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**THE AIRCRAFT VEHICLE'S FATIGUE
CRACKS EARLY DETECTION AND
INSPECTION METHODS OUTLOOK
DURING BENCH TESTS**

Doctoral Thesis

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ANOTĀCIJA

Promocijas darba satur:

- anotāciju (latviesu un angļu valodā), ievadu, pieci nodaļas, sledzienu, 138 lapas, 55 attēlas, 45 tabulas, 9 pielikumus, 82 literatūras atsauces.

ATSLĒGU VĀRDI: *nogurumizturības raksturojumi, lokāla zonu hipotēze, noguruma plaisas, noguruma atkarība bojājumi iekrāšanu, stohastiskas saitas, noguruma atteices kritēriju, matemātiskas modeļus, drošuma konstrukcijas analīzi, tehniska nosacījuma un droša postījuma principi.*

Promocijas darbā veikti pētījumi par bojājumu (plaisas) uzkrāšanos uz paraugiem ar ievērojamu koncentratoru skaitu. Pieņemot, ka noguruma bojājumu uzkrāšanās noris stohastiski atkarīgi, tiek piedāvāta lokālo zonu hipotēze (LZH). Lielizmēra konstrukcijās bojājumu koncentrācijas vietu var uzskatīt kā atsevišķu lokālo zonu apvienojumu, kas nosaka visas konstrukcijas ilgizturību (ilgmūžību).

Uz divpakāpju eksponenciālās sadalījuma funkcijas bāzes ar logaritmisko ilgizturības aprēķinu (DEDF) izstrādāts četru parametru matemātiska modelis noguruma pretestībai. Pieņemtais modelis apvieno noguruma ilgizturības sadalījuma funkcijas aprakstu, noguruma līknes, izturības robežas, ciklu jūtības sliekšni. Uzskata, ka noguruma līknes fragmentāri-lineārais attēlojums ar dažādiem pakāpes rādītājiem saistīts ar fizisku mehānismu izmaiņām noguruma bojājumu uzkrājumam materiālā pēc sloņojuma līmeņa izmaiņām. Pie šādiem uzskatiem dots piemērs resursa (ilgizturības) aprēķinam helikoptera Mi-26 ķīļa sijai.

Lokālo zonu Hipotēzes esamības pierādījums dot piemērā liela izmēra konstrukciju stendu izmēģinājumiem. Parādīts, ka labākās nasagraujošās kontroles metodes, veicot tehniskā stavokļa monitoringu lokālajām zonām, ir akustiskās emisijas un termovīzijas metodes. Kontrolēta kritiskās lokālās zonas sagraušana, kura, šajā gadījumā, ir indikators noguruma bojājumu uzkrāšanai visai konstrukcijai, ļaus izstrādāt konstrukciju ekspluatācijas metodes pēc principa-droša tās sagraušana (Fail Safe Concept).

"Ja es būšu mazrunīgs – mani nesapratīs"

Horace (Quintus Horace Flak).

ANNOUNCEMENT

The Doctoral Theses consists of:

- announcement (Latvian and English), introduction, 5 chapters, conclusion, 138 pages, 55 figures, 45 tables, 9 attachments, 82 references to authorities.

KEY WORDS: *fatigue strength characteristics, local zone hypothesis, fatigue cracks, dependence of fatigue damages' accumulation, stochastic relationships, fatigue test, mathematical models, reliability design analysis, principles of technical condition and safe destruction.*

In Doctoral Thesis the researches of dependence of fatigue damages' (cracks) accumulation are carried out on samples with numerous concentrators. The supposal that the fatigue damages' accumulation happen dependent, the *Hypothesis of Local Zones* (HLZ) is proposed. On the full (large)-size Designs it allows to consider an area of damages' concentration as a pool of some Local Zone, which defines durability of total Design.

On the basis of *double exponential probability distribution of fatigue durability with logarithm of loading cycles* (DEDF), the four-parametrical mathematical model of Local Zone's fatigue strength is developed. The model unites parameters of fatigue durability's *distribution function* (CDF), CDF of endurance limits, S-N fatigue curve and a sensitivity threshold on cycles. It is supposed that piecewise and linear description of S-N fatigue curve with various degree "m" is connected with various mechanisms of *fatigue damages' accumulation* (FDA) in a material (design) when loading level is changing. With this provision, the example of a *Safe Life Time* (SLT) evaluation is given for a keel beam of the Mi-26T helicopter.

The existence of HLZ postulates is shown on examples of the Bench Tests and Operation Work of full-size Designs. It is shown that the best method of *nondestructive test* (NDT) for monitoring of Local Zones' technical condition is the method of *acoustic emission* (AE) and thermo vision.

Advancing controlled destruction of the critical Local Zone, presented as indicator of FDA on total Design, will allow developing operation methods by the principle of safe destruction of a Design (Fail Safe Concept).

“

“If I am speaking short - that I will be unclear”

Horace (Quintus Horace Flak).

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*“Travel to one thousand lee begins
with the one first step.”*

The Chinese Book of Dao

INTRODUCTION

The Relevance of Research

The increase in the sizes of technical Designs, particular in Civil Aviation, results with need to investigate properties and nature of accumulation of fatigue damages (FDA) and promotion of a number of the hypotheses based on phenomenological bases of Fatigue Strength. The impossibility of natural large-size Designs' Bench Tests at operational loadings, and also the lack of possibility of their repetition (repeated reproduction of results of tests) demands to develop the methodology of the management of such Tests for obtaining the maximum information on tested Design. It leads to gradual changeover from the principle of operation on a Design technical condition to the principle of Operation on safe Design destruction (Fail Safe Concept). The Fail Safe Concept allows to begin operation of aviation Designs at an early stage of their production.

However such approach demands introduction and adaptation to specific conditions of Service Operation those NDT methods which are capable to catch fatigue damages (cracks) and to track their propagation "on-line" mode. In the proposed Thesis an attempt of Research of FDA nature for justification of methodology of carrying out fatigue tests of full-size Designs and their monitoring by NDT methods is made.

The Goal of Research

Research objective is to take a look more closely at nature of emergence and interaction of fatigue damages of large-size Designs at Bench Tests for justification of the principles of early failure detection at an initial stage of Tests and to develop the methodology of their monitoring on the basis of the offered Hypothesis of Local Zones (HLZ) with dependent accumulation of fatigue damages.

The Objectives of Research

To achieve this goal, it is necessary to solve the following tasks:

- a) to develop mathematical (phenomenological) models
 - of accumulation of fatigue damages in some local area;
 - of the size of a local area;
 - of reliability of the local area including definition of criterion of its destruction (refusal), characteristics of Fatigue Strength (probability distribution function of durability, DF of limits of endurance, a mode of S-N fatigue curve, a sensitivity threshold on cycles);
- b) to check the made hypothesis (HLZ) and mathematical (phenomenological) models on experimental data, for this purpose:
 - to make experiment on samples with several concentrators, as *on model analogue of FDA in a uniform body material or in a Design*;
 - to process the experimental data obtained by other authors;
 - to use the data obtained at tests of real aviation Designs;
- c) to identify methods of NDT that most adapted for use in the large-size Designs' tests;
 - to develop a methodology (sequence of actions (algorithm) of selected methods of NDT) for early detection of initial damages;
 - to put forward some recommendations on the application of the results obtained in Bench Tests of large-size aircraft Designs.

Research Methods

The following methods were applied to the solution of the objectives:

- mathematical (phenomenological), including probabilistic, modeling of processes of emergence of damages on large-size Designs;
- the statistical analysis of results of specimen's tests, including correlation analyses, the analysis with application of the Order Statistics theory; nonparametric statistics;
- the analyses of test results of real large-size Designs.

The Novelty of the Results

The novelty of this Research is the approach to the study of the FDA on the hole concentrators systems on specimens as the counterpart model-analogue of engineering Designs or solid body that:

- confirm the main parcels of a Hypothesis of Local Zones (HZZ) with stochastic FDA in them;

- confirm the main preconditions of the mathematical model of reliability of the Local Zone uniting all parameters of Fatigue Strength characteristics (DF of fatigue durability, a DF of endurance limit, a S-N fatigue curve, a sensitivity threshold on cycles);
- develop methodology of carrying out Bench Tests of full-size Designs with application of NDT methods.

The Practical Relevance of Research

Basic provisions of the proposed Research were applied during Bench Test in SIA RZEC „Aviatest LNK” (Riga, Latvia) at:

- the analysis of Safe Life Time of the stabilizer of the Mi-8 helicopter;
- the estimation of reliability of *Structural Significant Elements* (SSE) of the main landing gear of the Tu-154 plane;
- an assessment of Limited Safe Life Time and cracking of tail and keel beams of the Mi-26T helicopter;
- the developing of fatigue tests programs of helicopter SSE and a choice of a method of NDT to monitoring of fatigue tests of SSE of the Ka-62 helicopter.

It is shown that in the presence of several areas of the accelerated FDA (for example, at tests of a tail and keel beam of the Mi-26 helicopter) it is desirable to supervise consecutive destruction of these areas by other methods (a tenzoreview, acoustic emission, eddy-current, ultrasonic etc). At achievement of the critical level of cracking it is necessary to repair total surface of damage area completely.

The methodology of the monitoring management of a technical condition of large-size Designs are developed.

The Thesis submitted for Discussion

The following theses are submitted for discussion:

- the Hypothesis of Local Zones (HZL) with dependent FDA, as with positive, as negative stochastic relationships;
- mathematical models of FDA on large-sizes Designs and determination of the sizes of Local Zones;
- probabilistic models of characteristics of reliability (durability) of Local Zones as weak places of total Design: double exponential distribution function (DEDF) of a logarithm of number of cycles until failure and the piecewise and linear description of S-N fatigue curve in double logarithmic coordinates;
- the methodology of large-sizes Designs test and NDT monitoring.

*“No problem will seem
impracticable if to break
it into small tasks”.*

Henry Ford,
American industrialist

CHAPTER 1 QUESTION STUDING. PROBLEM STATUS DEFINITION. VIEW TO FUTURE

1.1 Description of the Problem

The status of the Aviation technology is characterized by growing increase in size, complexity and cost of technical systems. In particular, considering the aircraft designs of Civil Aviation, growing trend is visible of the size of the aircraft, their carrying and passenger capacity.

This trend with the need leads for:

- the increasing attention of application of new constructive (composite) materials;
- to increase the commercial work time of aircrafts;
- the breakaway from the principle of Design's Service Life Limit to the principle of Design's technical condition. The study on the principle of the safe destruction of Design components (Fail Safe Concept) has begun as indicators of Design degradation;
 - a search for the possibility of *non-destructive testing* (NDT) in “on-line” mode directly in Operation with immediate delivery of information about Design technical condition for its analysis on Service Centers (continuous Diagnostics) with following recommendations on aircraft's board or repair and maintenance enterprises.

The rapid development of computer software, miniaturization and performance components and electronic micro devices allows solving a lot of problems; some of them are solved right now. However, the investigation of peculiarities of nature of complex and large technical Designs, especially their degradation during Service Life, noticeably lags behind the development of high-tech information technology.

Still the following issues are not fully explored and resolved:

- the influence of scale effect to complex large-size technical Designs. The size increasing leads to change their properties as a result of non-additive property of system's elements, such as the fuselage, wings and other Structural Significant Elements;
- virtually unknown mechanical properties of composite materials, which, owing to their diversity and variety, as well as heightened sensitivity compared to metals in the fabrication technology and operating conditions do not allow for Developers and Designers to finish the database of their mechanical and other properties. All of these difficulties obstruct the changeover to principle of real technical condition operation as not fully covers all the elements of the technical system monitoring;
- develop methods for monitoring of technical condition of full(large)-size Designs in the process of its Operating include or check-up the entire structure in "on-line" mode that leads to significantly growth of costs, or leading to selectively checking only some parts without sufficient justifying it.

Lack of this knowledge almost negates the advantages of high-tech information technology and makes either remain on the principles of Limit Safe Life Time for major SSE based on partial tests of individual patterns, or try to get information from the test of full-size Designs. But it is obvious that the Fatigue Tests of expensive technical Designs itself are expensive and also requires new approaches to methodology of testing. So far the Test of full-size Designs remains the only MANAGEABLE way to obtain useful information about Design of technical state of degradation under various conditions of loading, as well as the search for methods of monitoring of the degradation of the NDT in "on-line" mode. In the search for ways of reducing the total cost of such Tests, attempts to maximize the information comes from Test are made by changeover the different scenario of Test on the same Design.

This approach to the testing is more useful, than a usual acquisition of information from Service. Let's to realize different scenarios of behavior and Design's loading performances and the nature of her degradation (exhaustion) estimation in the process trying out various options for the use of costly NDT methods. Only during such Branch Tests there is the ability to optimize and minimize costs to adapt the various NDT methods with the subsequent recommendations on their implementation and application in Operation. Testing of different options NDT checking of full-size Designs allows not only dramatically improve

the quality and reliability of tests, but also develop techniques for their application in Operation. But to maximize the use of NDT, it is necessary to know the peculiarities of the exhaustion of the SSE of complex technical Design when it's loaded, more precisely evaluate the conditions of occurrences and the accumulation of damage (degradation) and investigate the nature of their interaction and extension.

Due to the fact that during the expensive large-size Designs' testing many different in nature tasks are decided, a particularly important issue is the early detection of damage, followed by her CONTROLLED propagation. This reduces the probability to finish tests at an early stage of the study due to accelerated extension of damages. Thus the Explorer will have the chance to subsequently modifies the SSE research scenario (conditions of loading, test programs, test sequence of SSE) to obtain further information. At an early stage of research such approach will trying out various principles of management of on-condition and/or the Safe Destruction.

Aware that the first and most important test studies are *identifying the weaknesses (trouble spots) of the Design*. It is obvious that at large-size Design such weaknesses (spots) amount may be more than one. Then, the degradation of the Design will depend on the nature of the emergences and propagation of damage in these spots. This damages may arise dependently (or whatever) one from each other, depending on the circumstances and character of loading as in Tests and in Operation.

The next item is *the definition of parameters of reliability* (reliability, survivability, residual strength etc.) identified weaknesses (trouble spots) in the Design.

Related factors are the *definition of maintainability and efficiency of repair* of detected vulnerabilities and *the ability to control their operation*.

The primary research of dynamic characteristics (frequency) and stress review of the Design under various conditions of loading does not provide a complete image of Design's loading. While the dynamic characteristics evaluation, the attached mass of actuators and Bench Rig are distort the image. Also, the strain gauges show only local deformation, which often may not necessarily be followed by initiating the destruction. Preliminary study on stress state of the Design does not allow enough to accurately determine the location of the weaknesses. The local stresses shall be measured at a point of unsolved impact stress concentration, often large stresses are not critical to the reliability of the whole Structure. Besides major stress review is hampered by limited number of sensors. Another task of study is most important and difficult – to identify their installation's seats. Similarly, same

difficulties are come about from applying the selection and installation of NDT sensors for check-up. Often these problems are solved by analogy with the testing of individual SSE of Design, which in fact have the only weak point (trouble spot), or as the result of the test, or as the result of an artificial damage similar to that found in the Service Work. At a known location and nature of the damage it is easy to define NDT method.

Testing of full (large)-sized Designs, especially when studying degradation of its SSE, it can lead to some errors, which in this case are unacceptable. In fact, when testing of individual SSE has inaccuracies that test can be repeated by replacing the destroyed element with new one. In the case of large-size Design even the damaged element's repair can lead to offset trouble spots compared to apart.

1.2 Ways to Solve the Problem.

Thus the problem requires answers to the following questions:

- what is the possible nature of damage emergences and their propagation on a large-size Design, i.e., what is the nature of trouble spots' origin;
- what is the nature of their propagation, that is, what is the nature of the trouble spot degradation;
- what NDT for trouble spots monitoring is the most suitable and how to choose seat location of NDT sensors for detection of damage emergence at an early stage of test;
- how to limit the amount of check-up items by selecting zone(s) of special attention;
- how results obtained in the Tests may be applied to large-size Design during its Service Life Work.

For answers to these questions, it has to investigate the processes of accumulation of damage in such zones and try to determine the size and location of these zones and adopt the most effective NDT for these terms.

Relevance of carrying out such researches is connected with need of development of the principles of carrying out fatigue tests of large-size Designs, which demand continuous monitoring behind a technical condition of a Design during Tests. Periodic visual surveys, different installations of NDT sensors in critical (potentially weak) places, on purpose of early detection of initial damages, without knowledge of the basic phenomenological principles FDA in Design, bring to big temporary, material and human expenses: great time of a visual inspection of entire Designs leads to idle standing at the Bench Test (a low

availability factor of the bench rig), insufficiently exact choice of an installation seat of NDT sensors, demanding their reinstallation in other places, increases probability of the admission of initial critical damages, increases labor input at installation and setup of the equipment, more number of NDT inspectors and experts are demand to attract the Research attention.

Practice test of large-size Designs arise the need for tighter monitoring of technical condition during testing. This results in a dramatic reduction of Bench Rig's utilization. The need the complete inspection of the Design, often technically unjustified, required the involvement of a large number of operators and takes several times longer than the time of actually testing. It may passes to jump on the "chain fan-shape" Design's inspections that actually have some risk to skip damage¹. The most effective option for monitoring is the ongoing monitoring of the technical state of Design during its Tests. The NDT sensors have to be installed in advance known critical (weak) locations or in areas close to them.

In respect that *on the set of homogeneous and inhomogeneous components of alone full-size technical Design can exist a certain interaction of appearance of damaged*, the nature of the degradation of this areas of the Design is to be appreciated and then the NDT method are to be selected for monitoring of these areas. Thus the human factor is eliminated and the cost of its implementation is reduced. Knowing the characteristics of degradation of these areas, the parameters of reliability, maintainability can be estimated. After that the method of Operation of the Design can be detached as technical condition or Fail Safe Concept.

1.3 Problems are need to be solved To-Day and in Future

At an assessment of the reliability of large-size aviation Designs the main complexity is caused by the following problems:

- there is impossibility of repeatability of tests (experiment) results for the estimating of statistical characteristics of a reliability. Often only the individual (unique) Design pattern is subjected to Test, and sometimes Operation time of Design in actual Service replaces the time of Test;

- for the purpose cutting explores Bench Time of the Design, the fatigue tests are carried out at the raised values of loading. After that test results subsequently are extrapolated to the area of operational loadings without changing the mode and kind of fatigue strength

¹ *So for the Tu-154 aircraft main landing gear Branch Test, the availability stand factor down from 0.75 to 0.30 (without monitoring by acoustic emission) and was promoted to 0.65 for it later use). Also testing the tail beams of the MI-26 was forced to go on the "fan-shipped" of visual inspections that has led to a significant risk to skip the fatigue cracks[71].*

characteristics' distribution function. But thus it is not taken into consideration the almost proved [32] changing of the nature of emergence of fatigue damages at the lowered levels of loading. It is obvious that character and a kind of the fatigue strength distribution functions - fatigue durability, a limit of fatigue endurance, S-N fatigue curve – have to be other at operational loadings, than at tests loadings;

- entire total Design is hard to be monitored by NDT during "on-line" mode.

The specified problems with need lead to

- studying of nature of FDA (emergence of cracks) in a real large-size Design;
- develop the ways and methods of their detection at Tests and at Operation (methodology of NDT inspection) and the scenario of carrying out Tests;

- develop the Design so that a some amount of its SSE may safely collapsed at Tests and/or at Operation work time, thereby warning about saved up fatigue damages in the total Design. (It's may be shown the analogy with built-in sensors of FDA, which haven't gained the extensive use in aviation due to complexity of their installation, control and interpretation of data²);

- develop phenomenological reasonable mathematical and physical models of FDA and characteristics of Fatigue Strength. `Such models allow to describe processes of fatigue damages' emergences and failure outside a framework of the made experiments and Bench Tests and physically to prove extrapolation the Test results to area of operational loadings. Certainly these results can't be confirmed directly with experiment, but have to be confirmed by operating experience.

Owing to uncertainty of a location of initial emergence of fatigue damage and thereof impossibility monitoring of its propagation, there is a problem to define a possible place of emergence of a fatigue crack (damages) by integrated NDT methods. After the determination a zone of emergence of a crack it is possible to use the subsequent methods of NDT to evaluate the propagation of the found crack.

One more problem is how to definite (formalized) the criterion of Design's refusal (the Operation or Test termination moment [45]) of critical value of some formalized size is achieved: cracks' amount in some zone, a certain density by them per unit area or time

² So for example, at tests (and in operation) of the main landing gear of the Tu-154 plane, the top link of the spline joint appeared have been such analogy -sensor that perceiving the total torque. Thus this element becomes the natural safe indicator of torque fatigue loading to the whole main landing gear as the arisen crackin the joint extended to the area of the increasing thickness of a joint [72].

interval, quantities of the destroyed SSE, the main crack tops a certain size and other criteria. For a concrete zone this problem as may be supposed is solved by introduction of the notorious concentrator behind which monitoring by NDT methods is conducted.

In the proposed Research the attempt of studying and formalization of nature of FDA in large-size Designs is taken shot for the subsequent research of possibility to crossover to designing of aviation Designs on the Fail Safe Concept.

It is necessary to emphasize that the existing understanding of early detection of fatigue cracks generally consists in development of more sensitive methods and sensors of NDT. But during the working condition of large-size Designs to the forefront there is a need of definition of some zone where fatigue damage has to appear the first. Thus development of methodology and NDT tooling of carrying out NDT gains importance. This allows to carry out the subsequent monitoring the revealed cracks and failures.

In the long term probably compulsory entering of artificial damage into the certain (failure safe) zone of a Design. This artificial damage will play a role of the sensor for definition of level of FDA to whole Design. In this case the monitoring management is facilitated behind emergence and the subsequent development of fatigue damage.

"One Designer appealed to the Lord to get him the chance to design the equal-stressed airplane. The Lord gave him such opportunity. Some years the Designer flew on such airplane without any service and repair while once his airplane was scattered on a set of identical small parts in flight.

The parable of aircrafts' Designers.

CHAPTER 2 MODEL OF FATIGUE DAMAGES ACCUMULATION IN THE LARGE-SIZE DESIGNS. HYPOTHESIS OF LOCAL ZONES

2.1 Three models of fatigue damages accumulation to a uniform Design

For simplicity and without loss of a community let's consider three ways of fatigue damages accumulation on a uniform Design. Let the considered Design consists of a set of identical components and a damage of one of which doesn't cause destruction of total Design. But its destruction happens when damages achieve a certain density.

Known system models of accumulation of damages, generally for static and fatigue destructions, are based on similar preconditions – from "a hypothesis of a weak link" - destruction of the first micro component leads to destruction of entire System [44], asymptotic expression of this hypothesis [47], specifying hypotheses when destruction happens at achievement of some count "k" of damages [35, 38], up to destruction of all elements – "model of a classical bunch" [43].

In difference from given system models [25] parametrical models describe fatigue damages accumulation, as processes of various type, which bringing to logarithmic normally distribution [18], diffusive [40], to double exponential distribution with logarithmic durability [64] and others [4].

However practically all models consider only QUANTITY of the saved-up damages, but don't consider their DISTRIBUTION on System forming elements. If it didn't bring large

error by consideration of small Systems³, but in case at the description of big full(large)-size Designs it can already leads to great errors (for example, influence of a scale factor, a floating location of "a weak link" (trouble post), redistribution of stress after partial damages).

Besides the increased survivability of large-size Designs, that demands monitoring their technical condition, its rather effective management, especially behind those zones where there is an accelerated accumulation of damages.

Let's consider three possible situations (scenario) of accumulation of damages to a hypothetical uniform Design.

2.1.1 Uniform distribution of damages on making elements of the Design

Let's assume that the flow of damages on the Design occurs under Poisson's law, both on time, and on her elements (space) – evenly.

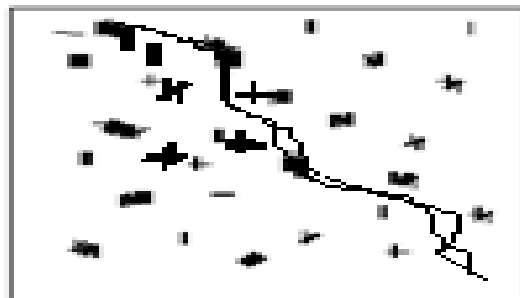


Fig.2.1 Uniform distribution of damages on making elements of the Design

The view of uniform distribution of damages to the Design is given in Fig. 2.1 – it is visible that such character is most dangerous to Design functioning. Actually, it isn't known where to install sensors of monitoring by NDT – there is not pronounced “a weak spot” and actually no damages’ of mode of fatigue crack or her significant defined initial center. Destruction of the Design happens almost instantly at achievement in any zone of critical density of damages.

