic optimization of the resources distribution among the units of the Logistic centre was presented. The problem of resources distribution in optimal way with maximal profit receiving among the units of the Logistic centre which has the treelike structure was solved with help of this model.
5. The third one - Logistic centre Supply Chain optimization model allows solve the task of the decision-making process for Logistics centre Supply Chain optimization through the creation of the new supply and sales channel in optimal way. Two aspects, such as maximum probability of the best effect achievement and average profit maximization are taking as the criterions for effectiveness of making decisions.
6. On the basis of the obtained results a number of guest lectures on the subject "Transport aspects in logistics" and "Practical ERP using in business" for the second year students of bachelors' studies programme of the Riga Technical University Mechanical Engineering faculty has been prepared.
7. It is important that all the results of my research were used in practical area that improves the work of several commercial companies and allow them to achieve better business results. During research time all used business statistic data were given from the real Latvian companies. Using the real data gives an opportunity to check and analyse the gotten results in practice. There are three companies: SIA "66North Ice-Balt", SIA "66North Delivery and Logistics" and SIA "EURECO" took part in research. After the new models based on dynamic programming method were implemented into the work of logistic companies the financial indices significantly increased.
8. The main results of this research are published in 5 articles and presented at 11 scientific conferences, at which the author presented 6 reports on the subject of the promotion work. The list of articles and reports at conferences are given bellow.

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