The durability of such Design is determined by results of tests, and its operation is made by the principle of "a assigned life time" (Limited Safe Life Time). To evaluate of durability

³ At research of an eye - fork destruction of a Mi-8 helicopter main gear frame, in cross section was revealed nonmetallic inclusions. The GOCT 1778-70 [42] allowed damages density found. However concentration them in one seat under a surface had led to premature destruction of the fork.

of such Systems usually applies *logarithmic normal distribution* (NDF) [27, 30, 49], that received under assuming of summation of a large quantity of micro damages.

$$F(\lg N) = \Phi\left(\frac{\lg(N - N_0) - a}{b}\right) \quad (2.1)$$

where $F(\bullet)$ – *cumulative distribuon function* (CDF);

$\Phi(\bullet)$ – cumulative function of normal (Gauss) distribution (NDF);

N – number of load cycles;

N_0 – sensitivity threshold of load cycles;

a, b – respective location "a" and scale "b" parameters of CDF.

Usually such principle applies to especially important SSE for which application of any monitoring of NDT in Operation is complicated. Inspection is exercised at periodic repair and/or operational forms of Service. If the flow of damages passes from a stationary Poisson flow to its sudden increase in Operation, when repair of separate damages becomes economically inefficient, Operation of the Design comes to stop [63].

Such nature of accumulation of damages is usually observed at high levels of loads (multifocal damages), in the presence in Design of numerous uniform concentrators (rivet connections etc), large-size uniform Designs like building constructions.

2.1.2 Concentration of damages to one certain zone of the Design.

Let the same Poisson flow of damages arise in the Design in one some local area (Fig.2.2).

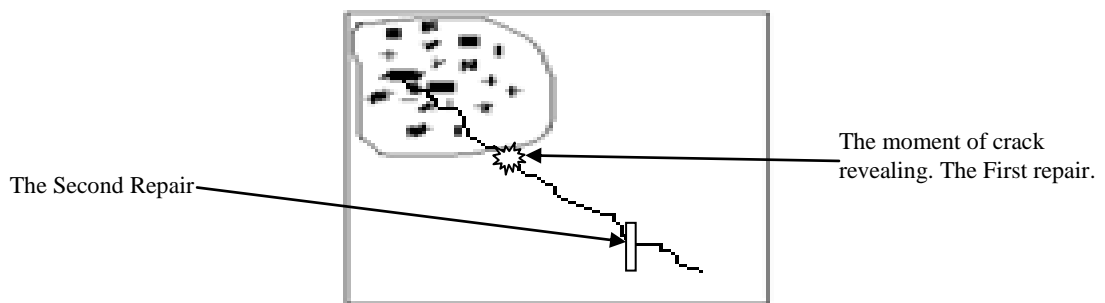


Fig. 2.2 The spreading of damages concentrated in some area of the Design

Considering reliability of this Design it is possible to claim that in this case as "the weak place" (trouble spot) is defined it is easy to manage monitoring of its technical condition. If in this place check-up of FDA is readily available, it is possible to carry out almost continuously. Limitation of the area by the sizes and its certain arrangement allows carry out

repair work. If destruction of this area, for example, in the form of formation of a main fatigue crack, leads to destruction of total Design (the crack isn't found in an Operation time), the durability of such Design is well described by model of "a weak link" or Weibull distribution [48] as *Smallest Extreme Value* (SEV).

$$F(N) = 1 - \exp\left[-\left(\frac{N - a}{b}\right)^m\right] \quad (2.2)$$

where N – number of load cycles;

a, b, m – respective location "a", scale "b" and mode „m" parameters of distribution function (CDF).

If monitoring of such zone allows to carry out crack repair, this damages' spreading is corresponds as work-life distribution of between-repairs practices. Such model is usually characteristic for airplane designs [36]. Such nature of destruction is most suitable for operation on a technical condition and Fail Safe Concept as NDT checking of the only lonely "weak place" allows not only to apply useful methods of NDT, but also to carry out repair work in time.

2.1.3 Concentration of damages to separate areas of the Design.

The damages' accumulation models considered above describe extreme cases of the general model of accumulation, though they are often observed in Operation. Most often, especially for large-size Designs the intermediate nature of the accumulation, which isn't relating to one of given examples, is observed.

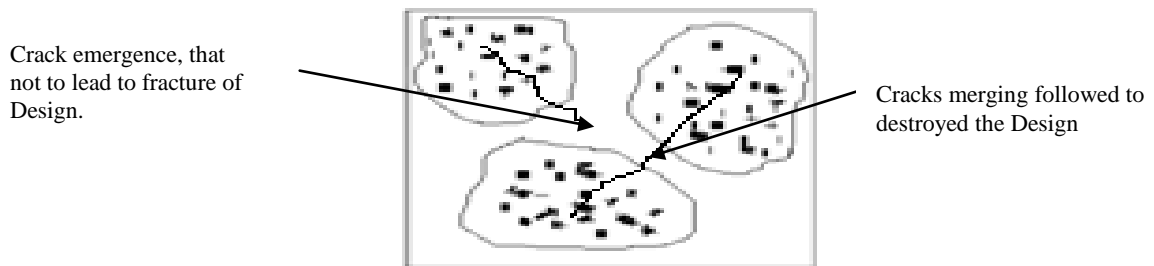


Fig. 2.3 Localization of damages on separate areas

On the one hand intermediate nature of FDA on the Design allows to apply the principles of operation on technical condition and safe destruction as localization of damages gives the chance to define "weak places" with after that their monitoring by NDT methods and maintainability. On the other hand it is necessary very precisely to identify a location of

these "weak zones". Usually such zones are defined previously or by results of surveys in Operation, or by results of Bench Tests.

However the nature of emergence of such zones, and also nature of their development didn't receive the detailed consideration and justification yet. Destruction of a local zone (in the form of emergence of fatigue cracks) usually doesn't lead to immediate destruction of total Design (owing to her survivability), especially if monitoring of this zone allows to prevent emergence of the main crack or her merging, with its well-timed repair. The Bench Tests of the Tu-154 airplane full-size main landing gear showed possibility of some SSE operation on Fail Safe Concept when destruction of separate SSE didn't lead to total destruction of landing gear and its intended application [72].

Then it is possible to accept that Design reliability can be described as reliability of cross-coupling several such areas (zones).

The assumption of existence of such zones allows carrying out consecutive research of reliability of each of these zones during fatigue Test of one Design. Indeed the monitoring of local zones in a mode "on-line" is already economically available at Bench Tests. Besides, there is an opportunity to probe the techniques of adaptation of methods of NDT and its tools to Service conditions.

Obviously, in the assumption of existence of such zones it is necessary to study the processes of cracking arising in them, to estimate characteristics and parameters of reliability (durability), to pick up the most effective methods of NDT, to prepare recommendations about frequency and areas of surveys, to prepare recommendations about repair of these zones and development of the Design by the principles of operation on a technical condition and/or Fail Safe Concept.

It is possible to notice that at emergence and localization of zones, processes of FDA on separate zone are similar to processes of the first model of accumulation, and reliability of entire Design can be described by the second model ("weak link") or SEV.

At first sight it is obviously possible that real FDA and the subsequent destruction happens on consecutive involvement of all given mechanisms.

At first in all potential zones of a Design damages accumulated according to paragraph 2.1 schemes. This at a certain quantity of cracks passes to the scheme Fig.2.3. Destruction of a Design comes after accumulation of damages will reach critical level in one of a zone and destruction of a Design will happen according to item Fig.2.2 scheme – "a hypothesis of a weak link" (SEV).

Earlier it was shown [64] that distribution function of fatigue durability of such zone, (if there is no immediate destruction of whole Design in case of destruction of local zone) has to be described by CDF in the form of double exponential distribution with logarithmic durability (DEDF) or *Largest Extreme Values* (LEV):

$$F(\tau) = \exp[-\exp(-\frac{\tau - a}{b})] \quad (2.3)$$

where $\tau = \lg N$ – logarithm of load cycles;

a, b – respective location „a” and scale „b” parameters of CDF.

It is possible to assume that in uniform Design the formation of such zones is possible only at *dependent* accumulation of damages in this zone. Then emergence of damage can provokes emergence of new damages near initial ones, localize an area of emergence and relaxe thereby other points of damages generation due to loading (stress) redistribution. This relationships has to extend on some distance, weakening in process of its increase.

The given reasons lead to a opportunity to formulate the "Hypotheses of Local Zones with dependent accumulation of fatigue damages"(HLZ).

2.2 The basic principles of the Hypotheses of Local Zones

Basic provisions and preconditions of a Hypothesis of Local Zones consist in the following:

- the full (large) Design can be presented as grouping of several Local Zones;
- in Local Zones the accumulation of fatigue damages happens stochastic dependent, and beyond its limits –independent;
- with increase in distance from initial damage, stochastic relationship between the moments of emergence of damages decreases;
- some Design (or her SSE) may be represented as one Local Zone;
- characteristics of durability, survivability of entire Design are defined by characteristics of Local Zones as „weak place”:

a) if the uncontrolled failure of Local Zone leads to failure of the total Design the Safe Life of the Design is described as Logarithmic Normal CDF;

b) if failure of Local Zone is controlled by NDT or others methods, and the wrecking of Local Zones leads to destruction of total Design the Safe Life of the Design is described by Weibull CDF as „weakest link” or SEV;

c) if the failure of Local Zone do not leads to failure the total Design the Safe Life of the Design (Local Zone) in operation is described by DEDF or SEV;

d) the damaged Local Zone may be presented as indicator of the fatigue damage accumulation in total Design if Operation is carry out by Safe Failure (destruction) principle.

Confirmation of a Hypothesis of Local Zones will allow to

- limit application of methods of NDT on certain zones, instead of on entire surface of Design;

- lower time and labor input of carrying out surveys at diagnostics of a large-size Designs at Bench Tests and it's operation;

- increase efficiency of repair of a Design;

- expand possibilities of Bench Tests, allowing to war-game various scenarios of loading of a Design at reliable early detection of fatigue cracks;

- define characteristics of reliability of the Design more precisely as a reliability of Local Zones;

- make recommendations about Tests not natural total-size Design, but its only critical Zones (elements);

- determine possibilities of coming over the principle of operation by a technical condition to the Fail Safe Concept;

- make recommendations about development of scenario and programs of Bench Tests and methodology of the management for NDT monitoring as in Bench Test as in Operation.

For the analysis of nature of FDA we will make the experiment consisting in fatigue tests of specimens with concentrators of a various mode and an arrangement on a uniform surface. Then it is possible to present these specimens as the model-analogue imitating emergence of damages on the areas of continuous section and we will try to estimate nature of interaction of such damages.

“It isn't matter with Data, but that You are doing with Them”.

K.F.C. Manderville

CHAPTER 3 RESEARCH OF DEPENDENCE OF ACCUMULATION OF FATIGUE DAMAGES ON SYSTEMS OF CONCENTRATORS

3.1 Goal of the Test

The goal of Test is experimental studying of interdependence of fatigue damages' accumulation on flat specimens with several concentrators in the form of identical holes with different diameters, a various arrangement relatively each other and concerning the direction of the applied loading. These specimens present the model-analogue of process FDA in solid body or uniform Design.

3.2 The objects-to be-tested

As objects-to be-tested flat specimens (5 mm thickness plate of aluminum alloy of D16T) were chosen differing configurations, number, arrangement and size of stress concentrators, executed in the form of circular holes of 5 mm diameters. At production and preparations of samples and specimens, the principle of TOTAL randomization [21] was carry out throughout: out of an aluminum alloy plates were applied preparations which in a casual order according to the table of random numbers [6] gathered in packages, were cut and processed on the milling machine. Also in a casual order the specimen's type for each preparation, an order of production of concentrators and an installation order on the test rig were defined.

Concentrators were put each from other and from the radius of a curve of transition of working part of a specimens to test machine seals at large distance 40 mm (more than eight diameters), which allowed applying an assumption about a stability and uniform of a field of stresses about the concentrator. Thus it was excluded the influence of other concentrators or captures of the test machine [22]. Production of concentrators was accompanied by careful processing of an internal working surface: multipass drilling of holes by drills of different diameters (from smaller to bigger) with the subsequent chamfering and expansion of openings on the 7th surface finish class.

Experiment and the analysis of the received results were carried out on specimens of two configurations:

- specimen of 5 mm thick and with four concentrators' types in the form of identical holes (diameter 5 mm). The specimens differ each from others by number and an arrangement of concentrators: a configuration of "V"-type (Fig. 3.1 a, b, c, d); two types specimens of 2 mm thick differ by number and size of concentrators: - a configuration of "W"-type (Fig. 3.1 e, f).

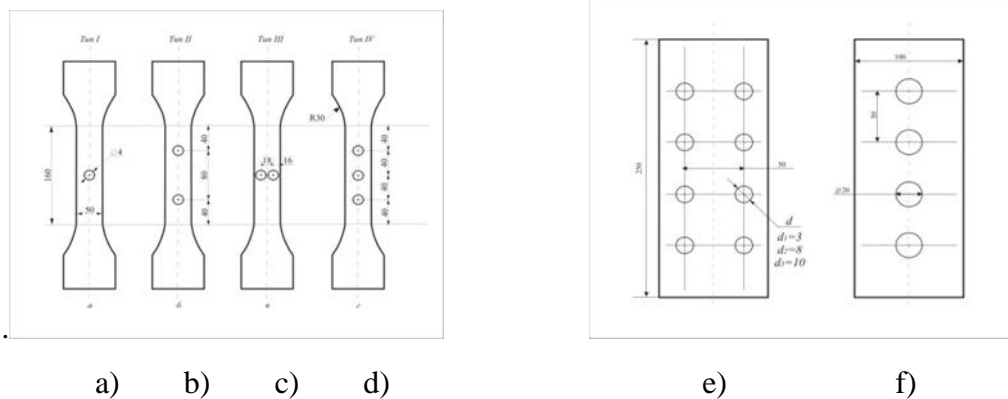


Fig.3.1 Specimens of “V”-type configuration: a), b), c), d) and “W”-type configuration⁴: e), f)

3.3 Test machine

Fatigue Tests of “V”-type specimens were conducted by test hydro pulsation machine of the YPC-10 type (production of Armavir plant, Russia). The spherical bearings installed in places of a joint of levers of transitional knots with captures of the Test Machine, excluded any possibility of emergence of any bending moments on working part of specimens.

Control of loading was checking on the screen of the monitor and automatically regulated by a control system of the test machine with feedback. The feedback was under control of strain gagedynamometer of Test Machine. The additional control carries out watching the strains measured by strain gages pasted on a working surface of specimen. For this class of test machine the relative error of applied loading value doesn't exceed 1%.

3.4 Program and procedure of fatigue tests

Tests were conducted until a fatigue crack emergence of around the concentrator (from hole edge). The system of wire sensors was applied to objective definition of the moments of emergence of cracks. The wire with a diameter of 0,01 mm, pasted round concentrators by glue on technology of strain gages seats. The wire sensor was turned on into a control electro

⁴ Results of tests of "W-type" samples have been received with Dc.H.Sc.Ing., prof. V.Pavelko and presented in the Appendix 5, Table 2.5

chain. At the time of sensor rupture (disconnection of the electro chain) the signal was given on an alarm board, registering the moment of crack's emergence on concentrator of circular hole kind. The inspection checking of controlling sample parties showed that such sensor worked at achievement of a fatigue crack of length about 0.5 mm from the hole side edge.

The "V"-type specimens were loaded with the reverse stress cycle close to pulsating, with parameters:

- maximum stress⁵: $\sigma_{\max}=120$ MPa
- minimal stress: $\sigma_{\min}=12$ MPa
- frequency of cycles: 8,3 Hz
- stress ratio: 0,1

At fatigue tests of specimens with several concentrators for emergence cracks on each of the concentrators, the first and the next cracks stopped by plastic deformation of a zone before the mouth of the arisen crack. The plastic deformation was carried out by cave-in the punch with the diameter of 4 mm. The punch was pressed into a specimen's body by load of 24 kN on a technique [66]. Such artificial area of plastic deformation stopped the revealed crack for the time being necessary for formation of cracks on others concentrators.

Thus, on "V"-type specimens it was received the one value of the moment of emergence of crack, but on "W"-type specimens - two values on each of concentrators. Further, the moments of emergence of cracks at side edges of the holes, fixed by a rupture of wire sensors, we will determine by the term "*durability of the concentrator*".

3.5 Results of fatigue tests of "V"-type specimens

Let's designate the concentrator located in the middle of working part of a specimen by the term "*inner*", and the concentrators located closer to captures of the test machine – the term "*outer*"⁶.

⁵ Theoretical stress concentration factor, taking into account cross-section wrap [2]:

$$\alpha_{\sigma} = 2 / (d/2h + 1/\beta)$$

where $2d$ – diameter of the hole concentrator,

$2h$ – width of specimen working part,

β – cross-section wrap factor.

⁶ Further in the text the "in" and "ot" index belong respectively to "inner" and "outer" concentrator, and the following index (/ a, / b, / c, / d) – indicates specimen's kind due to Fig. 3.1.

Let's take a logarithm of number of cycles N before emergence of a crack on hole as durability τ of the concentrator. It is obvious that $\tau = \lg N$ is a random variable (RV), cumulative distribution function (CDF) of which let's accepted as NDF:

$$\Phi\left(\frac{\tau - a}{b}\right) \quad (3.1)$$

where $\Phi(\bullet)$ - cumulative Normal (Gauss) distribution function,

a - location parameter of CDF (mathematical expectation),

b – scale parameter of CDF (mean square deviation).

3.5.1 Results of fatigue tests of “V”a)-type specimens (“inner” single concentrator)

To estimate location $a_{in/a}$ and scale $b_{in/a}$ parameters of durability NDF (3.1) for “inner” single concentrator, the sample size $n_{in/a}=20$ was tested. Results of tests are given in Table 3.1 (Appendix 1).

The estimations of NDF $a_{in/a}$ and $b_{in/a}^2$ parameters, obtained by method of matching moments [15]:

$$\begin{aligned} \hat{a}_{in/a} &= 5,27230; & \hat{b}_{in/a}^2 &= 0,00582; \\ \hat{b}_{in/a} &= 0,07278; & \hat{v}_{in/a} &= \hat{b}_{in/a}/\hat{a}_{in/a} = 1,38\% \end{aligned} \quad (3.2)$$

where symbol \hat{x} notes the statistical sample estimate of parameter x .

In Fig. 3.2 the experimental points that put on probabilistic graph of three main distributions: normal, smallest extreme (Weibull-SEV) and largest extreme distributions (DEDF-LEV). It is seen the satisfactory matching of experimental data with DEDF. But for comfort of our investigation's analyses of stochastic ties we will use normal distribution.

The check-up of the experimental points matching with predicted normal by Shapiro-Wilk criterion, recommended ISO 5479-97 [41], showed

$$S = \sum_k a_k (\tau_{(n+1-k)} - \tau_{(k)}) = 0,32397 \quad (3.3)$$

where $k = 1 \dots n/2$,

n – sample size,

a_k – see table [41]

The Test Statistics

$$W = S^2 / \sum_{i=1}^n (\tau_i - \bar{\tau})^2 = 0,32397/n \hat{b}_{in/a}^2 = 2,78 \quad (3.4.)$$

get into the hypothesis-acceptance region $\Omega_W \in \{W > 0,905\}$ for NDF of concentrator durability [41] at significance level $\alpha = 0,05^7$

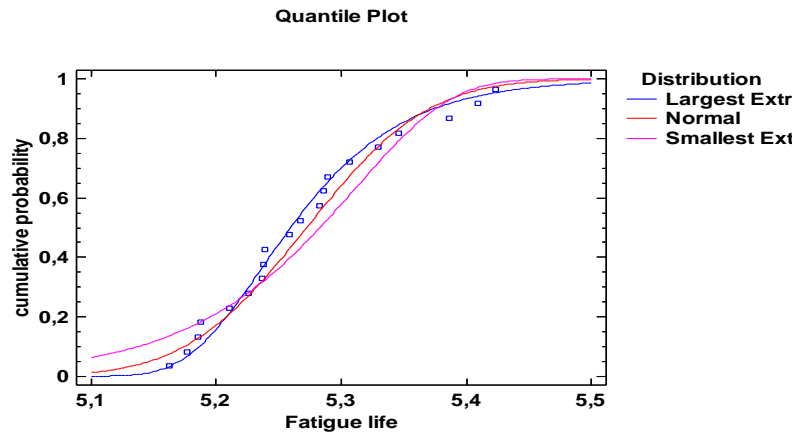


Fig.3.2 Cumulative Normal Distribution Functions for Experimental fatigue life Values Specimen type „a” (isolated inner concentrators)

In addition let's check a matching of experimental distribution with normal (with known parameters) by criterion of Kramer-Mizes-Smirnov [6]

$$W_n^2 = \sum_{i=1}^n (F_0(\tau_i) - \frac{2i-1}{2n})^2 + 1/12n = 0,0450 \quad (2.5)$$

where n – sample size,

i – rank of plot

The hypothesis-acceptance region $\Omega_{W_n^2} \in \{0...0,461\}$ for criterion W_n^2 suggest the null-hypothesis about the NDF of experimental data.

3.5.2 Results of fatigue tests of “V”b)-type specimens (two outer concentrators)

To estimate location $a_{ot/b}$ and scale $b_{ot/b}$ parameters of durability NDF (3.1) for “outer” two concentrators (Fig.3.1b), the sample size $n_{in/b}=14$ was tested. Results of tests are given in Table 3.2 (Appendix 2).

⁷ Further the testing of statistical hypotheses will be made at significance level $\alpha = 0.05$ (probability to reject a true null-hypothesis), both one side- and two side areas of test criteria.

On each “V”b)-type specimens two values of the moments of emergence of a crack (concentrators’ durability) were received consecutive. The received values of concentrator’s durability’s may be as stochastic independent each from others, so some stochastic relationship (dependence) among themselves.

Using the Kramer-Mizes-Smirnov Test (3.5.) have $W_n^2 = 0,058$ and null-hypothesis-acceptance region $\Omega_{W_n^2} \in \{0...0,461\}$. So the experimental data shows the satisfactory matching the experimental data with GDF (3.1) with parameters $a_{ot/b}$ and $b_{ot/b}$ (Fig.3.3).

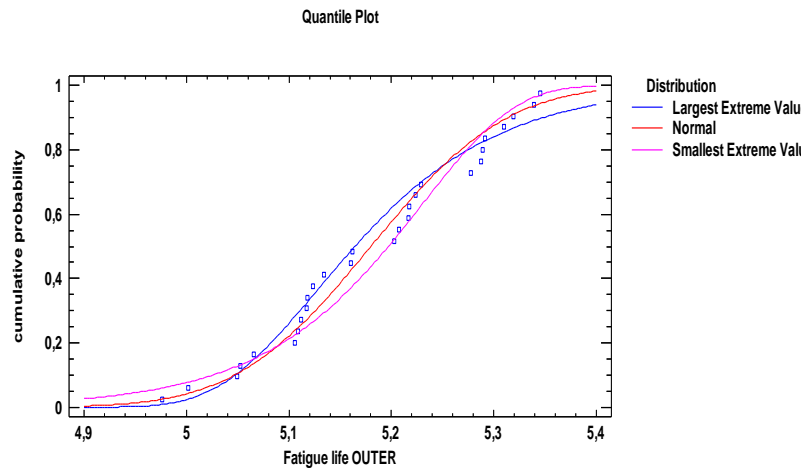


Fig.3.3. Cumulative Normal Distribution Function (NDF) with Experimental standardize durabilities for „V”b)-type „b” Specimens (isolated „outer” concentrators)

Put forward the null-hypothesis that the accumulation of fatigue damages (moments of emergences of fatigue cracks) on the System with two “outer” concentrators is stochastically independent. Next it is possible to pool these two samples together with 28 plots size. So have the estimation of “outer” concentrator durability $\tau_{ot/b}$:

$$\hat{a}_{ot/b} = 5,18012 \quad \hat{b}_{ot/b}^2 = 0,01081 \quad \hat{b}_{ot/b} = 0,10397 \quad \hat{v}_{ot/b} = 2,0\% \quad (3.6.)$$

So the two values of durability on each specimen may be considered as 1st and 2nd Order Statistics of NDF (3.1) with location $a_{ot/b}$ and scale $b_{ot/b}$ parameters.

The Order Statistics CDF for System with two independent concentrators with NDF (3.1) and parameters (3.6) are expressed [7]:

- the CDF of the 1st Order Statistic:

$$F_{(1)}(\tau_{ot}) = 1 - \left[1 - \Phi\left(\frac{\tau - a_{ot}}{b_{ot}}\right) \right]^2 \quad (3.7)$$

- the CDF of the 2nd Order Statistic:

$$F_{(2)}(\tau_{ot}) = \Phi^2\left(\frac{\tau - a_{ot}}{b_{ot}}\right) \quad (3.8)$$

It is known, that the Mathematical Expectation and Variance of the 1st and 2nd Order Statistics' CDF are linked to parameters a_0 and b_0 of the initial CDF as [7]:

$$a_{o(1,2)} = a_0 \pm 0,5642 b_0 \quad (3.9)$$

$$b_{o(1,2)}^2 = 0,6816 b_0^2 \quad (3.10)$$

Then the theoretical values of parameters of the Order Statistics' CDF:

$$a_{ot(1)} = 5,12146 b_{ot(1)}^2 = 0,00737 \quad b_{ot(1)} = 0,08584 \quad (3.11)$$

$$a_{ot(2)} = 5,23878 b_{ot(2)}^2 = 0,00737 \quad b_{ot(2)} = 0,08584 \quad (3.12)$$

The experimental data gives the following statistical estimates:

- for the 1st Order Statistic's CDF

$$\begin{aligned} \hat{a}_{ot/b(1)} &= 5,12165 & \hat{b}_{ot/b(1)}^2 &= 0,00899 \\ \hat{b}_{ot/b(1)} &= 0,09482 & \hat{v}_{ot/b(1)} &= 1,67\% \end{aligned} \quad (3.13.)$$

- for the 2nd Order Statistic's CDF:

$$\begin{aligned} \hat{a}_{ot/b(2)} &= 5,23860 & \hat{b}_{ot/b(2)}^2 &= 0,00610 \\ \hat{b}_{ot/b(2)} &= 0,07808 & \hat{v}_{ot/b(2)} &= 1,63\% \end{aligned} \quad (3.14.)$$

Also it is known [15], that the Order Statistics' CDF asymptotically converge to NDF. Suppose that the plots are stochastically independent and have taken away from the general sample having CDF with parameters (3.11) and (3.12).

Put forth the null-hypothesis:

$$H_0: b_{ot/b(1,2)}^2 = b_{ot}^2 \quad (3.15)$$

and alternative one

$$H_1: b_{ot/b(1,2)}^2 \neq b_{ot}^2 \quad (3.16)$$

where $b_{ot/b(1,2)}^2 = 0,6816 b_{ot/b}^2 = 0,00737$.

The Test Statistic χ^2 [23] is expressed by

$$\chi^2 = (n-1) \hat{b}_{ot/b(1,2)}^2 / b_{ot/b}^2 \quad (3.17)$$

and have the χ^2 – distribution with (n-1) degree of freedom, where n=14.

The null-hypothesis-acceptance region is $\Omega\chi^2 \in \{5,01\dots24,74\}$.

Experimental data gives:

$$\chi^2_{(1)} = 13 \cdot 0,00899 / 0,00737 = 15,86 \quad (3.18)$$

$$\chi^2_{(2)} = 13 \cdot 0,00610 / 0,00737 = 10,76 \quad (3.19)$$

There is no any base to reject the null- hypothesis, so values of criteria statistics belong to the set of acceptance of null hypothesis.

Let's similarly check equality of averages with a sample means. The Test Statistic is [23]:

$$t_{(1,2)} = \sqrt{n} (\hat{a}_{ot/b(1,2)} - a_{ot/b(1,2)})/b_{ot/b(1,2)} \quad (3.20)$$

and have the Student's CDF, also with (n-1) degree of freedom.

Put forth the null- and alternative hypotheses similar (3.15) and (3.16) to check the equation of sample means to mathematical expectations order statistics CDF. The hypothesis-acceptance region is $\Omega_t \in \{-2,16\dots2,16\}$, but statistical sample estimation is:

$$\hat{t}_{(1)} = \sqrt{14} (5,12165 - 5,12146)/0,09482 = 0,007 \quad (3.21)$$

$$\hat{t}_{(2)} = \sqrt{14} (5,23860 - 5,23787)/0,07808 = 0,035 \quad (3.22)$$

There is also no any base to reject the null- hypothesis, so values of criteria statistics belong to the set of acceptance of null hypotheses.

On Fig.3.4 is shown the graphs of Order Statistic's CDF (initial CDF is (3.1) and parameters value is (3.6)) and experimental plots.

A satisfactory matching of experimental data with theoretical is confirmed. It additionally confirms the independence of emergence of fatigue cracks on “outer” concentrators.

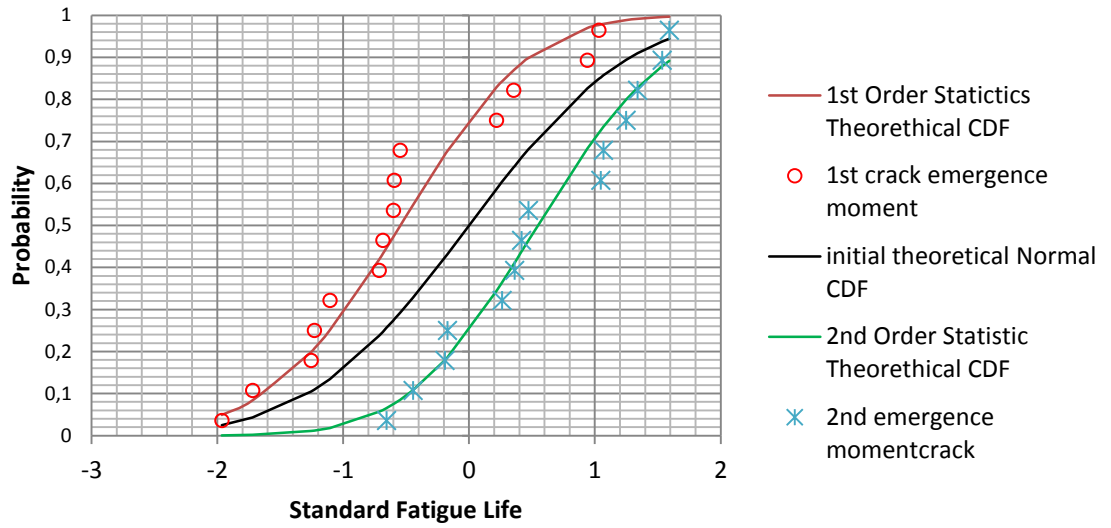


Fig.3.4 The 1st and 2nd Order Statistics' CDF of the fatigue life till the 1st and the 2nd cracks have been appealed consequently on “outer” concentrators

Thus, it is shown that the moments of fatigue cracks emergences on concentrators of “V”b)-type specimens are stochastic independent. However in the methodological purposes, let’s carry out the additional analysis, which will be required in further research of fatigue damages’ accumulation dependences on other System of concentrators.

As shown in [10] that in case of initial NDF theoretical value of correlation coefficient (CC) $\rho_{0(1-2)}$ between the 1st and the 2nd Order Statistics is equal

$$\rho_{0(1-2)} = 0,467 \quad (3.23)$$

The statistical estimation $\hat{\rho}$, calculated by the test results of “V”b)-type specimens

$$\hat{\rho}_{(1-2)} = 0,466 \quad (3.24)$$

Let check the null hypothesis

$$H_0: \hat{\rho}_{(1-2)} = \rho_{0(1-2)} \quad (3.25)$$

that sample value of CC is equal to theoretical value against the alternative hypotheses

$$H_1: \hat{\rho}_{(1-2)} \neq \rho_{0(1-2)} \quad (3.26)$$

Apply the Test Statistic T of Fisher [31]:

$$T = \sqrt{(n-3)} (z-z_0) \quad (3.27)$$

where $n = 14$ – sample size,

$$z = \frac{1}{2} \ln\left[\frac{1 + \rho_{(1-2)}}{1 - \rho_{(1-2)}}\right] \quad (3.28)$$

$$z_0 = \frac{1}{2} \ln\left[1 + \frac{(1 + \rho_{0(1-2)})}{(1 - \rho_{0(1-2)})}\right] + \frac{\rho_{0(1-2)}}{2(n-1)} \quad (3.29)$$

Test Statistic T have the standard NDF and null hypotheses-acceptance region is $\Omega_p \in \{-1,96...+1,96\}$.

The test data give the following results:

$$z = \frac{1}{2} \ln[(1+0,466)/(1-0,466)] = 0,505 \quad (3.30)$$

$$z_0 = \frac{1}{2} \ln[(1+0,467)/(1-0,467)] + 0,467/26 = 0,524 \quad (3.31)$$

$$T = \sqrt{11}(0,505-0,524) = -0,063 \quad (3.32)$$

There are no any bases to reject the null- hypothesis, so values of criteria statistics belong to the set of acceptance of null hypothesis. The moments of emergences of fatigue cracks are stochastically independent on “outer” concentrators of “V”b)-type specimens.

As one more proof of stochastically independence of the fatigue cracks emergences moments on “outer” concentrators is proximity to zero the intraclass correlation coefficient (ICC)⁸ [17]:

$$\bar{r} = \frac{1}{k-1} \left(k \frac{\overline{b_{\tau}^2}}{\bar{b}^2} - 1 \right) \quad (3.33)$$

where \hat{r} - sample ICC,

k - the number of elements in class, in our case – the number of concentrators on the specimen, is equal to 2,

$\bar{\tau}_k$ - the mean durability in class,

\hat{b}^2 - the full sample Variance relative the common mean of concentrator’s durability $\bar{\tau}$

\hat{b}_{τ}^2 - the sample Variance of mean values $\bar{\tau}_k$ of separate classes.

The experimental data give the following results:

$$\hat{b}^2 = \hat{b}_{ot}^2 = 0,01121 \quad (3.34)$$

$$\hat{b}_{\tau}^2 = 0,00557 \quad (3.35)$$

$$\hat{r} = -0,006 \quad (3.36)$$

⁸ *The application of interclass correlation coefficient is caused by that in this case the interfaced moments of emergence of cracks on a specimen are considered irrespectively about their order emergence. Otherwise the correlation relationship will be established between Order Statistics, but not actually the moments of emergence of cracks [17]. As an example it is possible to give the correlation analysis between growth of brothers.*

To put forth the null-hypotheses

$$H_0: r = 0 \quad (3.37)$$

and alternative one

$$H_1: r \neq 0 \quad (3.38)$$

Applies the Test Statistics T [31]:

$$T = (z - z_0) \sqrt{(2n-3)/2} \quad (3.39)$$

$$\text{where } z = \frac{1}{2} \ln(1+r^{\wedge})/(1-r^{\wedge}) \quad (3.40)$$

$$z_0 = 0 \quad (3.41)$$

$n = 14$ – the sample size.

If the quantity of elements in class is equal and $k = 2$ the Test Statistic T has the standard NDF [31] and null hypothesis-acceptance region $\Omega_T \in \{-1,96...+1,96\}$.

Have $z^{\wedge} = -0,006$ and $T = -0,006 \sqrt{12,5} = -0,021$, so may make the conclusion, that *There is no any base to reject the null- hypothesis, so values of criteria statistics belong to the set of acceptance of null hypothesis. The moments of emergences of fatigue cracks are stochastically independent on “outer” concentrators of “V”b)-type specimens.*

It is seen that the mean durability of “outer” concentrator a_{ot} is some lower than the durability of “inner” concentrators a_{in} .

Test the null-hypothesis $H_0: a_{in} = a_{ot}$ that the durability’s means are equal against the alternative $H_1: a_{in} > a_{ot}$.

The experimental data of fatigue test of “V”a)-type and “V”b)-type represent:

$$\hat{a}_{in/a} = 5,27230 \quad \hat{a}_{ot/b} = 5,180125 \quad (3.42)$$

$$b_{in/a}^{\wedge 2} = 0,00582 \quad b_{ot/b}^{\wedge 2} = 0,01081 \quad (3.43)$$

$$n_{in/a} = 20 \quad n_{ot/b} = 28 \quad (3.44)$$

Previously let’s compare estimates of Variances of two samples. Also test the null-hypotheses of Variances equality against alternative one of her differences. The Test Statistic is Fisher F-criteria [15]:

$$F = b_{ot/b}^{\wedge 2} / b_{in/a}^{\wedge 2} \quad (3.45)$$

That has the F-distribution (the CDF of Fisher) with $(n_{ot}-1)$ $(n_{in}-1)$ degrees of freedom. The null hypotheses-acceptance region is $\Omega_F \in \{0... 2,16\}$.

Calculating (3.45) shall obtain:

$$\hat{F} = 0,01121/0,00582 = 1,93. \quad (3.46)$$

There are no any bases to reject the null- hypothesis about the equality of the Variances of durability's of "inner" and "outer" concentrators.

If the Variances of two different samples are equal we may to use the Test Statistics t , that have the Student distribution with $(n_{in} + n_{ot} - 2)$ degrees of freedom [23]. So we have

$$t = (a_{in} - a_{ot}) \sqrt{(n_{in} + n_{ot} - 2) / \sqrt{(1/n_{in} + 1/n_{ot}) [(n_{in} - 1)b_{in}^2 + (n_{ot} - 1)b_{ot}^2]}} \quad (3.47)$$

If suppose the one side alternative hypothesis $a_{in} > a_{ot}$. the null hypothesis-acceptance region is $\Omega_t: \in \{0 \dots 1,68\}$.

Calculating (3.47) shall obtain:

$$\hat{t} = (5,27230 - 5,18079) \sqrt{46 / \sqrt{(1/20 + 1/28)(19 \cdot 0,00582 + 27 \cdot 0,01121)}} = 3,29 \quad (3.48)$$

and accept the alternative hypothesis that *the durability of "inner" concentrator is significantly greater than the durability of "outer" concentrators.*

This unexpected result leads to an important conclusion that under all equal other conditions the "weak place" (trouble spot) of a Design has to settle down in the area close to transferring from one some irregularity to another or from some one characteristic area to another, on any parameters differing from the first.

In our case it is possible to assume the following: *this is influence of closeness of "outer" concentrators to captures of the test machine, or to area where a load stream passes from one object to another.* Tests of numerous objects, both specimens and constructive elements, confirm this assumption: often the destruction happens in junctions of various constructive elements or even continuous *transferring of one rigidity to another.* The influence of closeness of test machine captures of the insufficiently correctly designed specimens also is observed during tests.

3.5.3 Results of fatigue tests of "V"c)-type specimens (two inner concentrators)

Let's subject results of fatigue tests of twenty "V)c)-type specimens (Fig. 3.1c) from two "inner" the concentrators located on width of a specimens to the similar (paragraph 3.5.2)

analysis. It is known that stresses at edges (sides) of holes for specimens of “V”a)- and “V”c)-type are identically and equally [22].

The Test results are listed in Attachment 3 (Table 3.3).

The fatigue durability CDF of joint two “inner” concentrators specimens (n=40) is shown on Fig. 3.5. It is seen the satisfactory matching with NDF: Kramer- Mizes-Smirnov Test Statistic (3.5) $W_n^2 = 0.071$

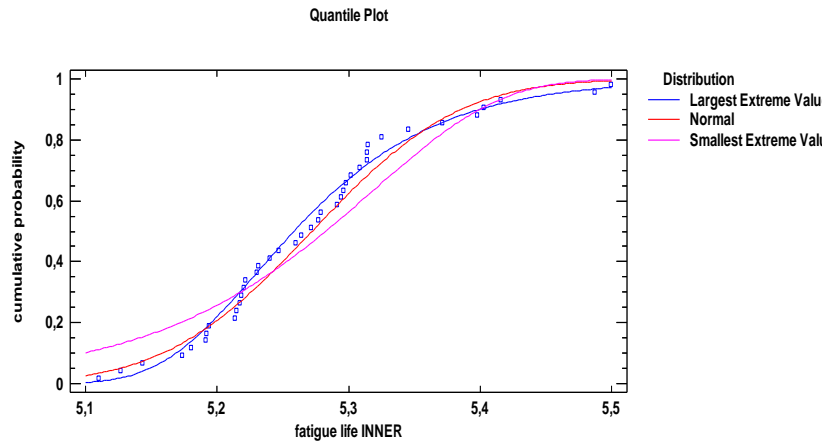


Fig.3.5 Cumulative Distribution Functions with Experimental fatigue durability of „V”c)-type specimens (two „inner” holes).

Let equate the sample estimation of durability’s NDF parameters for isolated “inner” concentrators (“V”a)-type) and twin “inner” ones (“V”c)-type). So we have:

$$\hat{a}_{in/a} = 5,27230 \quad \hat{a}_{in/c} = 5,27214 \quad (3.49)$$

$$\hat{b}_{in/a}^2 = 0,00582 \quad \hat{b}_{in/c}^2 = 0,00781 \quad (3.50)$$

$$n_{in/a} = 20 \quad n_{in/c} = 40 \quad (3.51)$$

As before, equate the Variances of two samples using Fisher F-Criterion (3.45) with $(n_{in/c} - 1; n_{in/a} - 1)$ degrees of freedom and null hypotheses-acceptance region $\Omega_F \in \{0 \dots 2,16\}$:

$$F = \hat{b}_{in/c}^2 / \hat{b}_{in/a}^2 = 0,00781 / 0,00582 = 1,34 \quad (3.52)$$

There is no any base to reject the null- hypothesis about the equality of the Variances

Then apply the two-side t- criterion of Student (3.47) with $(n_{in/c} + n_{in/a} - 2)$ degrees of freedom and null hypotheses-acceptance region $\Omega_t \in \{-2, 0 \dots +2, 0\}$:

$$\hat{t} = (5,27230 - 5,27214) \sqrt{59} / \sqrt{(1/20 + 1/40)} (19 \cdot 0,00582 + 39 \cdot 0,00781) = 0,007 \quad (3.53.)$$

Also there are no any bases to reject the null- hypothesis about the equality of the Means

Then these two samples we may consider as similar and have a right to pool them together. So get the unit sample of durability's of "inner" concentrator with NDF parameters:

$$a_{in} = 5,27219; \quad b_{in}^2 = 0,00703; \quad b_{in} = 0,08387; \quad v_{in} = 1,59\%; \quad n = 60 \quad (3.54)$$

The durability NDF of "inner" concentrators is shown on Fig. 3.6.

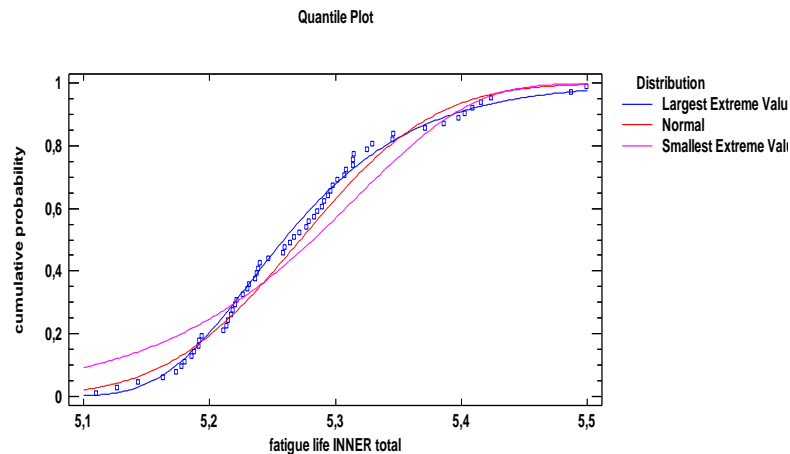


Fig.3.6 Cumulative Distribution Functions with Experimental Fatigue durability for Specimen with „inner” concentrators (total data).

The NDF with parameters (3.54) is considered as initial NDF of "inner" concentrator. The Kramer-Mizes-Smirnov Criterion (3.5.) is equal $W_{n,}^2 = 0,086$.

Note:

On Fig.A3.2(Attachment 3) it is possible to see, that the experimental values of fatigue durability CDF are not so much as theoretical ones in the area of small probabilities of failure.

This effect is known, as "a sensitivity threshold on cycles". But this area is important for determination of safe durability (Safe Life, resource) of Designs. To refine the matching data often in NDF (3.1) and others CDF enter the third parameter $\tau_0 = \lg N_0$ [30, 34]:

$$\Phi \left\{ \frac{(\tau - \tau_0) - a}{b} \right\} \quad (3.55)$$

However, it is known that the increase in number of parameters sharply reduces the accuracy of their statistical evaluation [15].

As well as in the previous case (paragraph 2.5.2), suppose, that independence fatigue damages of accumulation on system of specimen's concentrators takes place.

Then if NDF of „inner” concentrators durability (3.54) and Order Statistics CDF (3.9), (3.10) parametrs are known lets’s plot the CDF of the 1st an 2nd Order Statistics for the moments of crack emergences on the twin „inner” concentrators.

The theoretical values of the Order Statistics CDF parameters:

$$b_{in/c(1,2)}^2 = b_0^2 = 0,6816 \quad b_{in}^2 = 0,6816 \cdot 0,00703 = 0,00479 \quad (3.56)$$

$$a_{in/c(1)} = 5,27219 - 0,5642 \cdot 0,08387 = 5,22487 \quad (3.57)$$

$$a_{in/c(2)} = 5,27219 + 0,5642 \cdot 0,08387 = 5,31951 \quad (3.58)$$

respectively its experimental values (Table 3.3 Attachment 3)

$$n_{in/c(1)} = n_{in/c(2)} = 20 \quad (3.59)$$

$$a_{in/c(1)}^{\wedge} = 5,24662; \quad b_{in/c(1)}^{\wedge 2} = 0,00840; \quad b_{in/c(1)}^{\wedge} = 0,09163; \quad v_{in/c(1)}^{\wedge} = 1,75%; \quad (3.60)$$

$$a_{in/c(2)}^{\wedge} = 5,29766; \quad b_{in/c(2)}^{\wedge 2} = 0,00626; \quad b_{in/c(2)}^{\wedge} = 0,07909; \quad v_{in/c(2)}^{\wedge} = 1,49% \quad (3.61)$$

As before, to compare Variances take into use the χ^2 Test Criterion. This Criterion has the χ^2 -distribution with $(n_{in/c}-1)$ degrees of freedom (3.17) and null hypotheses-acceptance region $\Omega_{\chi} \in \{8,91...32,85\}$. So have sample estimations:

$$\chi_{(1)}^{\wedge 2} = (n-1) b_{(1)}^{\wedge 2} / b_0^2 = 19 \cdot 0,00840 / 0,00479 = 33,32 \quad (3.62)$$

$$\chi_{(2)}^{\wedge 2} = (n-1) b_{(2)}^{\wedge 2} / b_0^2 = 19 \cdot 0,00626 / 0,00479 = 24,83 \quad (3.63)$$

The criterion $\chi_{(1)}^{\wedge 2}$ for the 1st Order Statistics doesn't get to region of null-hypotheses acceptance, so the assumption of equality of experimental Variance of the 1st Order Statistics and theoretical ones is rejected. *Thereby is called in question and a hypothesis of independence of fatigue damages accumulation on system of twin “inner” concentrators.*

For check of hypotheses of equality of sample averages take into use the t-criterion of Student (3.20) with $(n_{in/c}-1)$ degree of freedom and null hypotheses-acceptance region $\Omega_t \in \{0...1,729\}$. The computations lead to

$$t_{(1)}^{\wedge} = (5,24662 - 5,22487) / 0,09163 \times \sqrt{20} = 1,06 \quad (3.64)$$

$$t_{(2)}^{\wedge} = (5,31951 - 5,29766) / 0,07909 \times \sqrt{20} = 1,23 \quad (3.65)$$

There are no any bases to reject the null- hypothesis about the equality of the Means of the 1st and 2nd Order Statistics of two “inner” concentrators’ durability’s.

On Fig. 3.7 the theoretical 1st and 2nd Order Statistics CDF similar to Fig. 3.4 are presented with experimental plots.

On Graphs it is seen the significant deviation of empirical plots from theoretical curves towards initial NDF. That allows making the assumption of existence of stochastic dependence between the moments of fatigue cracks emergence on “V”c)-type specimens i.e. *that accumulation of fatigue damages on this system of two “inner” of concentrators takes place stochastic dependent.*

Check this assumption.

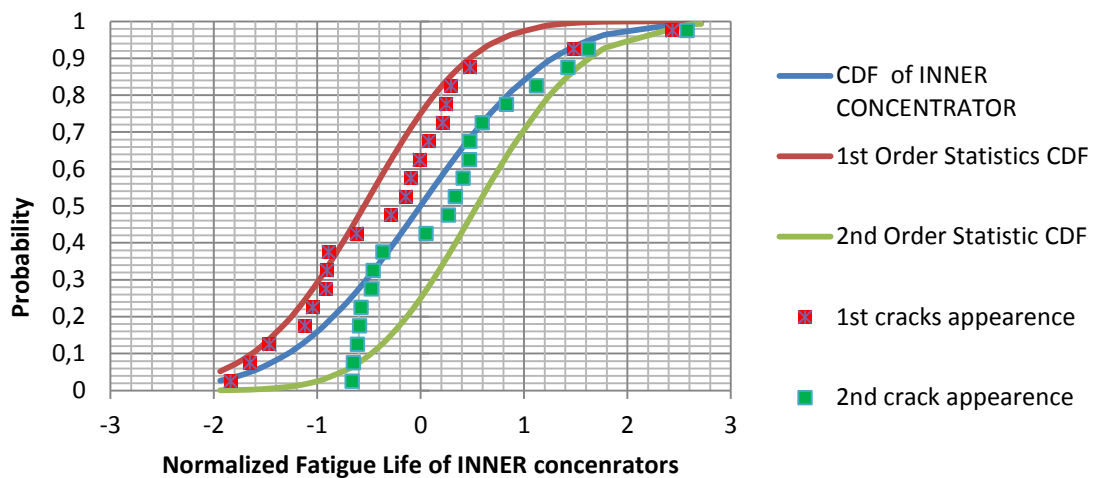


Fig.3.7. Standard Order Statistics Cumulative Distribution Function with Experimental Normalized Fatigue durability for Specimen with twin “inner” concentrators .

Let's continue the analysis of estimated dependence of fatigue damages accumulation of by comparison of experimental CC between the 1st and the 2nd Order Statistics with theoretical CC, that is equal $\rho_{0(1-2)} = 0,467$ (3.23).

As usually let's make a null hypothesis of the equality of experimental CC with theoretical one against alternative hypothesis $\rho_{in/c} \hat{=}_{(1-2)} > \rho_{0(1-2)}$. Results of tests and computation from (3.27), (3.28), and (3.27) give

$$n_{in/c} = 20 \quad (3.66)$$

$$\rho_{in/c} \hat{=}_{(1-2)} = 0,95541 \quad (3.67)$$

$$z \hat{=} = \frac{1}{2} \ln \left[\frac{1 + 0,95541}{1 - 0,95541} \right] = 1,89 \quad (3.68)$$

$$z_0 = \frac{1}{2} \ln \left[\frac{1 + 0,467}{1 - 0,467} \right] + 0,467/2(20-1) = 0,519 \quad (3.69)$$

$$T \hat{=} = \sqrt{(20-3)(0,95541-0,51852)} = 1,80 \quad (3.70)$$

The one side null hypotheses-acceptance region for the Fischer's T - criteria statistics (3.27) is equal $\Omega_p \{0...1,64\}$. It is seen that sample value of test statistic doesn't get to region of null hypothesis-acceptance. So the sample CC between the 1st and the 2nd Order Statistics is statistically significant and is more than the theoretical one.

It means that *significant positive stochastic relationship exists between the 1st and the 2nd Order Statistics*, so it is possible to draw a conclusion of dependent fatigue damages' accumulation on "V)c)-type specimen's concentrators.

Similar the estimate the ICC (3.33) presents:

$$b^2 = b_{in}^2 = 0,00781 \quad (3.71)$$

$$b_{\bar{\tau}}^2 = 0,00709 \quad (3.72)$$

$$r^{\wedge} = (2 \cdot 0,00709 / 0,00781 - 1) = 0,812 \quad (3.73)$$

$$z^{\wedge} = \frac{1}{2} \ln(1+0,812)/(1-0,812) = 1,133 \quad (3.74)$$

$$z_0 = 0 \quad (3.75)$$

$$T^{\wedge} = 1,133 \sqrt{(2 \cdot 20 - 3)/2} = 4,873 \quad (3.76)$$

The null-hypothesis-acceptance region for T-Criterion (2.17) is $\Omega_p \in \{-\infty, +1,64\}$. As the null hypothesis also is rejected in favor of alternative one, so we may draw a conclusion that the positive sample value of ICC is statistically significant more than zero.

So, the initial assumption of independence of fatigue damages' accumulation on twin "inner" concentrators located on width of working part of a specimen is rejected.

On such system of concentrators statistically significant positive correlation relationship is observed between the moments of fatigue cracks' emergence.

3.5.4 Results of fatigue tests of "V"d)-type specimens (two outer plus one inner concentrators)

Let's carry out the analysis of results of fatigue tests of 17 specimens like "V)d)-type (Fig. 3.1, d) with three concentrators located along a specimen (in direction of applied load) – one "inner" and two "outer". Results of fatigue tests are given in Table 3.4 (Appendix 4).

Such system of concentrators can be considered as association of two systems: systems with one "inner" the concentrator (with durability NDF parameters (3.54)) and systems with twin "outer" concentrators (with durability NDF parameters (3.6)).

Before starting research of dependence of fatigue damages accumulation on a "V)d)-type specimen, let's carry out a comparison of parameters estimates for the received NDF.

Analysis of data listed in Table 3.4 shows that the first fatigue crack appeared ONLY on “outer” concentrators of all 17 specimens. At the same time 11 out of 17 values of fatigue durability of the 3rd Order Statistic belongs to “inner” concentrators.

Let’s compare the fatigue durability’s of “inner” concentrators.

We have the evolution of parameters NDF (3.54) for “inner” concentrator and new test results for “inner” “V”d)-type concentrator (3.77):

$$a_{in} = 5,27219; \quad b_{in}^2 = 0,00703; \quad b_{in} = 0,08387; \quad v_{in/d}^{\wedge} = 1,59\%; \quad n_{in} = 60 \quad (3.54)$$

$$a_{in/d}^{\wedge} = 5,30935; \quad b_{in/d}^{\wedge 2} = 0,00625; \quad b_{in/d}^{\wedge} = 0,07908; \quad v_{in/d}^{\wedge} = 1,49\% \quad n_{in/d} = 17 \quad (3.77)$$

F-criterion Fisher (3.45) and null hypotheses-acceptance region $\Omega_F \in \{0... 2,11\}$ with $(n_{in} - 1; n_{in/d} - 1)$ degrees of freedom lead to:

$$F = b_{in}^{\wedge 2} / b_{in/d}^{\wedge 2} = 0,00703/0,00625 = 1,125 \quad (3.78)$$

and acceptance of the null hypotheses of Variables’ equality.

Sample t – statistic has the Student CDF with $(n_{in/d} - 1)$ degrees of freedom (3.20):

$$t^{\wedge} = (5,30935 - 5,27219)/0,07908 \times \sqrt{17} = 1,937 \quad (3.79)$$

decline the null hypotheses as her acceptance region is $\Omega_t: \in \{0...1,746\}$.

So the fatigue durability of “inner” concentrator on “V”d)-type specimens is significantly great than that of the isolated or twin “inner” ones.

For the fatigue durability of “outer” concentrators “V”d)-type specimens we have:

$$a_{ot/d}^{\wedge} = 5,17258; \quad b_{ot/d}^{\wedge 2} = 0,00916; \quad b_{ot/d}^{\wedge} = 0,0840; \quad v_{ot/d}^{\wedge} = 1,62\% \quad n_{ot/d} = 34 \quad (3.80)$$

compared it with (3.6):

$$a_{ot/b}^{\wedge} = 5,18012; \quad b_{ot/b}^{\wedge 2} = 0,01081; \quad b_{ot/b}^{\wedge} = 0,10397; \quad v_{ot/b}^{\wedge} = 2,00\%; \quad n_{ot/b} = 28 \quad (3.6)$$

Null-hypotheses-acceptance region is equal $\Omega_F \in \{0...1,76\}$ with $(n_{ot/b} - 1; n_{ot/d} - 1)$ degrees of freedom and the F-criterion (3. 45):

$$F^{\wedge} = b_{ot/b}^{\wedge 2} / b_{ot/d}^{\wedge 2} = 0,01081/0,00916 = 1,42 \quad (3.81)$$

do not decline hypothesis about the equality of Variances of durability of „outer” concentrators.

Similar if to compare the Means (3.20):

$$\hat{t} = (5,18012 - 5,17258)/0,0840 \times \sqrt{34} = 0,523 \quad (3.82)$$

The null hypothesis-acceptance region included the sample t-statistic, which does not reject a hypothesis about Means equality.

To pool these samples (3.6) and (3.80) together we get the parameters of durability NDF for “outer” concentrators:

$$a_{ot} = 5,17599; \quad b_{ot}^2 = 0,00934; \quad b_{ot} = 0,09665; \quad v_{ot} = 1,87\%; \quad n_{ot} = 62 \quad (3.83)$$

The Kramer-Mizes-Smirnov criterion (2.5) of matching experimental CDF to theoretical NDF is $W_{n.}^2 = 0.0987$.

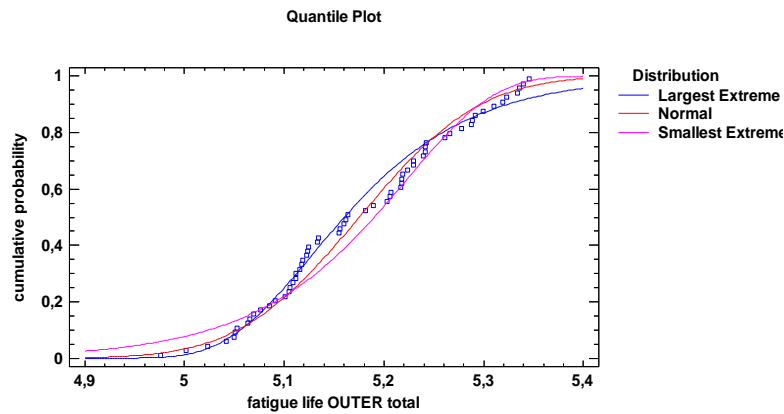


Fig.3.8 Cumulative Distribution Functions with Experimental fatigue durabilities Values for Specimen with „outer” concentrator (total n=62).

So we have received the estimations of parameters for durability NDF (3.1) of „outer” (3.83) and „inner” (3.54) concentrators. Now let’s carry out an correlation analysis of data for „V”d)-type specimens.

As it was already mentiond, the first moments of emergence of cracks ALWAYS were on „outer” concentrators (the 1st Order Statistic). Excluding from consideration specimens where the 2nd Order Statistic was observed on „inner” concentrators (NrNr 5, 10 ·13, 15, see Table 3.4 of the Appendix 4). The estimation of CC $\hat{\rho}_{ot(1-2)/d}$ between the 1st and the 2nd Order Statistics’ CDF for durabilities of „outer” of the concentrators:

$$\hat{\rho}_{(1-2)/d} = 0,587; \quad n = 11 \quad (3.84)$$

The theoretical value of CC is equal 0,467 (3.23).

Using the T-criterion (3.27) and null hypothesis-acceptance-region as $\Omega_p \in \{-1,96...+1,96\}$:

$$z^{\wedge} = \frac{1}{2} \ln[(1+0,587)/(1-0,587)] = 0,673 \quad (3.85)$$

$$z_0 = \frac{1}{2} \ln[(1+0,467)/(1-0,467)] + 0,467/2(11-1) = 0,530 \quad (3.86)$$

$$T^{\wedge} = \sqrt{(11-3)(0,673-0,530)} = 0,405 \quad (3.87)$$

see that

there is no bases to reject the null-hypothesis about the equality of the sample CC to theoretical for 1st and 2nd Order Statistic.

The theoretical parameters of the 1st Order Statistic's CDF for durability "outer" concentrators are known (3.9), (3.10):

$$a_{ot(1,2)} = a_{ot} \pm 0,5642 b_{ot} \quad (3.9)$$

$$b^2_{ot(1,2)} = 0,6816 b^2_{ot} \quad (3.10)$$

are equally (3.83):

$$b^2_{ot(1,2)} = 0,6816 \cdot 0,00934 = 0,00637 \quad b_{ot} = 0,07979 \quad (3.88)$$

$$a_{ot(1)} = 5,17599 - 0,5642 \cdot 0,07979 = 5,13097 \quad v_{ot} = 1,56\% \quad (3.89)$$

The sample parameters estimation by experimental data are:

$$a^{\wedge}_{ot/d} = 5,11363; \quad b^{\wedge}_{ot/d} = 0,00367; \quad b^{\wedge}_{ot/d} = 0,06062; \quad v^{\wedge}_{ot/d} = 1,19\%; \quad n = 17; \quad (3.90)$$

Compare (3.90.) with (3.88) and (3.89).

Variances' comparison by using the Test-Criterion χ^2 (3.17) [23] (χ^2 – variable value, having χ^2 CDF with (n-1) degree of freedom, in our case n=17) and applied the null hypothesis-acceptance region $\Omega_{\chi^2} \in \{0...; 26,30\}$ depicts

$$\chi^2 = (n-1) b^2_{ot(1)} / b^2_{ot/d(1)} \quad (3.17)$$

$$\chi^2_{(1)} = 16 \times 0,00934 / 0,00367 = 40,72 \quad (3.91)$$

Similar the Mean's comparison by using the t-criterion of Student (3.20):

$$t^{\wedge}_{(1)} = \sqrt{17} (5,13097 - 5,11363) / 0,00367 = 19,48 \quad (3.92)$$

that belongs to Student CDF with (n-1) degree of freedom and where the null-hypotheses-acceptance region is $\Omega_t \in \{0...1,746\}$ allows to made a conclusion that an alternative hypothesis of a divergence of experimental parameters estimates compare with theoretical ones is accepted.

So including into the twin „outer” concentrators system the third, „inner” one, leads to significantly decrease in Mean and Variance of he first crack durability.

Earlier it was shown that *the Mean durability of „inner” concentrator significantly increased in such system.*

This fact leads to the assumption about the existence of stochastic dependence on this system of concentrators.

Let's carried out the analysis of Order Statistics' CDF for the system of three concentrators (“V”d-type concentrators): two “outer” and one “inner” ones. As well as early, we will assume independence of fatigue damages accumulation on such system. Then this system can be considered as conjunction of two independent Systems having durability NDF (3.1) for each concentrator, i.e. durability NDF of two “outer” concentrators, with parameters (3.83) and systems with durability NDF of one “inner” concentrator, having parameters (3.54).

In the Attachment 6 the formulas' of the 1st, 2nd and 3rd Order Statistics derivation are presented for the observed system of concentrators:

$$F_{(1)}(\tau) = 1 - [1 - \Phi\{\frac{\tau - a_{in}}{b_{in}}\}][1 - \Phi\{\frac{\tau - a_{ot}}{b_{ot}}\}]^2 \quad (3.93)$$

$$F_{(2)}(\tau) = \Phi^2\{\frac{\tau - a_{in}}{b_{in}}\} + 2\Phi\{\frac{\tau - a_{ot}}{b_{ot}}\}\Phi\{\frac{\tau - a_{in}}{b_{in}}\}[1 - \Phi\{\frac{\tau - a_{ot}}{b_{ot}}\}] \quad (3.94)$$

$$F_{(3)}(\tau) = \Phi\{\frac{\tau - a_{in}}{b_{in}}\}\Phi^2\{\frac{\tau - a_{ot}}{b_{ot}}\} \quad (3.95)$$

where the initial NDF (3.1) parameters signified as (3.54) for “inner” and (3.83) for “outer concentrators are presented.

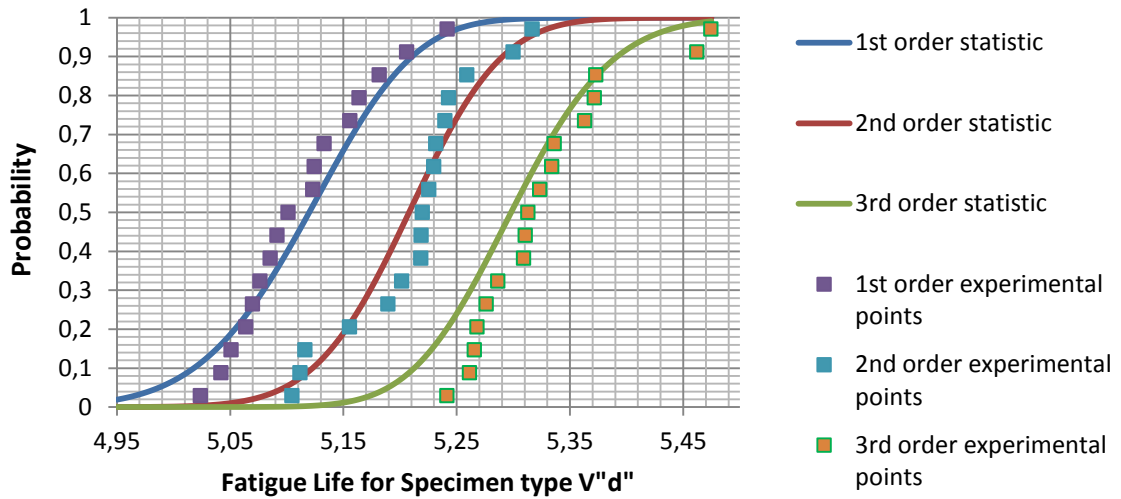


Fig.3.9 The Cumulative Distribution Functions of 1st, 2nd and 3rd Order Statistics of Fatigue Durability for twin “outer” and isolated “inner” Concentrators System

On Fig.2.9 it is seen that the experimental plots are deviate from theoretical Order Statistics’ CDF. It is may confirms an assumption of the stochastic dependence of fatigue damages accumulation on “V”d)-type specimen.

To evaluate this stochastic relationship let’s making the multidimensional correlation analyses [29].

In the general case the matrix of experimental plots is as follows:

$$X = \begin{pmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1k} \\ \dots & \dots & \dots & \dots & \dots \\ x_{n1} & \dots & x_{nj} & \dots & x_{nk} \\ \dots & \dots & \dots & \dots & \dots \\ x_{m1} & \dots & x_{mj} & \dots & x_{mk} \end{pmatrix} \quad (3.96)$$

If we have the sample of “n·k” plots x_{nk} for k-dimensional NDF with parameters vector (\bar{x}_i - means, s_i - variances) and correlation matrix R of CC_{ik} :

$$\bar{\mathbf{x}} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_k \end{pmatrix}, \quad \mathbf{s} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_k \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 1 & r_{12} & \dots & r_{1k} \\ r_{21} & 1 & \dots & r_{2k} \\ \dots & \dots & \dots & \dots \\ r_{k1} & r_{k2} & \dots & 1 \end{pmatrix}, \quad (3.97)$$

where the CC matrix R is symmetrical ($|r_{ij}| = |r_{ji}|$) and positive defined.

For the fatigue Test data of “V”d)-type specimens the CC matrix (k=3) is as follows:

$$\mathbf{R} = \begin{pmatrix} 1 & \rho_{ot/1-in/d} & \rho_{ot/1-ot/2} \\ \rho_{ot/1-in/d} & 1 & \rho_{in/d-ot/2} \\ \rho_{ot/1-ot/2} & \rho_{in/d-ot/2} & 1 \end{pmatrix} \quad (3.98)$$

and includes the paired CC r_{xy} [31]: “1st outer – inner”; “2nd outer – inner”; “1st outer -2nd outer”:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{(n-1)s_x s_y} \quad (3.99)$$

The CC $\rho_{ot/1-ot/2}$ have been evaluate early and theoretical is equal to CC of the 1st and 2nd Order Statistics for NDF with parameters (3.83) and independent of “inner” concentrator.

$$\hat{\rho}_{ot/1-in/d} = -0,151 \quad (3.100)$$

$$\hat{\rho}_{ot/1-ot/2} = 0,455 \quad (3.101)$$

$$\hat{\rho}_{in/d-ot/2} = -0,483 \quad (3.102)$$

It is seen that the $\hat{\rho}_{ot/1-ot/2} = 0,455$ is practically equal to theoretical value for CC of 1st and 2nd Order Statistics $\rho_{0(1-2)} = 0,467$ (3.23).

For the evaluation of others CC make use of Test Criterion t of Student. The t- statistic belongs to CDF of Student with (n-2) degree of freedom [31] and the null-hypothesis region $\Omega_t \in \{0 \dots 2,12\}$:

$$t = \rho \sqrt{(n-2)/(1 - \rho^2)} \quad (3.103)$$

The estimations are:

$$t_{ot/1-in/d} = -0,151 \sqrt{(17-2)/(1 - (-0,151)^2)} = 0,592 \quad (3.104)$$

$$t_{in/d-ot/2} = -0,483 \sqrt{(17-2)/(1 - (-0,483)^2)} = 2,13 \quad (3.105)$$

The statistical significant summation (3.105) leads to very important output: if negative correlation relationship is observed, the *earlier the crack is arise on system of concentrators, there the later will be cracks on other concentrators. And vice versa, the later the first crack i.e. arise, the earlier there will be cracks on the remained concentrators, i.e. at late emergence of the first crack it is possible to expect the avalanche stream of the subsequent cracks ("a Design's fatigue Overheat")*.

The partial CC (PCC) [31]:

$$\rho_{12|3} = (\rho_{12} - \rho_{13} \rho_{23}) / \sqrt{(1 - \rho_{13}^2)(1 - \rho_{23}^2)} \quad (3.106)$$

contrary to of pair CC ρ_{ik} (3.99), indicated the stochastic relationship between two random values (RV) if to reject the influence of the 3rd RV.

So we have:

$$\rho_{ot/1-ot/2|in} = (0,455 - (-0,151)(-0,483)) / \sqrt{(1 - (-0,151)^2)(1 - (-0,483)^2)} = 0,441 \quad (3.107)$$

$$\rho_{ot/1-in|ot/2} = (-0,151 - (0,455)(-0,483)) / \sqrt{(1 - (0,455)^2)(1 - (-0,483)^2)} = 0,088 \quad (3.108)$$

$$\rho_{ot/2-in|ot/1} = (-0,483 - (0,455)(-0,151)) / \sqrt{(1 - (0,455)^2)(1 - (-0,151)^2)} = -0,471 \quad (3.109)$$

The analysis of partial CC reveals that the moment of first crack emergence on “outer” concentrator actually is agreed with the theoretical 1st Order Statistics CDF (3.11), i.e. the durability of specimen doesn't depend on entering “inner” concentrator in to System. But at emergence of two cracks on “outer” concentrators the durability of “inner” the concentrator significant increases.

The t-criterion of Student (3.103)

$$t = -0,471 \sqrt{(17-2)/(1 - (-0,471)^2)} = -2,07 \quad (3.110)$$

if compared with the null-hypothesis region $\Omega_t \in \{0 \dots 2,12\}$ practically pointed to existence of negative correlation relationship on “V”d)-type specimen.

The analysis of multiply CC (MCC) $\rho_{1|23}$ [13] testifies the similar output (the MCC point to stochastic relationship of the single random variable (RV) with the group of all others). The MCC is always positive:

$$\rho_{1|23} = \sqrt{(\rho_{12}^2 + \rho_{13}^2 - 2\rho_{12}\rho_{23}\rho_{13})/(1 - \rho_{23}^2)} \quad (3.111)$$

Evaluations of MCC:

$$\begin{aligned} \rho_{ot/1 | ot/2-in} &= \sqrt{(\rho_{ot/1-ot/2}^2 + \rho_{ot/1-in}^2 - 2\rho_{ot/1-ot/2}\rho_{ot/1-in}\rho_{ot/2-in})/(1 - \rho_{ot/2-in}^2)} = \\ &= \sqrt{(0,455^2 + (-0,151)^2 - 2 \cdot 0,455(-0,151)(-0,483))/(1 - (-0,483)^2)} = 0,462 \end{aligned} \quad (3.112)$$

$$\begin{aligned} \rho_{ot/2 | ot/1-in} &= \sqrt{(\rho_{ot/1-ot/2}^2 + \rho_{ot/2-in}^2 - 2\rho_{ot/1-ot/2}\rho_{ot/1-in}\rho_{ot/2-in})/(1 - \rho_{ot/1-in}^2)} = \\ &= \sqrt{(0,455^2 + (-0,483)^2 - 2 \cdot 0,455(-0,151)(-0,483))/(1 - (-0,151)^2)} = 0,619 \end{aligned} \quad (3.113)$$

$$\begin{aligned} \rho_{in | ot/2-ot/1} &= \sqrt{(\rho_{ot/1-in}^2 + \rho_{ot/2-in}^2 - 2\rho_{ot/1-ot/2}\rho_{ot/1-in}\rho_{ot/2-in})/(1 - \rho_{ot/2-ot/1}^2)} = \\ &= \sqrt{(-0,151^2 + (-0,483)^2 - 2 \cdot 0,455(-0,151)(-0,483))/(1 - (-0,455^2))} = 0,489 \end{aligned} \quad (3.114)$$

The checking-up the null hypothesis of equality the experimental values of MCC to zero by F-Criterion of Fisher [13]:

$$F = \rho_{1|23}^2 / (k-1) / (1 - \rho_{1|23}^2) / (n-k) \quad (3.115)$$

So is finding:

$$\hat{F}_{ot/1} = 0,462^2 / (3-1) / (1 - 0,462^2) / (17-3) = 1,90 \quad (3.116)$$

$$\hat{F}_{ot/2} = 0,619^2 / (3-1) / (1 - 0,619^2) / (17-3) = 4,35 \quad (3.117)$$

$$\hat{F}_{in} = 0,489^2 / (3-1) / (1 - 0,489^2) / (17-3) = 2,20 \quad (3.118)$$

The null hypothesis-acceptance region for Fisher's F-Criterion (3.115) with (k-1; n-k) degree of freedom is $\Omega_t \in \{0...3,74\}$. The null hypothesis is rejected for the MCC of second "outer" concentrators. Thus existence of negative correlation relationship on "V"d)-type specimens receives one more confirmation. Emergence of cracks on the first concentrators SLOWS DOWN, (if negative stochastic relationship exists) on other concentrators. Thereby the first emergence of cracks on the outer concentrators forming some zone of the accelerated generation of fatigue cracks (positive correlation relationship) and a zone of their delay.

The additional analyses we have make using the intraclass correlation coefficient (ICC) (3.33). In the case of the "V"d)-type specimen we have the class quantity k =3 (the number of concentrators on specimen).

So:

$$b^2 = 0,01140; \quad b^2_{\bar{\tau}} = 0,00125; \quad r^2 = \frac{1}{2} (3 \cdot 0,00125 / 0,01140 - 1) = -0,336^9 \quad (3.119)$$

If $k > 2$ the theoretical CDF (or its transforming) of ICC is hard to describe. But it is known the proportional approximation [17]:

$$dF(r) \propto \frac{(1-r)^{\frac{n(k-1)}{2}-1} [1-(k-1)r]^{(n-3)/2}}{[1-\rho + \rho(k-1)(1-r)]^{(nk-1)/2}} dr. \quad (3.120)$$

where r – the sample estimate of ICC;

ρ – the theoretical value of ICC;

k – the number of classes;

n – the number of experimental plots.

In our case if $\rho = 0$, $k = 3$, $n = 17$ the expression (3.120) takes on:

$$dF(r) \propto (1-r)^{16} (1-2r)^7 dr. \quad (3.121)$$

Computing dF/dr and normalizing the area under a curve of CDF to 1, we may receive that critical value ICC is equal to $r_\alpha = -0,10$ at a significance level $\alpha = 0,05$. Then the experimental value of the sample ICC (3.119) lies outside a null hypothesis-acceptance region.

⁹ It is necessary to notice that the interclass coefficient of correlation is asymmetrical concerning zero. At the same absolute values of ICC negative values express stronger dependence in comparison with positive ones. As shown in [17] the region of ICC existence is presented: $-1/(k-1) \leq r \leq 1,0$

Thus, for System of three concentrators on “V”d)-type specimen, two “outer” and one “inner”, the stochastic dependence of the moments of fatigue cracks’ emergences is confirmed. The failure of the concentrators located in the direction (along) of operating loading is characterized by negative correlation relationship. It means that concentrators on System (if there stochastic dependent relationship is existed) make a whole united body. If the additional concentrator is introduced into System, the durability of such System happens differently, than on a hypothesis of the weakest link.

3.6 The analysis of correlation relationships for Order Statistics’ CDF of multidimensional normal distribution

It is known, that the density of probability multidimensional NDF is presented as:

$$f(T) = \frac{1}{\sqrt{(2\pi)^n [R]}} \exp(-1/2 \cdot T R^{-1} T') \quad (3.122)$$

where $T' = \{\tau_1; \tau_2; \dots \tau_n\}$ – column-vector of standard values;

R – correlation matrix (2.97);

R^{-1} – matrix inverse of the matrix R ;

$[R]$ – discriminant of matrix R .

At first let’s consider the simplest case of two-dimensional NDF ($n=2$).

If CC is equal to zero ($\rho = 0$), the Order Statistics’ CDF are expressed as (3.7) and (3.8).

If $\rho = +1,0$ the two-dimensional NDF degenerates into one dimensional NDF (3.1.) with Order Statistics’ CDF (3.7) and (3.8).

If $\rho = -1,0$ the Order Statistics’ CDF are expressed as [1]:

$$F_{(1)}(\tau) = 2\Phi(\tau) \quad \text{if } \tau \leq 0 \quad (3.123)$$

$$F_{(1)}(\tau) = 1,0 \quad \text{if } \tau > 0 \quad (3.124)$$

$$F_{(2)}(\tau) = 0 \quad \text{if } \tau \leq 0 \quad (3.125)$$

$$F_{(2)}(\tau) = 2\Phi(\tau) - 1 \quad \text{if } \tau > 0 \quad (3.126)$$

For the others values of CC ρ from $-1,0$ to $+1,0$ let’s use two-dimensional NDF [16, 28] with correlation matrix R (3.97) if $\kappa = 2$ and density of probability distribution function (3.122) if $n = 2$.

On Fig.3.10 there is illustrated the Order Statistics’ CDF if $\rho = -0,7$ and $\rho = +0,7$, as well if $\rho = -1,0$ и $\rho = +1,0$.

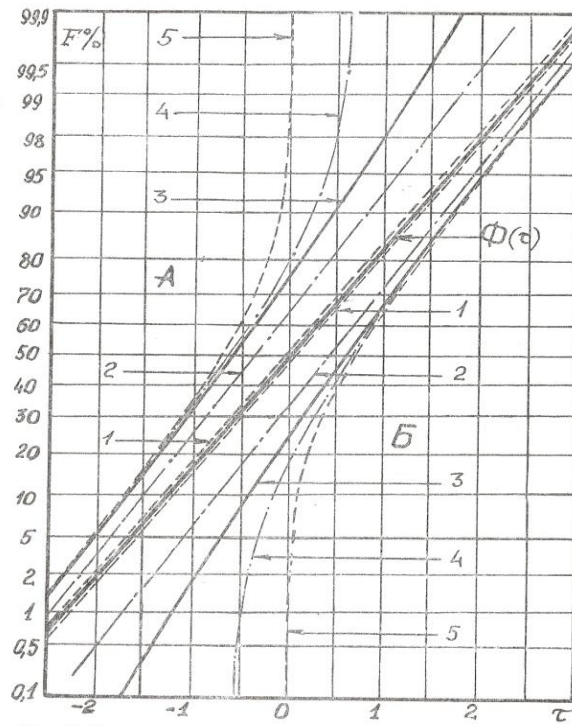


Fig.3.10 Initial Standard NDF $\Phi(\tau)$ and CDF of 1st (A) and 2nd (B) Order Statistics of two dimensional NDF with CC: 1) $\rho=+1,0$; 2) $\rho=+0,7$; 3) $\rho=0$; 4) $\rho=-0,7$; 5) $\rho=-1,0$

The Fig.3.10 illustrates, that in System with two elements (concentrators) if positive CC is increased the 1st Order Statistic's CDF converges to initial NDF, i.e. the probability of the first refusal decreases, the second - on the contrary - increases. At negative values of CC on the contrary - probabilities of the moment for the first refusal increases, and the second - decreases. It is possible to make the assumption that *early emergence of damages localizes a place of it development and origin the others, whereas the later emergence of damages promotes the emergences of the new.*

The expression of the Order Statistics of multidimensional CDF, even for family of NDF, in an analytical format is interfaced to great difficulties, therefore for the analysis of such distributions let's applies methods of statistical modeling (the principle of "Monte-Carlo") [12, 79].

Let's consider positive defined two correlation matrix with three elements having positive and negative CC:

$$R_1 = \begin{bmatrix} 1 & 0,5 & 0,5 \\ 0,5 & 1 & 0,5 \\ 0,5 & 0,5 & 1 \end{bmatrix} \quad (3.127)$$

$$R_2 = \begin{bmatrix} 1 & -0,5 & 0,5 \\ -0,5 & 1 & -0,5 \\ 0,5 & -0,5 & 1 \end{bmatrix} \quad (3.128)$$

and two correlation matrix with five elements:

$$R_3 = \begin{bmatrix} 1 & 0,5 & 0,5 & 0,5 & 0,5 \\ 0,5 & 1 & 0,5 & 0,5 & 0,5 \\ 0,5 & 0,5 & 1 & 0,5 & 0,5 \\ 0,5 & 0,5 & 0,5 & 1 & 0,5 \\ 0,5 & 0,5 & 0,5 & 0,5 & 1 \end{bmatrix} \quad (3.129)$$

$$R_4 = \begin{bmatrix} 1 & -0,5 & 0,5 & -0,5 & 0,5 \\ -0,5 & 1 & -0,5 & 0,5 & -0,5 \\ 0,5 & -0,5 & 1 & -0,5 & 0,5 \\ -0,5 & 0,5 & -0,5 & 1 & -0,5 \\ 0,5 & -0,5 & 0,5 & -0,5 & 1 \end{bmatrix} \quad (3.130)$$

For each of positions there were formed 10 samples with volume of 25 plots.

The results of statistical simulation is illustrated on Fig.3.11

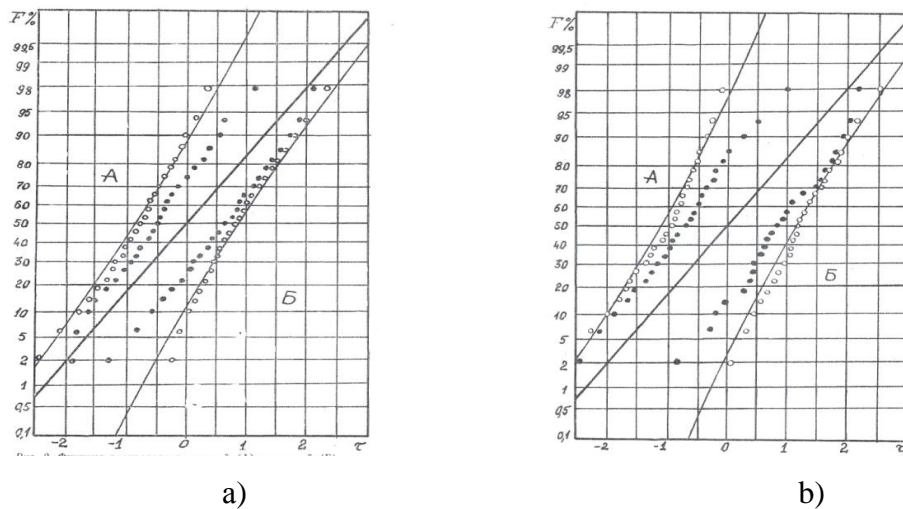


Fig. 3.11 Order Statistics' CDF a) the 1st (A) and the 3rd (B) for correlation matrixes of R1 - • (3.127) and R2 - ◦ (3.128), b) the 1st (A) the 3rd (B) for correlation matrixes of R3 - • (3.129) and R4 - ◦ (3.130).

Compared together the Order Statistics two dimensional CDF with three- and five dimensional CDF there is not the principal differenced of its view.

3.7 Mathematical model of stochastic relationship's influence area on System of concentrators

3.7.1 Mathematical model of stochastic relationship's zone dimensions.

Early it was shown the existence of stochastic relationships between the moments of fatigue cracks' emergence on the System of concentrators. Also the direction of these relationships - lengthways (negative correlation relationships) and across (positive correlation relationships) applied loading are shown. Thereby some zone in which accumulation of fatigue damages happen stochastic dependent is formed. It is interesting to determine the sizes of such zone.

For an evaluation of distance in System of concentrators on which stochastic relationships act, the following mathematical model of this area is offered.

The assumption about the existence of such zones have been proposed by the Hypothesis of Local Zones (paragraph 2.4) and proved of fatigue tests of "V"-type specimens results.

Let we consider some area S_0 of the object interested. This area is formed by equal magnitude micro elements ΔS_i . There is interdependent accumulation of the fatigue damages in each of micro elements under the action of the cyclic loads. We make the following suggestions:

- the distance between the centers of microelements is Δl ;
- for simplicity and without loss of generality, we consider the case of one dimension l ;
- the durability of whole zone S_0 is determined by the longevity of its constituent micro elements due to the loaded standby model [46].

Let suppose that fatigue damages are gradually accumulated in micro elements along dimension l .

Assume that the correlation coefficient $R(l)$ of fatigue durability for two micro elements at distance l from each other, so as $R(0) = 1$, if $l = 0$, but the correlation coefficient $\rho(\bullet)$ of fatigue durability for two adjoining micro elements at distance Δl from each other and at distance l from the first damaged micro elements is dependent of number $n = l/\Delta l$ micro elements along dimension is where

$$\rho(l + \Delta l) = 1 - \mu(l)\Delta(l) \quad (3.131)$$

where $\mu(0)=0$.

Assume that this relation is correct :

$$R(l + \Delta l) = \rho(l + \Delta l)R(l) \quad (3.132)$$

or inserting (3.131) to (3.132)

$$R(l + \Delta l) = R(l) - R(l)\mu(l)\Delta l \quad (3.133)$$

and transforming (3.133):

$$\frac{R(l+\Delta l)-R(l)}{\Delta l} = -R(l)\mu(l) \quad (3.134)$$

Assuming Δl in (3.134) going to zero, we have

$$\frac{dR(l)}{dl} = -R(l)\mu(l) \quad (3.135)$$

Solving the resulting differential equation (3.135) for $R(l)$, we have

$$R(l) = \exp\left\{-\int_0^l \mu(x)dx\right\} = e^{-\varphi(l)} \quad (3.136)$$

Thus, we receive an expression how the force of the stochastic relationship determined by CC, is depended on l distance. For an estimation of parameters of model (3.136) we will consider results of "W"-type specimens (Fig.3.1 e, f), where constrain stochastic force and its direction we will determine by change of interclass coefficients of correlation (333).

3.7.2 Analysis of fatigue tests results for "W"-type specimens

3.7.2.1 Conditions of carrying out tests

Unlike the previous tests (paragraphs 3.2, 3.4), fatigue cracks were formed at the both sides of the hole concentrators with different diameters (Fig.3.1 e, f) - on the external and the internal side of the specimen (two cracks in each of the hole concentrator). Number of specimens of each type: - with a hole diameter of 3, 10 mm (eight concentrators) - five specimens, and with a diameter of 8 mm (eight concentrators) and 20 mm (4 concentrators, Fig.3.1 f) - four specimens. The fatigue cracks were emerged on the holes sides both as internal and external ones, perpendicular to the action of applied loading. Tests were carried out in the hydraulic testing machine, generates the cyclic tension loading with frequency of about 12 Hz.

Two regimes of the regular cyclic load have been chosen [67]:

$$\begin{aligned} \sigma_{\max} &= 120 \text{ MPa}, & \sigma_{\min} &= 0 \text{ MPa}, \\ \sigma_{\max} &= 120 \text{ MPa}, & \sigma_{\min} &= 60 \text{ MPa}, \end{aligned}$$

The results of fatigue test are listed in Table 3.5 (Attachment 5).

3.7.2.2. *The value of the correlation coefficient versus the diameter of the concentrator*

In the previous paragraph it is shown that the most effective device of the analysis of stochastic relationships in Systems of concentrators is the interclass coefficient of correlation (3.33).

We'll investigate the diameter of the hole (Fig.3.1.e) versus the value of ICC. To do this, the values of durability (the logarithm of the number of cycles) until the fatigue cracks origin's moment on both sides of the hole concentrator is studied as one class $k=2$ (the number of concentrators)

As for testing the hypothesis that the ICC is significantly different from zero, let's apply a criterion statistics T (3.39):

$$T = (z - z_0) \sqrt{\frac{2n-3}{2}} \quad (3.39)$$

where $z = \frac{1}{2} \ln \frac{1+r}{1-r}$;

$z_0 = 0$;

n – number of observations;

\hat{r} – sample value of ICC.

Table 3.6

ICC between the fatigue cracks origin's moments versus the diameter of the hole concentrators

The diameter of the hole concentrator, mm	The sampling ICC (3.33) \hat{r}	Sample size	Criterion statistic (3.39) T	The critical value of the criterion statistic [28]
3	0,91	40	9,48	2,02
8	0,76	32	5,50	2,04
10	0,63	40	4,60	2,02
20	0,28	16	1,10	2,13

As seen from Table 3.6, the values of the sampling of ICC shows significant positive interdependence of the emergences of fatigue cracks on the sides of the holes, i.e. the first emergence of a fatigue crack on the side of the hole provoke second crack at the opposite side, but this dependence decreases with increasing hole diameter and virtually disappears at large diameter holes.

So we can draw two conclusions:

- for small diameter holes the second crack can be considered as a propagation of the first;
- for large diameter holes the both emerged cracks are independent and all cracks can be considered as one sample.

3.7.2.3. The distance between the concentrator's centres versus the force of stochastic relationships

To investigate how the distance between holes centres affects on the dependence of the emergence of fatigue cracks on them, an experimental data of fatigue test results of „W”f-type specimens (Fig.3.1 f) were analysing. As shown above, the moments of fatigue cracks origins at the hole sides can assume stochastic independence.

In the Table 3.7. are listed the statistical estimations of ICC for grouped of two-, three-, and four hole concentrators.

Table 3.7

The ICC between the fatigue cracks origin's moments versus the distance between the centers of hole concentrators.

Space between the hole's concentrators centers, mm	Number of concentrators in the class, (3.33) k	Sampling intraclass correlation coefficient (3.33) \hat{r}	Sample size	Criterion statistics (3.39) T	The critical value of the criterion statistics [28]
50	2	-0,80	24	4,99	2.07
100	3	-0,45	48	1,89	2,01
150	4	-0.29	32	1,13	2,04

As well as the research of stochastic relationships on „V”d-type specimens with the concentrators located along working part of specimen, the test results of „W”f-type specimen prove the existence of the negative correlation relationships decreasing in process of increase in distance.

3.7.3 Assessment of parameters of mathematical model of the sizes of a zone of stochastic dependence

Let's determine a kind of function $\varphi(l)$ in expression (3.136). For this purpose take the logarithm of both sides of equation (3.136), where in the left part there will be numerical values of ICC, and in right one - distances corresponding to them between points of emergence of fatigue cracks (centers of concentrators) (Table 3.10).

As can be seen from Table 3.7 with increasing distance between the centers of the holes (concentrators), the dependence of the emergences of fatigue cracks in them is weakened. Early, it have been noted, that the ICC is not symmetric about zero, so a negative relationship is more sensitive compared the same absolute values than the ICC for positive relationships. [17].

Table 3.8

ICC versus the distance between the fatigue cracks emergences.

	Perpendicular the applied load				Along the applied load		
Diameter l , mm	3	8	10	20	50	100	150
$\ln(l)$	1,10	2,08	2,308	3,0	3,91	4,61	5,01
$R(l)$	0,91	0,76	0,62	0,28	-0,80	-0,45	-0,29
$\ln(-\ln(R))$	-2,36	-1,29	-0,74	0,24	-1,50	-0,23	0,21

"**Bold**" values of interclass coefficients of correlation are specified as statistically significant other than "zero"

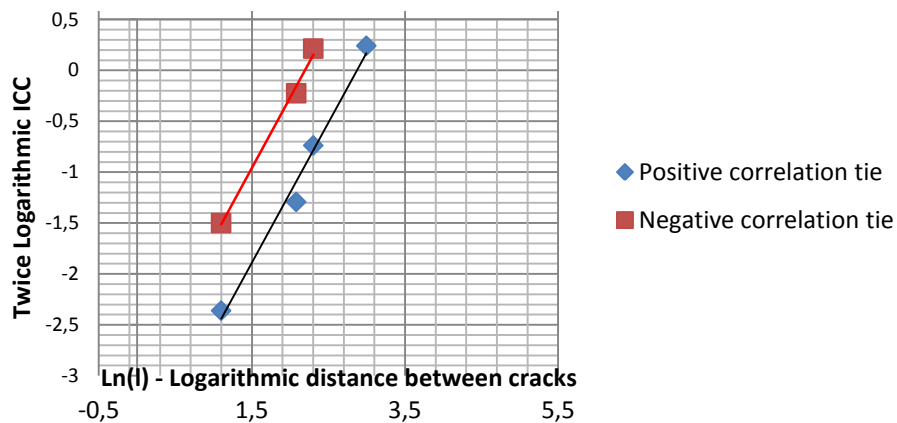


Fig.3.12 The Distance between cracks location versus the ICC values (logarithmic coordinates)

On both graphs, we see almost linear dependence of the ICC varying versus distance between the fatigue cracks origins in logarithmic coordinates.

When we can suppose the following:

$$\text{Ln}(-\text{Ln}(R(l))) = \text{Ln}C + m\text{Ln}(l) \quad (3.137)$$

where $\text{Ln}(\bullet)$ – is the logarithm operator

C, m - parameters of the linear functions Fig.3.12

Comparing (3.136) and (3.137) we get

$$R(l) = \exp(-\varphi(l)) = \exp(-Cl^m) \quad (3.138)$$

and thus determinate the function - $\varphi(l)$ (3.129):

$$\varphi(l) = Cl^m \quad (3.139)$$

So we may propose the following procedures of determination of distance, along with the fatigue damage emergences is occur dependently:

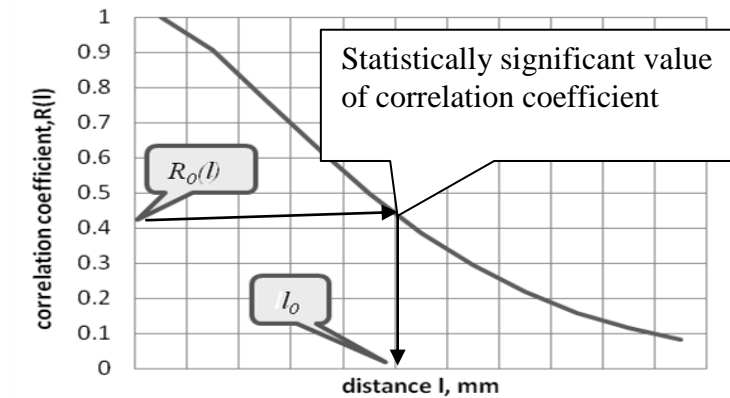


Fig.3.13 The ICC varying versus on the distance between the fatigue damages location according to the mathematical model (3.138)).

$R_0(l)$ - the ICC value *statistically significant different from zero*;

l_0 – the dimension of local zone with dependent fatigue damage's accumulation.

Analyzing the (3.139), we can show that the parameter m describes the curvature of graph or how the ICC is reducing, but the parameter C – shows the power of the stochastic relationships (as less as C - the relationship is expansioned over a greater distance).

Let's consider the (3.138) and determine the statistical estimation of parameters C and m for the linear regression model (3.137) with positive and negative ICC (Fig.3.12). In this case if the correlation coefficient have negative sign we take absolute values of this correlation coefficient.

So we have the following statistical estimations:

- for positive correlation relationship (perpendicular to applied force):

$$\hat{C} = 0,0192; \quad \hat{m} = 1,3716; \quad (3.140)$$

- for negative correlation relationship (along to applied force):

$$\hat{C} = 0,00047; \quad \hat{m} = 1,590; \quad (3.141)$$

As usual, let assume the significance level $\alpha = 0,05$ for testing hypothesis that the correlation coefficient is significantly is different from zero. Then the critical value of the criterion statistics $T = 1,746$ (Student T-test criterion (3.39) with degree of freedom $df=16$) for $R_0(l)$:

$$R_0(l) = 0,458; \quad (3.142)$$

Substitute $R_0(l)$ from (3.142) and \hat{C} with \hat{m} from (3.140) to (2.137). Solving the resulting equation for l , we determinate the size l_0 of the zone in the horizontal direction:

$$l_{0hor.} = 14,9 \text{ mm}$$

Similarly, to determine the vertical size of the zone (substitute $R_0(l)$ from (3.142) and \hat{C} with \hat{m} from (3.141) to (3.137). and solving the resulting equation for l):

$$l_{0vert} = 106,0 \text{ mm.}$$

Thus we may suppose that inside this zone where are dependent accumulation of fatigue damage.

It should be noted that this result can be referred only to fatigue tests results of "W"-type specimens and has more qualitative character. Nevertheless, it is possible to claim, as in a

case with “V”-type specimens, that the negative stochastic relationship (along the applied load) extends on the greater distance, than positive (across the applied load) one.

3.8 Some conclusions by the stochastic analysis of fatigue tests results of specimens

By the results of fatigue tests it is possible to draw the following conclusions:

- a mathematical model for the dependent fatigue damage zone is represented;
- as an analogue of the zone with numerous fatigue damage the specimens with multi-hole concentrators have been testing till fatigue cracks were originated on one or both sides of concentrators. The test results showed a satisfactory agreement of experimental data with theoretical predictions;
- the estimation of model parameters and the determination of the zone sizes with the dependent FDA are executed. Shown that the dependence of FDA has both positive (across the direction of applied load) and negative (along the direction of applied load) correlation relationship. Negative correlation relationship is more pronounced and extends over a greater distance;
- positive correlation relationship implies that when a first crack is originated the next positive correlated fatigue cracks may be considered as the previous (initial) one’s propagation, subsequently merging to one main crack (The “airplane fatigue destruction model” – one main crack propagation);
- negative correlation relationship implies that the probability of an occurrence of new cracks in the dependent zone drops, thereby locating the seat of first crack. But thus it is allowing the new cracks occur outside this zone. (The “helicopter fatigue destruction model” – as the flow of failures with the appearance of numerous cracks);
- existence of two compatible perpendicular correlation relationships of FDA leads to formation of some area or a Local Zone of dependent FDA.

The main conclusion of the fatigue tests analysis for specimen’s with numerous concentrators is seal of approval the Hypothesis of Local Zones with Dependent Accumulation of Fatigue Damages.

3.9 Results and consequences of the Hypothesis of Local Zones Existence

3.9.1 The correlation relationships on the System of concentrators

In the previous paragraphs the obtained experimental data showed existence of stochastic relationships on Systems of concentrators. Positive correlation relationship works in the direction perpendicular to action of the applied load, and negative correlation relationships

works along the applied load. The distance of action of negative relationship is several times more, than positive one. That forms some zone which confirms one of postulates of a Hypothesis of Local Zones (paragraph 2.2). Schematically action of these relationships is shown in fig. 3.13

3.9.1.1 Action of positive correlation relationship

Dependence of FDA on concentrators located on width of working part of a specimen, has nature of positive correlation relationship. Existence of positive correlation relationship between the durability's of concentrators' means, that the destruction of one (first) of concentrators accelerates destruction of others. Therefore, the increasing the number of concentrators on specimen width (in the direction the perpendicular of operating loading) sharply reduces durability of the specimen at destruction of the first of concentrators.

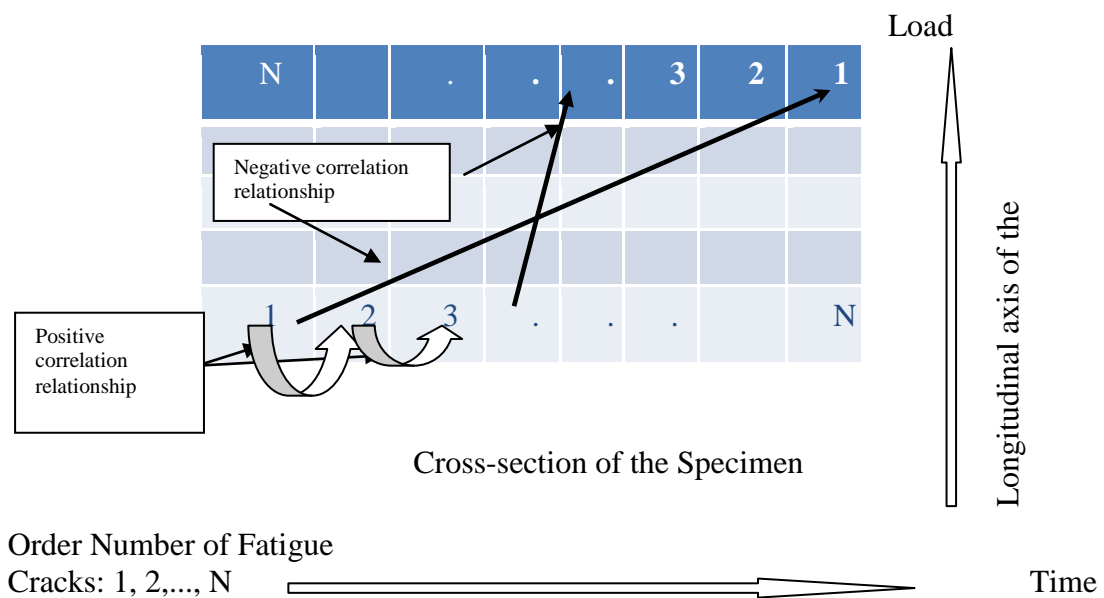


Fig 3.14 Stochastic cross-coupling of the fatigue damages origin in the Local Zone due to the direction of the applied loading

Consecutive destruction of the concentrators may be considered as propagation of the first arisen crack. Let in some micro zone the early on time damage first will arise (on the Fig.2.13 at number 1 on the bottom line of the provided scheme). In the presence of positive correlation relationship across a vector of applied load, the probability of the subsequent of damage, near the first one sharply grows. At emergence of the second (near by the first) crack a high probability also to arise to the third, etc. It is shown that in this case emergence of the subsequent, after the first, damages it is possible to consider as their merge, and this merge not necessarily has to be consecutive. The arisen flow of damages forms the local

center of future crack and defines (in the area of a congestion of damages) character and speed of its propagation.

It is obvious that early emergence of damages leads to fast formation of a stage of the main crack and the subsequent destruction of the loaded Design. Such stage of FDA evolution is characteristic for high level of studied Design loading. If the first damage to this area arises late, and the subsequent damages of this section will arise a little later, reducing probability of emergence of the main crack.

Positive cross-correlation relationship supposes that before there the fatigue crack will have been originated, the early the subsequent crack will be found out in direction perpendicular to the applied load. Such sequence of fatigue cracks' originations assumes a mode as a propagation of the first crack, converting it *to the only main spread out one*. So therefore this meeting crack's sequence will determinate the durability of all zone.

3.9.1.2 Action of negative correlation relationship

Dependence of FDA on the concentrators located along working part of a specimen, or along action of applied loading, has negative nature of correlation relationship.

In the considered area the early appeared damage provokes emergence of the subsequent damages, but existing stochastic negative relationship detains emergence of new damages in *other* areas, localize future centers nearby the first damage. It is a case that leads to formation of several zones, and at severe loading – to multifocal destruction of the considered section.

At late emergence of damage in considered area, negative stochastic relationship will provoke earlier emergence of crack in other areas. Emergence of the subsequent cracks along the direction of applied load, subjects less possibly, than earlier the first crack will be found.

Vice versa, the later the first crack will emerge, the probability of emergence of the subsequent cracks along the applied load that can lead to formation of numerous zones of emergence of cracks, is higher. It means that the extension of concentrators' number along the specimen length (in the direction of operating load) doesn't lead to essential change of durability of whole specimen. But there is expected the accelerated raise of damages flow throughout the Design loaded area with concentrated some local zones..

There is a paradoxical situation: *when the earlier crack emerges it is allows to provide monitoring of its propagation, then the crack arises later is may be the signal to the*

accelerated cracking of overall of Design. But this paradox allows to formulate the main basis of Fail Safe Concept.

3.9.2 Consequences of a Hypothesis of Local Zones

As a result of the above it is possible to draw the following consequences of HLZ:

- the Design durability is determined by the durability of its Local Zones. This zone has to be put during operation of Design under “on-line” NDT monitoring. Such monitoring only for one zone substantially reduces temporal, material and still human expenses during its maintenance, operation and service;
- the Local Zone needs repair entire its area as a one (first) crack is revealed. That crack will initiate other cracks in this Local Zone (as first crack’s propagation) due to positive correlation relationship of FDA;
- knowing the loading conditions of the Design and its “weak place” or finding out the first fatigue damages in her, it is possible to forecast their propagation, supposing expecting subsequent flow of damages, as propagation of one only main crack, that determines the endurance of this zone;
- the sizes of a Local Zone can exceed the extent of studied Design. In this case it is possible to determine minimum possible size of a tested specimen necessary for an evaluation of its reliability;
- considering full-size natural Designs as a pool of Local Zones with known nature of formation of fatigue cracks (see Chapter 1), there is possible to concentrate check-up and NDT monitoring for early detection of a crack resources on one (critical) zone, up to the possibility of its controlled destruction as Fail Safe Concept;
- the FDA allows maximizing application of NDT methods for early detection of cracks (especially in “on-line” mode at Bench Tests). Monitoring of these zones, under certain conditions of early detection and check-up of crack propagation (a strain review of stresses for a dangerous zone, the estimation of survivability of a Design with a controlled growth of a crack, an opportunity of Safe Destruction of this zone at operating time, propagation of a crack to natural cracks stoppers: stringer, the frames, the increased metal consumption etc) allows to transfer to the principle of Safe Destruction of separate strictly controlled zones;
- accepting the Hypothesis of Local Zones with a dependent FDA in them, it is possible to explain (until now unexplained) the phenomenon of scale effect at that it is less expressed on length of the specimens (along the applied force) when fatigue tested, what on his width

(across to the applied load) [5, 49], and the existence of S-N curves breaks [33, 51, 56, 70] – when the destruction of specimens is occur under the different mechanisms of FDA in different local zones), and the variety of fatigue endurance CDF expressions, frequently used only for experimental data interpolation and smoothing, but out of these fatigue tests data analyses of insensitive physical (stochastic) processes [55];

- it is also interesting, that if the destruction probability of Structural Significant Element is lowered, the total risk of destruction of the Design does not go down! - because, at presence of negative relationship, than later it will be found out the first damage, the early will be originated the next ones, that *results in speed-up and out-of-control destruction* (so-named “fatigue overheating”) after the protracted period of zero-defects operation;

- it is possible to draw conclusion from here, that in a full-size Designs the first arising up fatigue crack locks on itself a power (energy) flow and it is high probability to accumulate the fatigue damages in a Local Zone perpendicular to this power flow. But the probability of appearance of potential fatigue damages to direction along to the power flow is reducing and strongly enough on greater distance;

- the gradual FDA in one of Zones with positive correlation relationship allows to consider this Zone as ONE GLOBAL DAMAGE that means opportunity, and it is frequent also need, repair of ENTIRE INITIAL ZONE, instead of its separate cracks in it;

- the Hypothesis of Local Areas allows formulizing the one of the principle of Fail Safe Concept:

it is necessary to develop a Design so that in her certain part (Local Zone) a damages' accumulation happens as accelerated flow, but the destruction of this Zone didn't lead it to destruction of entire Design. It is desirable the available NDT monitoring of this Zone.

Note.

Generally speaking, the Hypothesis of Local Zones carries broader sense and an explanation. Accumulation of damages (degradation) of complex technical system has such phenomenological character: gradual failure of separate elements of System can be interdependent stochastic. Knowing the direction and a sign of these relationships it is possible to predict the subsequent refusals of elements. So it may be its localization in some Zones: the *Zones of Special Attention* in which also it is necessary to establish NDT monitoring of a technical condition an “on-line” mode.

Existence of such zones explains the nature of large-scale effect at increase in the sizes of system. If the element of System doesn't exceed the sizes of a Local Zone, characteristics of its reliability correspond to characteristics reliability of a Local Zone. If the sizes of an element exceed the sizes of a Local Zone, the interference of such zones CHANGES the characteristics of reliability of total System. And it can concern not only technical, but also other Systems where events have stochastically dependent character. Apparently the HLZ explains also results received [69], when the equal stressed design has durability below a similar design with concentrators.

It is necessary to take into account that arising up and spreading fatigue cracks may change a direction of power stream (due to the redistribution of stresses) and configuration of the Local Zone.

Accepting a hypothesis, that reliability of Local Zone determines reliability of whole Design, it is necessary to consider basic probabilistic characteristics of the fatigue strength and endurance of the Local Zone.

"Nobody trusts calculations, except the Estimator, and everybody believes in results of tests, except the Test Engineer"

The parable of Test Engineer

CHAPTER 4 MODEL OF FATIGUE DURABILITY OF LOCAL ZONE

4.1 Basic Probability Models

Usually to describe the fatigue strength and endurance as random variables, the Cumulative Distribution Function (CDF) is used. Different authors put to use the different type of CDF, however, as shown in [4, 45, 49], these choice often take concern to data smoothing without base the phenomenological nature of fatigue destruction in these models.

To the well-known models that describe the fatigue destruction phenomenological is given in Chapter 2, and that is why they are most applied at fatigue strength design, takes:

- Weibull model of “weak link” (or Smallest Extreme Value -SEV):

$$F(N) = 1 - \exp\left[-\left(\frac{N - a}{b}\right)^m\right] \quad (4.1)$$

where N – number of load cycles,

a, b, m –CDF parameters (respectively location, scale and mode parameters).

- model of random accumulation of numerous small fatigues damages (H.Kordonsky, M.Stepnov, V.Kogaev, B. Bolotin and other):

$$F(\lg N) = \Phi\left(\frac{\lg(N - N_0) - a}{b}\right) \quad (4.2)$$

where N - number of load cycles,

N_0 –cycle sensibility factor,

a,b - CDF parameters (respectively location and scale).

Evidently, that for satisfactory description of fatigue durability three parameters of CDF are required.

In papers [58, 64] it is shown that at the low levels of loading (large values of the fatigue endurance or the high level of reliability, low load conditions) the CDF is well described by *two-parameter* expression:

$$F(\tau) = \exp\left[-\exp\left(-\frac{\tau-a}{b}\right)\right] \quad (4.3)$$

where $\tau = \lg N$ – the logarithm of number of load cycles,
 a, b – CDF parameters of location “a” and scale “b”.

Comparing different CDF with fracture character of fatigue destruction it is possible to mark the following, that the model of Weibull “weak link” (4.1) describes brittle and semibrittle destruction, while the model of an accumulation of numerous small fatigue damages (4.2) is semiductile destruction.

As shown in [58] to get correct CDF of fatigue endurance at the Operating loading condition is not possible, therefore specify the kind of fatigue durability CDF to the Operating area of high fatigue endurance is not just possible.

In paper [46] it is shown that sizes of volumes of some loaded structures and their amount have influenced to the kind of CDF. Accepting the Hypothesis of Local Zones it is possible to show that the Local Zone with the dependent FDA have the ductile mode of destruction, when at positive cross-correlation relationship of origins of the fatigue damages meeting afterwards in a main spread out crack [59]. So it results in the model of maximums distribution [58] (Largest Extreme Values) or to CDF expression (4.3), where a – location parameter, b – scale parameter, $\Gamma'(1) = -0,5772\dots$ Euler constant tied with mathematical expectation $M\{\tau\}$ and variance $D\{\tau\}$:

$$M\{\tau\} = a - b\Gamma'(1) \quad (4.4)$$

where $\Gamma'(1) = -0,57721\dots$ Euler Constant;

$$D\{\tau\} = \pi^2 b^2 / 6 \quad (4.5)$$

Thus each of presented models of FDA is described by CDF with phenomenological physical reasonable basis.

4.2 The description of double exponential CDF with logarithmic transform of fatigue durability

The analysis of fatigue tests data of specimens, Structural Significant Elements of Designs, especially *at low levels* of loading or the *increased survivability*, when destruction of SSE of a Design doesn't lead to failure of total Design yet, shows that in this case it is preferable to use double exponential distribution to the description of fatigue durability with logarithm of load cycles (DEDF) or Largest Extreme Value (LEV).

The more Design's survivability, dictated by need of destruction a large number of micro components at low levels of loading, the better process of fatigue failure meets to the scheme of model of "maximum" CDF (LEV). Having only two parameters (actually minimum possible), this CDF well describes area of small probabilities of the destruction, just of interest to definition of a Design's Safe Life Time and in which the effect of a sensitivity threshold of cycles is brightly expressed (Fig.4.1):

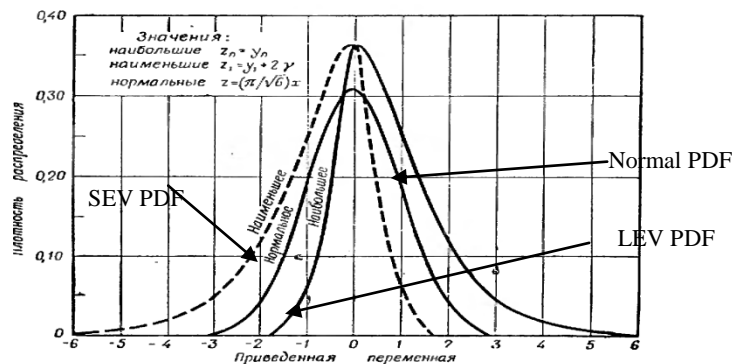


Fig. 4.1 The Comparison of Normal, LEV and SEV standard probability density distribution functions (PDF).

Really, the condition on the existence of a sensitivity threshold of cycles N_0 can be written down as impossibility of approach of an event of $N < N_0$. That it is actually possible to accept equivalent events with very small probability of approach. Also it is possible to assume that existence of a sensitivity threshold of cycles N_0 is caused by action of negative correlation relationships in process of FDA in specimen's body (Fig. 3.10).

At the description of fatigue durability by other CDF for the description of this effect enter the third parameter that considerably reduces the accuracy of statistical estimates of all parameters of CDF or leads to sharp increase in volume of test sample [15].

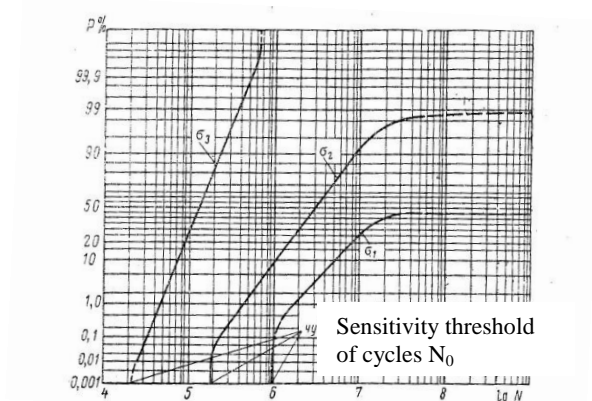


Fig.4.2. The existence of a sensitivity threshold of cycles N_0 during fatigue tests of aviation elements [8].

It is possible to compare this main distribution by method, given in [45], see Table 4.1:

Table 4.1

Calculation and comparison the values of RV for NDF, Weibull (SEV) and DEDF (LEV)

Quantile level p	0.001	0.01	0.1	0.2
Quantile NDF	-3,090	-2,326	-1,281	-0,841
Quantile Weibull (SEV)	-4,935	-3,136	-1,304	-0,719
Quantile DEDF (LEV)	-1,057	-0,741	-0,200	0,079

where quantile NDF = $\Phi^{-1}(p)$;

$$\text{quantile SEV} = (\log(-\log(1-p)) - \Gamma'(1))\sqrt{6/\pi};$$

$$\text{quantile LEV} = -\log(-\log(p)) - \Gamma'(1)\sqrt{6/\pi};$$

Again the DEDF is more fitted to describe fatigue life durability at low values of “fail” probabilities (a sensitivity threshold of cycles N_0). The same we can see for specimens’ fatigue durability, given in Attachments 1, 2, 3, 4 (Author data).

In Attachment 7 the Fatigue Test Data more than 2 000 specimens are represented. It is the most representative sample (till 600 specimens to one level of loading) was received by M.Stepnov [30]. There is shown that the DEDF is the most suitable CDF to fit the data in the Low Probability Area (small values of failure), especially for small loads or high reliability.

The main advantage of DEDF is to fit the experimental data without the addition to CDF one more parameter - the sensitivity threshold of cycles N_0 .

The main expression of DEDF (4.3) with parameters (4.4) and (4.5) may be presented in different forms:

$$F(N) = \exp \left[- \left(\frac{c}{N} \right)^m \right] \quad c = \exp(\alpha), m = 1 / \beta \quad (4.6)$$

$$F(\tau) = \exp[-\lambda \exp(-m\tau)] \quad \lambda = \exp(\alpha / \beta), m = 1 / \alpha \quad (4.7)$$

It is very importance to represent the expressions of the maximum likelihood function for the censored data. That allows to consider the samples which haven't failure at Tests or are being in Operation at statistical estimates of parameters (right censored data).

The system of partial derivatives of Maximum Likelihood Function for parameters α and β have the following expressions:

$$\begin{cases} \beta \left(\frac{n}{k} - 1 \right) \mu(T) - \exp(\alpha / \beta) \sum_k \exp(-\tau_i / \beta) / k + 1 = 0 \\ \beta \left(\frac{n}{k} - 1 \right) \mu(T) T - \exp(\alpha / \beta) \sum_k \tau_i \exp(-\tau_i / \beta) / k + \sum_k \tau_i / k - \beta = 0 \end{cases} \quad (4.9)$$

where

T – censor border (Test Base),

n – total sample size,

k – test sample size, when $\tau_i < T$, ($i = 1, \dots, k$),

$\mu(T)$ – value of the intensity function in point of T .

The expression of Intensity Function $\mu(\tau)$:

$$\mu(\tau) = \frac{\lambda m \exp(-m\tau) F(\tau)}{1 - F(\tau)} \quad (4.9)$$

$$\alpha = -\beta \ln \sum_k \exp(-\tau_k / \beta) / n \quad (4.10)$$

$$\beta = \frac{\sum \tau_i / n - \sum \tau_i \exp(-\tau_i / \beta) / n}{\sum_k \exp(-\tau_k / \beta) / n} \quad (4.11)$$

where $n = k$ (the full sample size without censoring)

α and β – location and scale parameters (4.3).

One of features of experimental Intensity Function of fatigue durability is her non-monotony [9, 49, 57]. It is possible to show that CDF (4.3) possesses this property also.

4.3 The statistical Model of the Local Zone fatigue strength

4.3.1 The premises of the Local Zone fatigue characteristics Model.

It is known that characteristics of fatigue strength consist of statistical descriptions of the fatigue endurance limits, fatigue life time and the S-N curve [49]. In the accepted practice parameters of all three fatigue characteristics are determined by results of separate experiment test plans, independent from each other. Thus an estimation of CDF parameters of fatigue strength characteristics is carried out by the principle of the best interpolation of the data obtained at tests with the following subsequent extrapolation of data results to the area of operational loading. However such extrapolation is carried out ignoring the change of physical processes of FDA at change of level loading of the object. A kind of CDF does not changes also [32].

At the best number of determined parameters usually is not less than six. Reduction of the number of determined parameters sharply reduces the volume of demanded tests or increases the accuracy of parameters estimation at the same quantity of the tested objects [15].

In engineering practice of the estimation of Safe Life Time (the technical system Limited Safe Life) it is interesting to define areas of small values of CDF (the "left tail"), see Fig.4.2 [8]. In works [58, 59, 64] it is shown that for the description of fatigue durability in operating conditions two-parametrical DEDF has the greatest preference. In the mathematical theory of reliability this probability distribution function is known as the distribution of "maxima" [18] or Largest Extreme Values (LEV). As fatigue tests data of specimens and objects of the high survivability shows, the CDF of "maxima" well describes fatigue durability and at the raised loadings, but at decrease in level of test loadings convergence of parameter's estimates to the theoretical improves (see Attachment 7; Attachment 4, table3.9). One of postulates of The Hypotheses of Local Zones is that the CDF of Local Zone' durability has the form of DEDF, as the failure of Local Zones occurs after accumulation a lot of fatigue damages in it.

So, the first premise of the submitted statistical Model is the mode of fatigue durability CDF, which has the expression (4.3).

Numerous researches show that S-N curves of fatigue have points of its breaks, a multimodality of the distribution density, non-monotonous change of some parameters (a dispersion of fatigue durability, the functions of intensity, etc) [51, 56].

Such effects it is possible to explain by change of the mechanism of fatigue damages accumulation due to change of the loading level of the object. The observed brakes of S-N curve are followed often by changing of the S-N curve's tilting angle. For practical convenience it is the best of all to describe a S-N fatigue curve as piecewise and linear approach [57] where the inclination of S-N fatigue curve (or exponent m defined by angle of linear piece) will depend on level of loading of Object.

So, the second premise of the submitted statistical Model is the mode of the S-N curve, which has the expression:

$$\sigma_i^{m_i} N_i = const \quad (4.12)$$

or

$$m_i \lg \sigma_i + \lg N_i = const \quad (4.13)$$

where

- m_i – the tilting angle of the S-N curve's sectional-linear part numbered i ,
- σ_i - the loading level (stress, force, load etc.),
- N_i – durability (number or loading cycles, life time, endurance etc).

The third premise of the submitted statistical Model is the following range of its definition:

- more the one million of loading cycles,
- exponent m value of S-N fatigue curve exceeds of six.

4.3.2 The uniform Model of the fatigue strength

In this paragraph it will be shown, how fatigue durability CDF is linked with limit of endurance CDF, S-N fatigue curve and sensitivity threshold of cycles N_0 . If CDF of fatigue durability is known, at some assumptions of a kind of S–N fatigue curve it is possible to receive expression for CDF of a limit of endurance [39, 52, 54].

Let's assume that at low load's levels CDF of fatigue durability belongs to self-same CDF assemblage, exactly to DEDF with logarithm of load cycles as durability $\lg N = \tau$.

Let's assume also, that CDF's exchanging is depends with load's level σ only with on changing of its location and scale parameters $a(\sigma)$ and $b(\sigma)$:

$$F(\tau|\sigma) = \exp\left\{-\exp\left[-\frac{\tau-a(\sigma)}{b(\sigma)}\right]\right\} \quad (4.14)$$

Let's introduce the note:

$$\varphi(\tau, \sigma) = \frac{\tau-a(\sigma)}{b(\sigma)} \quad (4.15)$$

Function (4.14) will have properties of CDF if the following conditions will have been satisfied:

$$\begin{aligned} & \text{if } \sigma \rightarrow 0 & \varphi(\tau, \sigma) & \rightarrow -\infty, \\ \text{or the } F(\tau|\sigma) & \rightarrow 0 \text{ and} & & \\ & \text{if } \sigma \rightarrow \infty & \varphi(\tau, \sigma) & \rightarrow +\infty \\ \text{or the } F(\tau|\sigma) & \rightarrow 1,0 & & \end{aligned}$$

It is easy to see that proportionality of function (4.15) to a logarithm σ meets this condition. Let's designate $s = \lg \sigma$ and

$$\varphi(\tau, \sigma) \propto s$$

It is known that at the domain of the Model (third premise) if the load is increased the Variance and the Coefficient of Variation are decreased [26, 54].

For CDF (4.14) the Coefficient of Variation $v(\tau, \sigma)$ is:

$$v(\tau, \sigma) = \frac{\sqrt{D\{\tau|\sigma\}}}{M\{\tau|\sigma\}} = \frac{\pi/\sqrt{6}}{\frac{a(\sigma)}{b(\sigma)} + \Gamma'(1)} \quad (4.16)$$

where:

$$D\{\tau\} = \frac{\pi^2}{6} b(\sigma)^2$$

$b(\sigma)$ – the scale parameter of CDF (4.14);

$M\{\tau\} = a(\sigma) - b(\sigma)\Gamma'(1)$ - mathematical expectation of life time τ ,

$a(\sigma)$ - the location parameter,

$\Gamma'(1) = -0,57721\dots$ Euler constant.

Then it is possible to assume:

$$b(\sigma) = C/s^m \quad (4.17)$$

$$\frac{a(\sigma)}{b(\sigma)} = \gamma s^m + \theta \quad (4.18)$$

Denote the following by:

$$\gamma C = \tau_0 \quad (4.19)$$

$$C[\theta - \Gamma'(1)] = P \quad (4.20)$$

$$M\{\tau\} = \tau_{av} \quad (4.21)$$

Substitute the expression of mathematical expectation in (4.16) and take account of (4.18); we receive the equation of S-N fatigue curve for average values of durability:

$$(\tau_{av} - \tau_0)s^m = P \quad (4.22)$$

Substitute (4.17) and (4.18) in (4.44) and twice take the logarithm we receive a fatigue S-N curve for quantile with probability p - level:

$$(\tau_p - \tau_0)s^m = C[-f(p) + \theta] \quad (4.23)$$

where

$$f(p) = \ln(-\ln p)$$

p - quantile of p - probability level.

Denote the following by:

$$A\{\tau\} = \frac{\theta C}{\tau - \tau_0} \quad (4.24)$$

$$B\{\tau\} = \frac{C}{\tau - \tau_0} \quad (4.25)$$

we receive

$$F(s|\tau) = \exp\left\{-\exp\left[-\frac{s^m - A\{\tau\}}{B\{\tau\}}\right]\right\} \quad (4.26)$$

So the endurance limit's CDF on the accepted base τ has also two-parametrical DEDF with double logarithm of load $s = \lg\lg\sigma$.

For an estimation of the offered Model parameters, let's will address to results of tests more than 2 000 of D16T alloy specimens [30, Table 2, p.11].

This Data are the most suitable for S-N fatigue curve analyses as about 500 specimen have been tested till their fracture on each high levels of loading stresses, and at least 50 specimens – on low levels of loading stresses.

4.3.3 The model validation on experimental data

The main statistical estimates of the fatigue test data CDF's parameters are presented in Table 4.2

Table 4.2

The results of fatigue tests of specimens

Stress load σ , kg/mm ²	$s = \lg \sigma$	Mean life time $\bar{\tau} = \lg N$	Std. Dev. of τ	Coeff. Variance $v(\sigma)$ (4.16)	Scale Parameter $b(\sigma)$ (4.17)	$a(\sigma)/b(\sigma)$ (4.18)
20	1,301	5,65	0,222	0,0393	0,1731	32,058
18	1,255	6,18	0,290	0,0483	0,2261	25,977
16,5	1,217	6,23	0,321	0,0515	0,2503	24,327
15	1,176	6,52	0,307	0,0471	0,2394	26,653
14	1,146	6,96	0,389	0,0559	0,3033	22,366
13	1,114	7,33	0,419	0,0572	0,3267	21,845

The primary experimental data of the specimen's fatigue tests allow to determinate all parameters of the offered statistical Model (4.14), (4.17), (4.18).

So it is necessary to estimate the following Model parameters:

m , C are determinated by (4.17), and γ , θ - by (4.18).

These four parameters contain all information about the fatigue strength: fatigue endurance limits, fatigue durability and the S-N fatigue curve.

Diagrammatically the S-N fatigue curve look as a straight line in Graph with double logarithmical coordinates " $s = \lg \sigma$ versus $\tau = \lg N$ ", see Fig.4.3:

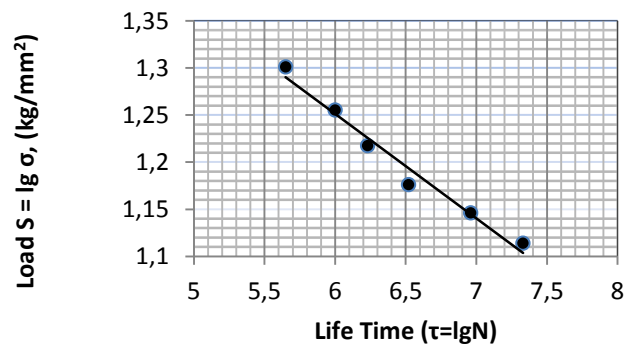


Fig. 4.3 S-N fatigue curve with double logarithmical coordinates "Load (stress) versus Life time (cycles)"

According (4.17) the more the value of load the less the value of parameter b(s).

On Fig.4.4 the decrease of the scale parameter is shown than the stress level is increasing. Also we can see a good straight line alignment.

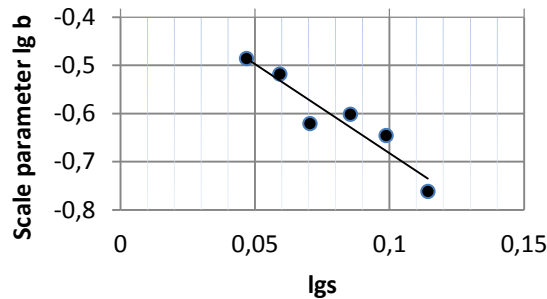


Fig. 4.4 The decreasing of scale parameter b(s) according the increasing of stress ($lgs = \lg \lg \sigma$)

As it is shown in (4.16) the Coefficient of Variance have to decrease under load increasing, or the same kind of thing – the ratio a (s)/b (s) in (3.18) have to also go up. Indeed, as have seen on Fig. 4.5 this dependence is near to linear one.

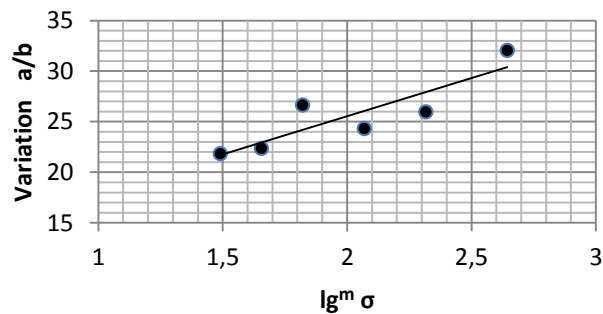


Fig. 4.5 The increasing the value of ratio a(s)/ b(s) according the increasing of stress $\lg^m \sigma$

Estimates of parameters C and m of the expression (4.17) we will determine by experimental data (Table 4.3) and Fig.4.4. So taking the logarithm of the left and right side of the expression (4.17) and applying a least square method, receive:

$$\lg b(s) = -0,313207 - 3,6936 \cdot \lg s \quad (4.27)$$

or

$$m = 3,6936; \quad \lg C = - 0,313207; \quad C = 0,4862; \quad (4.28)$$

Make the same to equation (4.18):

$$A(s)/b(s) = 10,43 + 7,55847 \cdot s^m \quad (4.29)$$

or

$$\gamma = 7,55847, \quad \theta = 10,43 \quad (4.30)$$

The adjustment of the received estimates of parameters with experimental data is shown on Fig. 4.4 and Fig. 4.5.

If we substitute the assayed estimates of parameters in (4.9) and (4.10), we find

$$\tau_0 = 7,55847 \cdot 0,4862 = 3,675 \quad (4.31)$$

$$P = (10,43 + 0,57742) \cdot 0,4862 = 5,352 \quad (4.32)$$

and also receive expression of S-N fatigue curve for average values of durability:

$$(\tau_{av} - \tau_0)s^m = P \quad (4.33)$$

Insert (4.7) and (4.8) to (4.4) and twice take the logarithm of both sides, we receive S-N fatigue curve for quantile of p - probability level:

$$(\tau_p - \tau_0)s^m = C(-f(p) + \theta) \quad (4.34)$$

where

$f(p) = \ln(-\ln p)$, p – fixed quantile value of CDF.

Side by side we calculate (4.14) and (4.15):

$$A\{\tau\} = 10,43 \cdot 0,4862 / (\tau - \tau_0) = 5,072 / (\tau - \tau_0) \quad (4.35)$$

$$B\{\tau\} = 0,4862 / (\tau - \tau_0) \quad (4.36)$$

$$\tau_0 = 3,675 \quad (4.37)$$

and when insert it in (4.16) deduce the CDF of endurance level:

$$F(\sigma|\tau) = \exp \left\{ -\exp \left[-\frac{\lg^m(\sigma) - 5,072 / (\tau - \tau_0)}{0,4862 / (\tau - \tau_0)} \right] \right\} \quad (4.38)$$

where τ is the accepted base of test time.

Note, that the offered statistical Model includes the sensitivity threshold of cycles N_0 at small values of probabilities as τ_0 value (4.27). So sensitivity threshold of cycles N_0 is equally $N_0 = 4731$ load cycles.

The parameters of A $\{\tau\}$ and B $\{\tau\}$ for different Test Cycles Bases are shown in the Table 4.3.

If we take the logarithm of (4.23) both sides, we get a new transformation of S-N fatigue curve (Fig.4.6) as double logarithmic coordinates and with account of sensitivity threshold of cycles N_0 It seems the better fitting prediction linear data with experimental ones, compared with Fig.4.3

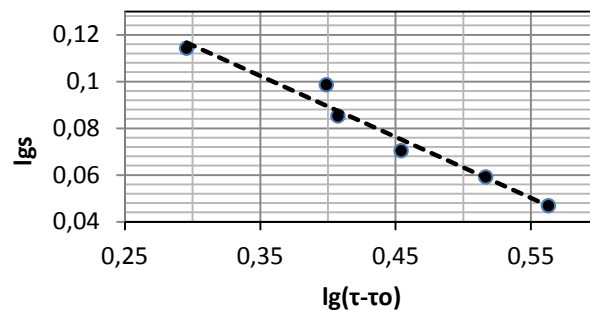


Fig. 4.6 The S-N fatigue curve for double logarithmic coordinates.

Now we consider the mode of Endurance Limits PDF for different Test Cycles Basis.

Table 4.3

The Fatigue Strength (Endurance Limit) CDF parameters

Test Base		A(τ)	B(τ)
N	$\tau = \lg N$		
$5 \cdot 10^5$	5,700	2,505	0,240
$1 \cdot 10^6$	6,000	2,182	2,209
$5 \cdot 10^6$	6,700	1,677	0,161
$1 \cdot 10^7$	7,000	1,525	0,146
$5 \cdot 10^7$	7,700	1,260	1,121
$1 \cdot 10^8$	8,000	1,173	0,112

The CDF of Endurance Limits for Test Cycles Basis are shown on Fig. 4.7 (Test Base -1 ml and 10 ml cycles), Fig. 4.8 (Test Base -100 ml cycles).

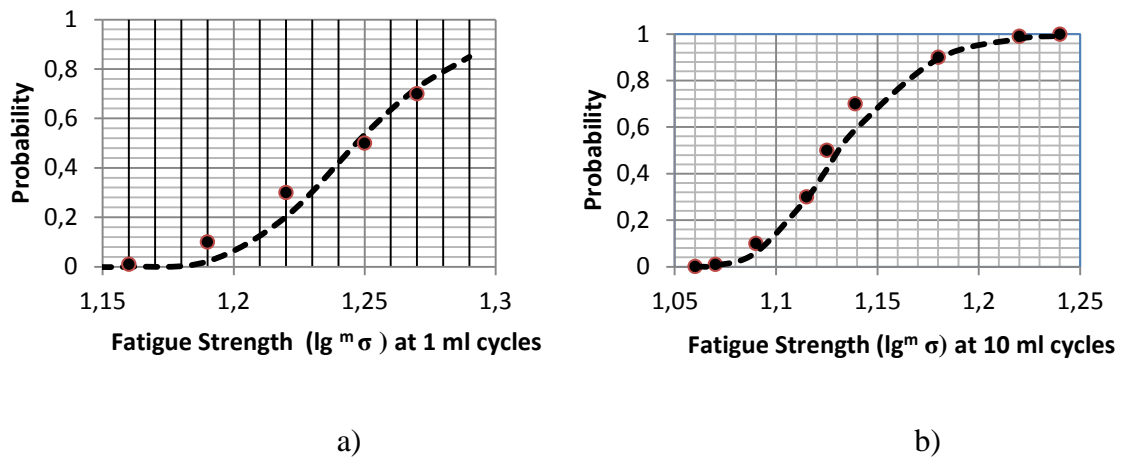


Fig. 4.7 The PDF of Endurance Limit for Test Base a) - of 1 ml cycles, b) – of 10 ml cycles.

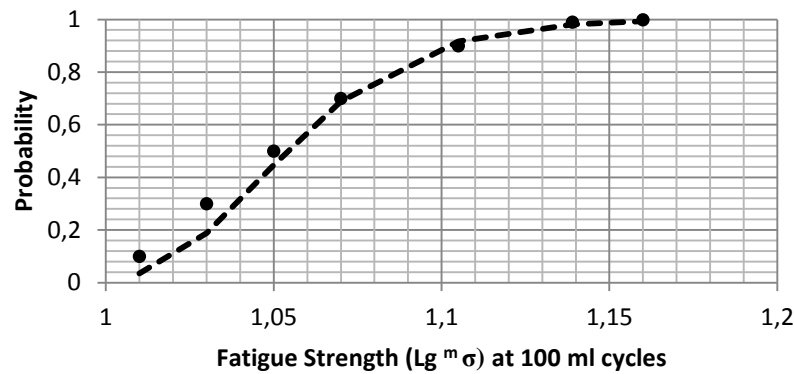


Fig. 4.8 The CDF of Endurance Limit for Test Base of 100 ml cycles.

Analyzing the provided Figures it is evident that in the offered Model the CDF of Endurance Limits are more satisfactory fitted with the experimental data in process of Test Cycles Basis increasing. The best data fitting is observed in the area of small probabilities, i.e. in the field of adoption of the engineering decision on durability of studied Object.

So, it is shown, that for a complete statistical description of characteristics of the fatigue strength (such as fatigue endurance limits, safe life durability, S-N fatigue curve, sensitivity threshold of cycles it is required only four interconnected parameters, namely: “C” – the parameter of the decreasing of the variance (dispersion) of the durability till failure, m – the parameter of the inclination for logarithmic S-N fatigue curve, γ and θ – parameters of the coefficient of a variation decreasing under the load is increased.

The offered Model minimizes costs of the estimation of fatigue strength parameters by the results of fatigue tests. It allows receiving the estimates of all main PDF parameters of

fatigue strength characteristics based at small number of the tested samples. It is especially important for the research of the large full-size technical Design's durability, tests of which are labor-consuming and expensive. It is necessary to emphasize that the range of definition of the received functions lies in a zone of great values of the Safe Life durability, the high survivability of Test Design of or low levels of its loading. It is characteristic for Operating conditions. The test reproduction of these conditions is difficult and often can't be received experimentally. The offered Model based on phenomenological reasonable CDF of "maxima" (LEV) and therefore is suitable for the description of composite material's specimens, correctly to extrapolate results of experiment.

4.3.4 The example the Local Zone operational durability prediction (the Safe Life evaluation)

It is known both as earlier researches [51, 56] carried out, as some of modern works [32, 70], show that the mechanism of the FDA strongly differs through the loading level, so it is necessary to describe the process of destruction by other phenomenological models in desired predicted area of long life time, which, unfortunately, cannot be confirmed experimentally due to its high duration and expensiveness. About the distinction of mechanisms of fatigue failure revealed during the fatigue tests it was marked in [14]. Moreover, even at one loading level various mechanisms of fatigue failure of objects were found out. In this case the distribution of durability is characterized by the mix of function of distributions, usually no more than two distributions. This mix distribution depends, as consider in [50], from initial quality of researched object and can be represent as:

$$F(N) = \chi F_1(N) + (1 - \chi) F_2(N) \quad (4.39)$$

where

$F_i(N)$ – the Design durability's CDF, which fatigue failure occurs on i - mechanism,
 χ - a share of the Designs which have collapsed on the first mechanism.

However such mixes of distributions should be absent practically due to increased quality assurance of production. In this case, presence of such mix of distributions (4.39) shows that on this level of loadings one mechanism of fatigue failure may transfer to another. Let's called this point of the transfers as N_{oi} . – the area of durability close to N_{oi} cycles, where the failure mechanism is changed – “point of changing.”

The changing from one mechanism of fatigue failure to another is accompanied by breaks of S-N curves, a multimodality of the distribution density, non-monotonous change of some parameters (a dispersion of fatigue durability, the functions of intensity, etc.). No one of any of the accepted fatigue models cannot explain and describe these phenomena.

As shown in [64] the two-parametric DEDF with logarithmic life time (4.3) suitable describes the life time under low loading or high reliability Designs.

Take the supposition that one of the proofs of the various mechanism of fatigue damage accumulation is the various values of S-N curve's parameter "m".

It is known, that the S-N curve is described by the expression (4.12) or (4.13).

More often the S-N curve (4.13) is described as linear function, with two linear parts (Fig.4.9a) or as exponential mode (Fig.4.9b) [39].

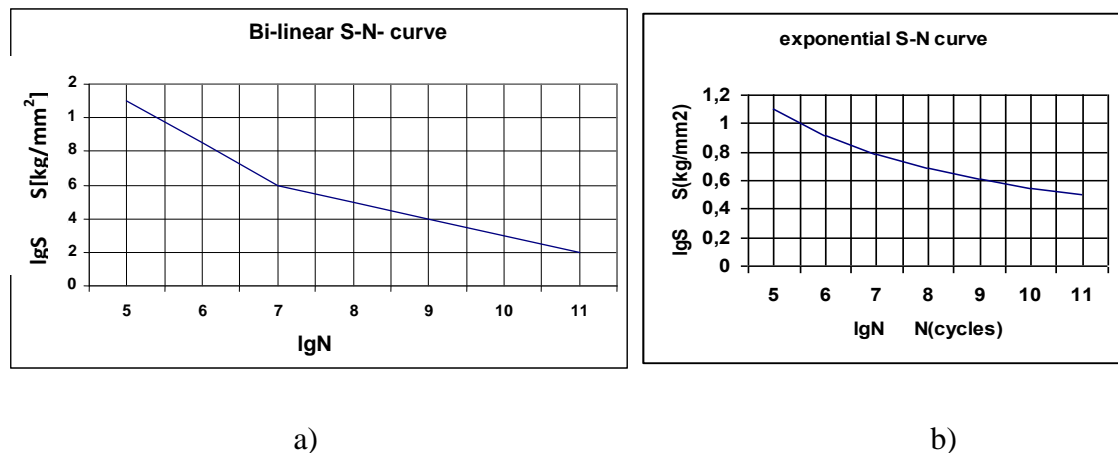


Fig. 4.9 The S-N fatigue curves kinds: a) – two linear parts, b) exponential

However, both in the first and in the second cases, different endurance areas (especially in its great values) are described usually with the extrapolation in area of great values by the same kind of CDF. That, especially in logarithmic coordinates, leads to an appreciable error.

In practice for the aluminum alloys there is applies a sectional-linear S-N curve with double logarithmic coordinates. We shall accept a hypothesis about changing of the mechanism of FDA as varying the parameter “m” of S-N fatigue curve. As an example let's consider the Table 4.4 at the following values of S-N curve turning-points (point of the transfers as N_{oi})¹⁰.

As stated above, it is inconvenient and expensive to determine experimentally a kind of the CDF for long life time's values of durability; however it is possible to determine

¹⁰ This kind of S-N fatigue curve was used for helicopter's components Safe Life evaluation.

changing of an expectation mean and a dispersion of the random variables in expression (4.14).

Next we show the determination of CDF parameters for any of considered linear (constant values of m -parameter) areas. (If it is have to shift to other area and after that to the area of operational working loads it is necessary to repeat the resulted calculations, based on changing point $\lg N_0$).

Table 4.4

The sectional-linear S-N fatigue curve description and point of transfers.

Range of durability, cycles c	Value of m parameter of S-N curve	A "point of changing" $\lg N_0$ when the mechanism of fatigue accumulation is changed	The distribution function of the durability
up 1 ml	4	1	Weibull (4.1)
from 1 ml up 10 ml	6	6	logarithmic-normal (4.2)
from 10 ml up 100 ml	8	7	logarithmic-normal (4.2)
from 100 ml up 1 000 ml	10	8	DEDF (4.3)
from 1 000 ml	12	9	DEDF (4.3)

Let us assume that the kind of the S-N curves and CDF of the durability and the fatigue limits are the same assumed sectional-linear area. It is shown [54], that for extrapolation of tests results in area of assumed loadings it is possible to take advantage of the following formula:

$$\begin{aligned}
 (\lg N_i - \lg N_{0i}) \lg^{m_i} \sigma_i &= \\
 &= (\lg N_j - \lg N_i) \lg^{m_i} \sigma_j
 \end{aligned}
 \tag{4.40}$$

where

$N_{i,j}$ - durability under $\sigma_{i,j}$ loading,

m_i, N_{0i} - S-N curve parameters, see Fig. 4.10 and Table 4.4.

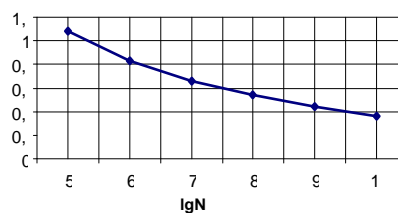


Fig. 4.10 The multi sectional-linear S-N curve

Let set $\tau = \lg(N/N_0)$ and $S = \lg\sigma$ and take the logarithm (4.40):

$$\lg \tau_j = \lg \tau_i + m \lg S_i - m \lg S_j \quad (4.41)$$

All random variables in (4.41) - are stochastic independent, therefore is true:

$$D\{\lg \tau_j\} = D\{\lg \tau_i\} + m^2 D\{\lg S_j\} \quad (4.42)$$

where

$D\{\bullet\}$ - the random variable's dispersion operator.

Let arrange the local linearization of $\lg S$ and $\lg \tau$ (4.42) in the area of expectation mean $E\{\bullet\}$ [15]:

$$D\{\lg \tau_i\} = \frac{D\{\tau_i\}}{E^2\{\tau_i\}} = \text{var}^2\{\tau_i\} \quad (4.43)$$

$$D\{\lg \tau_i\} = \frac{D\{\tau_i\}}{E^2\{\tau_i\}} = \text{var}^2\{\tau_i\} \quad (4.44)$$

$$D\{\lg S_j\} = \frac{D\{S_j\}}{E^2\{S_j\}} = \frac{\lg^2 e D\{\sigma_j\}}{E^2\{S_j\} E^2\{\sigma_j\}} = \frac{\lg^2 e \text{var}^2\{\sigma_j\}}{\lg^2 E\{\sigma_j\}} \quad (4.45)$$

where

$\text{var}\{\bullet\}$ - the operator of the variation coefficient,

$e = 2,7183$ - Napierian logarithm base.

Then use (4.43), (4.44) and (4.45) in (4.42):

$$\text{var}^2\{\tau_j\} = \text{var}^2\{\tau_i\} + \frac{m^2 \lg^2 e \text{var}^2\{\sigma_j\}}{\lg^2 E\{\sigma_j\}} \quad (4.46)$$

Apply the mean expectation operator to (4.41):

$$E\{\lg \tau_j\} = E\{\lg \tau_i\} + m \lg S_i - m E\{\lg S_j\} \quad (4.47)$$

Changing the logarithmic operator in (4.47) with expectation mean operator:

$$\lg E\{\tau_j\} = \lg E\{\tau_i\} + m \lg \frac{S_i}{E\{S_j\}} \quad (4.48)$$

or raising (4.48) to the power based 10:

$$E\{\tau_j\} = E\{\tau_i\} \left(\frac{\lg \sigma_i}{\lg E\{\sigma_j\}} \right)^m = E\{\tau_i\} \eta_\sigma^m \quad (4.49)$$

where:

$$\eta_\sigma = \frac{\lg \sigma_i}{\lg E\{\sigma_j\}} \quad (4.50)$$

- the load safety factor

Expressions (4.46) and (4.49) define the durability (Safe Life Limit) CDF parameters for the Design in operation through known loading parameters, measured in flight, and by results of tests.

Then parameter β (4.5), describing the Variance of the Design's Life Time in Operation, can be appreciated by results of tests and load measurements in Operation from the ratio (4.46) and (4.49)

$$\beta = \text{var}\{\tau_i\} E\{\tau_i\} \frac{\sqrt{6}}{\pi} \quad (4.51)$$

So, we know the parameters of the position α and the scale β , and can determinate the reciprocal distribution of durability under operational conditions.

Now, knowing the Variance of fatigue durability at operational loadings (4.51), it is possible to determine or to approve the Safe Life Time in Operation, confirmed with some assurance.

Setting normative probability of destruction P_{norm} , having n objects tested and observing the minimal operating time received at tests N_{min} , we may to address to ratio, offered in [8]:

$$P_{\text{norm}} = \int_{-\infty}^{+\infty} n [1 - F_0(\lg N_{\text{min}})]^{n-1} \int_{-\infty}^{\lg N_{\text{min}} - \lg \eta_n} dF(\lg N) dF_0(\lg N_{\text{min}}) \quad (4.52)$$

where

P_{norm} – normative Design's destruction probability,

n - the amount (number) of the tested Designs,

N_{min} - the minimal Design's durability under test condition,

η_N – cycle's safety factor, is in accordance with normative Design's destruction probability.

Take (4.3) and (4.41) to (4.52), transfer the changing variable

$$y = \exp\left[-\exp\left(-\frac{\lg N - \alpha}{\beta}\right)\right] \quad (4.53)$$

and note the following

$$\nu = \exp\left(\frac{\ln \eta_N}{\beta M}\right) \quad (4.54)$$

where $M = 2,303$ - the transition factor from natural logarithm to decimal.

Have [1]:

$$P_{norm} = n \int_0^1 (1-y)^{n-1} y^\nu dy = \frac{\Gamma(n+1)\Gamma(\nu+1)}{\Gamma(n+\nu+1)} \quad (4.55)$$

where $\Gamma(\bullet)$ – gamma-function

If taking the known rations [24], get:

$$P_{norm} \prod_{i=0}^{n-1} \left(\frac{\nu}{n-1} + 1\right) = 1 \quad (4.56)$$

where $\Pi(\bullet)$ – multiply operator.

Then the cycle's safety factor is determined with (4.54):

$$\eta_N = \exp(2,303\beta \ln \nu) \quad (4.57)$$

where β - is determined with (4.51)

ν - is determined with (4.54)

Thus, the operation loads σ_{work} , the test loads σ_j , defined assurance (4.54), n tested Designs, the minimal test durability $N_{min \text{ test}}$ obtained, the S-N curve mode (4.40) [27], exceedance of test load under operation ones (4.50), the kind of CDF (3.3) with its parameters (4.49) and (4.51) are known. Now it is need to result the minimal test life time $N_{min \text{ work}}$ to equivalent one at work loadings condition. Using the same S-N curved and (4.40) we determine the Variance of fatigue work durability β and set a level of safety (probability of failure) P_{norm} . From (4.54) and (4.52) we determine cycles safety factor η_N .

So the safety life time under work conditions R is

$$R = \frac{N_{\min work}}{\eta_N} \quad (4.58)$$

The expression (4.58) is based to determinate the Safe Life Time if test results are known.

In the Appendix 8 the Evaluation of Service Safe Life Time is deduced for keel beam of the Mi-26T helicopter by results of Bench Tests.

4.3.5 The determination of Failure Criterion of Local Zone

For an assessment of reliability of a Local Zone one of important points is to define and formalize the moment of refusal of the Zone – Failure Criterion. It is also necessary to define the Failure Criterion for the realization of the Fail Safe Concept of the Design. Some options of Safe Destruction are given [45, 73]. For example it was offered to accept the moment of increase in an intensity of emergence of fatigue cracks in centerwing the Tu-134 plane [63].

At an assessment of a Limited Service Life of Structural Significant Elements it is possible to consider the following options of Failure criteria of failure of a Local Zone:

- critical value of density of the found damages (cracks in stringers, a covering and other strength members and SSE in a certain Zone;
- critical number of the destroyed stringers in a certain Zone;
- critical number of fatigue cracks in a covering, extending in the direction of action of positive correlation relationship (Chapter 2) – especially aspiring subsequently merging to one main crack;
- the accelerated growth of one of the main trunk crack in a covering;
- increase in number of the subsequent repairs of a Design by the criteria stated above, that brings to increase idle times of Design's Tests.

At fatigue tests of the Mi-26T helicopter (The Appendix 8) for Failure Criterion - the increase in number of repairs of a keel and tail beam after achievement of the found main cracks of the accelerated growth and their concentration in the plane of action of positive correlation relationships.

*Snail climbs up on a tree of cherries. Bypass
flying birds tell her:*

*- But on this tree there are no any cherries
now !*

*Without interrupting the way, the Snail
answered:*

- Will be when I will reach there.

Anthony de Mello

CHAPTER 5 METHODOLOGY OF LOCAL ZONES LARGE-SIZE DESIGNS TECHNICAL CONDITION MONITORING

5.1 Review of nondestructive control methods

In the previous chapters it is shown that despite the uniform loaded condition in large-size Designs, accumulation of fatigue damages happens not uniform. During Bench Tests or Operation some Local Zones are observed on Design where the fatigue damages are concentrated.

Owing to practical impossibility to provide NDT monitoring of the total Design, such concentration of fatigue damages in advance known zones will allow to reduce the volume of NDT monitoring of total Design. Thereby we have an opportunity to find out the beginning of formation of the main destruction (a fatigue crack) at the earliest stage of its emergence. That considerably expands possibilities of further research of a Design at Bench Tests and increases its safe work in operation.

So far the most wide-spread NDT checking both at tests and in operation is visual. Despite high efficiency of this method of cracks' detection, she has low productivity and labor input doesn't allow increasing fully profitability of carried-out monitoring, automating her process and demands special preparation of NDT inspectors.

Besides efficiency of visual method sharply decreases in Operation because of adverse working conditions of NDT inspectors under bad weather conditions and during the work in Field Service.

In this case the knowledge of existence of Local Zones would allow examining only these Zones –so named "special attention areas" - first of all in Operation. Unfortunately character emergence of Local Zones at Bench Tests can differ from their emergence in Operation. In this case a big role existing NDT methods are signified.

Let's consider the main existing NDT methods to select the most suitable for their application, their main merits and demerits when monitoring a technical condition during Bench Tests.

Proceeding from the physical phenomena on which NDT is based, it is accepted to allocate its nine main types:

- the acoustic;
- eddy current;
- the magnetic;
- the electric;
- the radio wave;
- the thermal;
- the optical;
- the radiation;
- with application of getting substances.

Practically at once it is possible to refuse from magnetic, electric, eddy current (are unusable for diagnostics of composite designs), radiation and with application of getting liquids (require special conditions of application and comes to the end with the visual analysis of pictures).

The Radio-Wave method of NDT is based on registration of changes of parameters of electromagnetic waves of the radio frequency range interacting with object of testing. Waves of super high-frequency of range with a length from 1 mm to 100 mm are usually used. When using this type of NDT existence of defects in studied designs leads to emergence of additional reflections of an electromagnetic field which change an interferential picture and cause additional losses of energy.

For our case a shortcoming is to use of the powerful generator of electromagnetic waves and her movement or design for a scanning along the test surface. At application of this NDT method it is necessary to use of contact devices that complicates automation processes at continuous measurements and control of moving objects. Therefore this method is applied generally in testing of dielectrics, and also at research of a condition of a surface of conductive carrying-out bodies [19].

Also practically (need of scanning, availability of a powerful radiating unit, dependence of results of NDT on its seat, difficulty of an identification of damages at complex structure of a design) ultrasonic methods possess the same shortcomings [75].

The thermal method of NDT is based on registration of changes of thermal or temperature fields of testing objects. It is applicable to designs from any materials [76]. In thermal methods of NDT as trial energy the thermal energy is used extending in Design of testing.

The temperature field of a surface of Design is a source of information on features of process of a heat transfer which, in turn, depends on existence of internal or external defects. Distribution of temperatures in a Design depends on its properties: geometrical parameters, chemical composition, existence of defects and others.

Advantages of thermal control are: remotability of test, high speed of information processing; high efficiency of tests; high linear resolution, possibility of control at one - and bilateral approach to a Design; theoretical possibility of testing of any materials; multiple parameter nature of tests; possibility of a complementary combination of thermal NDT with other types of NDT; compatibility with standard information acquisition system; possibility of line control and creation of the automated control and management systems by technological processes. Thermal control can be exercised as highly automated and technological devices demanding high level of competence and knowledge from experts, and rather simple methods in application of the NDT, available to any user.

At contactless measurements there are rather rigid requirements to purity of environment. However at Bench Tests use of this method is represented useful and effective. At expeditious control use of the budgetary thermal imagers of producers of firms Testo, Fluke, FLIR, DALI is possible [76]

The optical method of NDT is based on supervision or registration of parameters of the optical radiation interacting with design of testing. This interaction is connected with absorption, reflection, dispersion, dispersion, polarization and other optical effects. This method applies to control of a condition of a surface and detection of superficial defects. Optical methods have very broad application thanks to a big variety of ways of obtaining primary information. Possibility of their application for external control doesn't depend on a material of object of parameters, but to essential shortcomings carry rigid requirements to a state of environment and purity of a surface of a product and a necessity of the visual analysis [77].

5.2 The short description of an acoustic emission method

Lack of the specified shortcomings does a method of the acoustic emission (AE-method) to one of the most effective for monitoring of a technical condition of a Design during a Bench Tests and, in the long term, operation of the large-size Designs which are especially in a static loading condition (any sort construction designs: buildings, bridges, pipelines, storages, band designs, etc.).

The main features of an acoustic emission test method that defines it of advantage:

- provides detection of defects on degree of their danger;
- possesses high sensitivity to growing defects and allows to define in operating conditions a crack increment to shares of millimeters;
- the integrity of a method provides check-up total Design with use of one or several sensors which have been unmovable installed on a surface of Design;
- the method allows to carry out check-up of the most various technological processes, and also processes of change of properties and a condition of materials;
- the orientation and the disposition of the Design doesn't influence detect ability of defects.

Advantages of the AE-method over traditional methods of NDT:

- AE method's integrity: using one or several sensors installed not movably on areas of Design, it is possible to check total Design. This property of AE-method is especially useful at research remote (not available) areas of testing Design;
- in difference from scanning methods of NDT, AE-method doesn't demand thorough preparation of Design surface. Therefore, running of AE-method and its results don't depend on a condition of a surface and quality of its processing and the environmental condition. The insulating cover (if it is available) is removed only in installation sites of sensors;
- detection and registration only expending defects that allows to classify defects not by the sizes (or by other indirect signs – a form, a disposition, an orientation of defects), but on degree of their danger (influence on durability) for testing Design;
- the high efficiency of the AE-method many times over surpassing productivity of traditional methods of NDT, such as ultrasonic, radio graphic, eddy-current, magnetic etc.;
- AE-method remotability gives possibility of carrying out AE-testing during considerable removal of the operator from studied Design. This feature of AE-method allows to use effectively it for check-up (monitoring) of responsible large-size Designs, extended or

especially dangerous Designs without their disable out of operation and harm for the personnel;

- possibility of tracking of various technological processes and assessment of a technical condition of Design in “real time” (on-line) that allows to prevent emergency destruction of testing Design.

The maximum ratio efficiency/cost does AE-method to the most suitable for its applicable at bench tests.

The AE-method consists in generation of elastic waves of stresses in solid bodies as a result of local dynamic reorganization of their structure. The AE-method is based on the analysis of parameters of these waves. The main sources of AE — the processes of plastic deformation connected with emergence, movement and disappearance of defects of a crystal lattice: emergence and development micro and macro cracks; friction (including surfaces of gaps). AE-method is shown in the form of separate acoustic impulses.

The basic parameters of AE-method are number N of impulses during supervision (total account of N) and the activity of AE-signals equal to quantity of impulses in some interval of time of supervision (account of N speed). Actually not all impulses of AE are fixed, but only exceeding a certain threshold is registered.

For the description of process of AE it is important not only quantity of impulses, but also their amplitude. The parameter, considering both values, is effective value of acoustic issue of V, proportional to product of activity (or account speeds) AE multiplies on average value of amplitude of signals of AE for a unit of time [78].

For AE-method approbation possibilities at tests of large-sizes designs and others Objects were carried out some experiments which have confirmed efficiency of AE-method for its use for monitoring of a technical condition.

5.3 Application of AE-method at bench tests of different Designs.

During the progress of application and adaptations of AE-method for bench tests in "LNK Aviatest" Ltd has been received some the results having practical significance. Application of AE-method as one of the most effective methods of detection of damages at an early stage of emergence it was confirmed with a large number of various experiments and tests, both static, fatigue and thermal. As examples we will give only some results of experiments.

5.3.1 Static tests of the lower link of the torque-link of the main landing gear of the Tu-154

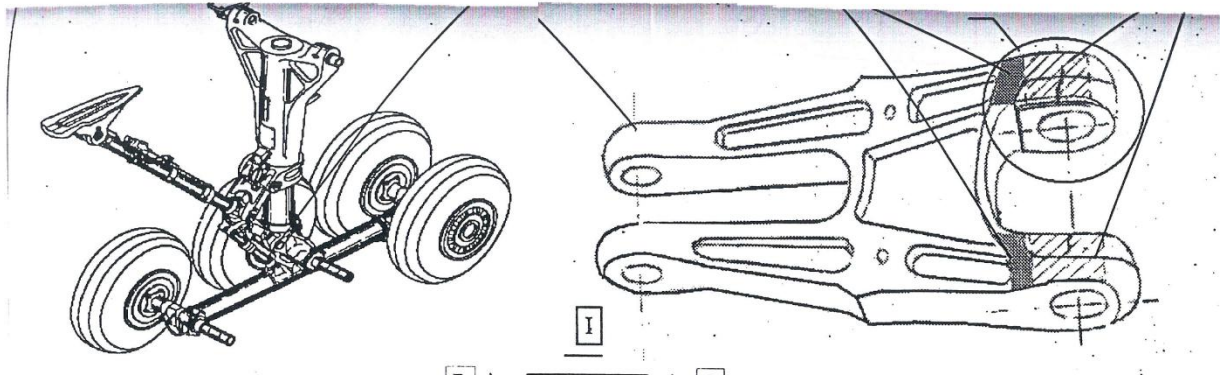


Fig.5.1 The lower-link of the torque-link of main landing gear of Tu-154 airplane

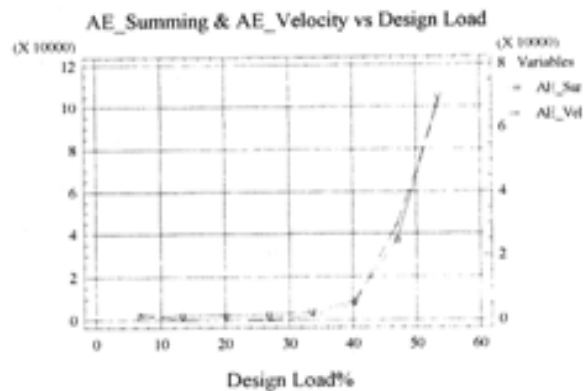


Fig.5.2 Growth of the total account of AE-signals after 40% of destroying static loading.

Similarly at static tests of various details and units it was shown that when loading to 40% from a static ultimate (destroying) load the accumulation of damages on Design it wasn't observed. After 40% of design loading it is necessary to continue the Test without stoppage—so-called "a non-return point".

5.3.2 Static loading of composite made specimens

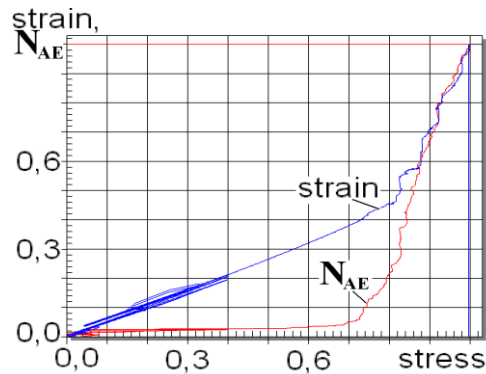


Fig.5.3 The diagram "stress" versus "deformation" of composite specimen's loadings.

On Fig.5.3 it is visible that growth of AE-signals begins much earlier, than starts changing the module of elasticity [58].

5.3.3 Dynamic tests of railway sleeper seat

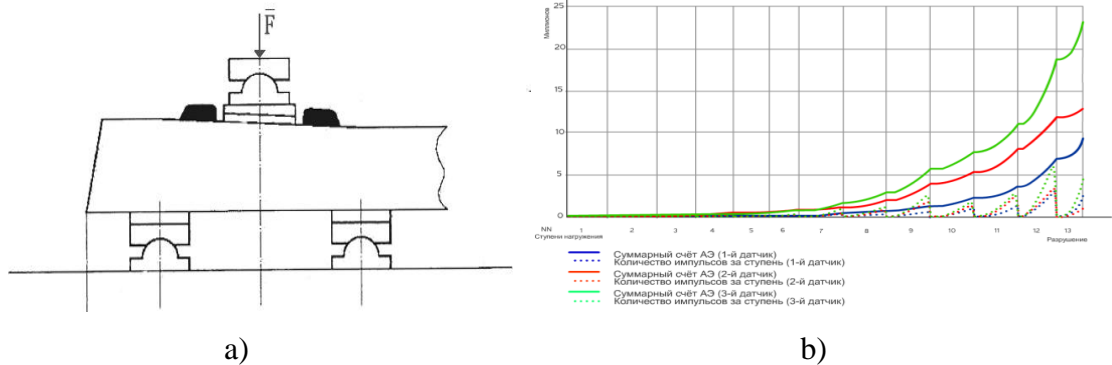


Fig.5.4 Static tests of railway sleeper's seat: a) the scheme of loading, b) AE-impulses summing

Fig. 5.4 demonstrates as at stage-by-stage step increasing dynamic loading of railway seat, lack of AE-signals (see b) at the beginning of a next stage (after previous step load is dropped) point out of "curing" of the sleeper with stressed armature [61].

5.3.4 The investigations of not pro-glove of a composite covering of the aileron after a lightning impact

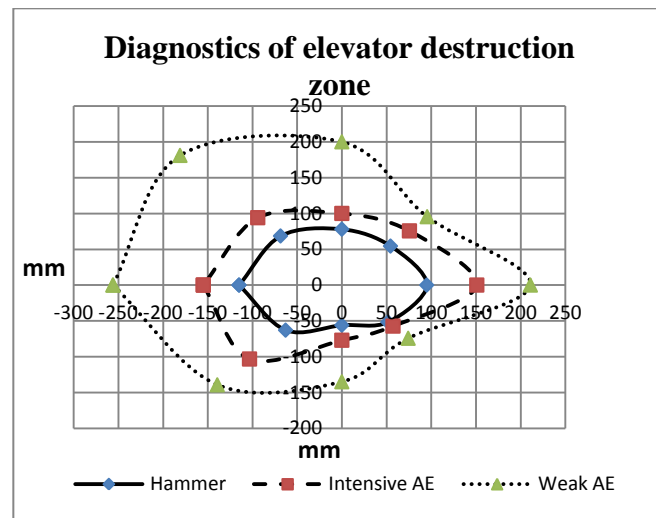


Fig.5.5 The zones of not pro-glove of a composite covering of the aileron after a lightning impact, defined by AE-method after local thermal heating of a covering.

The damaged covering of the aileron after a lightning stroke was exposed to local heating (no more than 100 degrees by hot water). At natural cooling of a covering AE-signals define the sizes of a zone of not pro-glove.

4.3.5 The application of AE-method for early detection of damages to local zones

The given examples show that at research of FDA in Local Zones the method of acoustic emission possesses the greatest efficiency in comparison by others. Its main shortcoming is need of preliminary loading of a Design that is complicated to carry out in Operation.

AE-method has useful property to predict the destruction beginning under various conditions of loading at an early stage of FDA. That allows predicting the beginning of fatigue failure even before emergence of fatigue cracks. This property does AE-method as almost irreplaceable means when monitoring Bench Tests and definitions of a location of Local Zones, as "special attention areas". One more advantage of AE-method is possibility of formalization of experimental data in directly digital form whereas installations of thermo vision ("thermal imagers") have not of such opportunity.

Nevertheless, convenience of use of "thermal imagers" in Operation and as the main advantage - opportunity to localize the most damaged areas, does this method as the arch-rival to AE-method.

It is obviously possible for early detection of fatigue cracks on large-size Designs previously to install AE-sensors in estimated local zones. When sharp increases in growth rate of AE-impulses accumulation is occurred it is possible consistently to apply methods of thermo vision and after the specification and localization of a position of potential destruction to use known methods ultrasonic, eddy-current and others NDT.

5.4 Main stages of the management of Local Zones technical condition monitoring

As an example we will consider the management of the Mil-26 helicopter tail and keel beam monitoring.

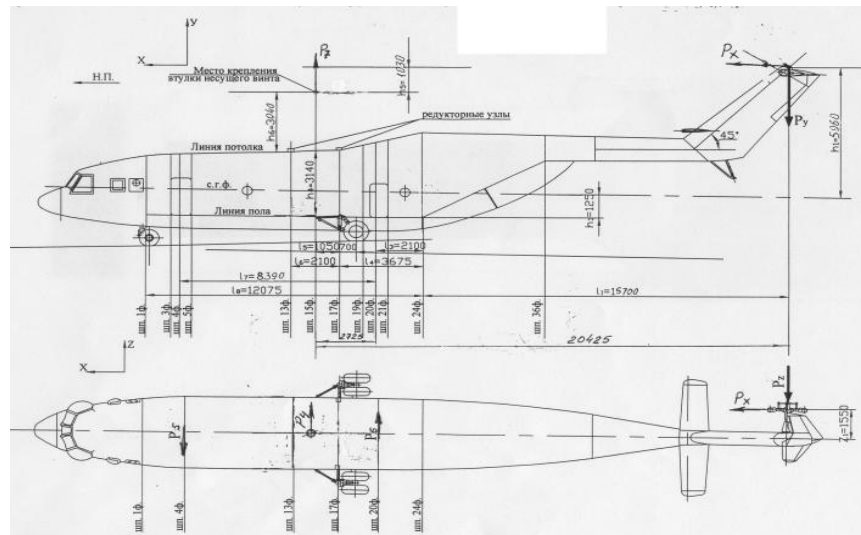


Fig.5.6 The load distribution of Mil-26 helicopter under bench fatigue tests

5.4.1 Stage 1. Preliminary studying of a stressed state of a Design

As shown earlier, the investigation of reliability of a large-size Design is connected with some difficulties at a monitoring of its technical condition in Bench Tests and at its Operation. Complexity consists in initial uncertainty of installation seats of sensors of NDT: acoustic emission, strain gages, wires of thermal imagers. Therefore one of the first stages it is considered to be a preliminary estimate of a stress distribution of Design by computational methods and, if there is an opportunity, a strain-gaging at static loading without entering damages into observed Design (paragraph 5.3.1).

The computational methods consist in creation of mathematical models by a method of finite elements with the subsequent their checks by results of a strain-gaging and other investigations (vibration, dynamic and others). It is possible to show that the indications of

local deformations measured by strain gages not always show the valid technical condition of a Design. Stress indications are influenced strongly by local seats of sensor's sticker, local coefficients of stress concentration, difference of stiffness and a point of transition of loadings from one area to another.

It is necessary to mark, that strain gages are sensitive to components of loading vector when due to the redistribution of stresses the local deformation (strain) remains invariable, but a contribution to it of various loads will qualitatively change.

Nevertheless the computational method of a finite element and the subsequent strain-gaging in a static loading allow estimating possible position of Local Zones. The following decision on an arrangement of the sensors seats for a technical condition monitoring of Design in "on-line"-mode, is based on investigations of a behavior of similar designs in Operation, the data of Bench Tests of identical Design, engineering and design intuition of the Developer.

At this stage the invaluable benefit is brought by use of AE-sensors which show zones of the accelerated accumulation of AE-impulses without Design will be overloaded at its static loading (paragraph 5.3.1).

These zones also are "special attention areas", and as a rule, are Local Zones of dependent FDA. These zones also become the first to start strain-gaging research of Design and installation of AE-sensors of NDT on them for initial monitoring in an "on-line" mode. Visual periodic control covers total area of research of a large size Design.

5.4.2 Stage 2. Correction of sensors places installation of and frequency of NDT

After installation of NDT sensors functioning in a "on-line" mode in the chosen seats of a Design, there is an information acquisition about parameters of AE- signals, change of nature of deformations on strain gages and places of emergence of fatigue cracks and damages. If there is at insufficient number of sensors, the "fan"-mode of information acquisition is organized. The subsequent correction and data processing, including correlation analyses of the moments of fatigue cracks emergences, AE-signals and if necessary of other methods of stochastic processes data processing are follows.

The purpose of such system analysis is to specify the locations of Local Zones with dependent FDA both as already formed by cracks as and assumed on change of indications of AE-signals, independent or stochastic related. After the analysis of the obtained data the correction of installation seats of sensors is carried out. Localization of Zones of the

checking, especially those zones where FDA are concentrated or/and the raised level of local deformations and intensity of AE- signals is observed, is identified whenever possible. Visual periodic NDT of these Zones is carried out more often and in larger scope, than in other, less damaged zones.

5.4.3 Stage 3. Monitoring of Local Zones technical condition and the investigation of speed of fatigue damages accumulation

At this stage it is necessary to carry out the analysis of the moments and places of emergence of fatigue cracks (Attachment 9), and to try to define the principle of formation of Local Zones.

Repeating stage 2 procedure with the analysis of the received preliminary results, localization of zones is defined completely. Then the attention is directed on survivability of this (these) of zone (zones): strain gages are re-stuck on a way of possible distribution of a crack (in principle carry out a role of sensors of cracks), functioning of AE-sensors passes in from a mode of quality standard of recorded of AE-signals to the quantitative – the purpose of both methods to prevent premature destruction of a Local Zone and to receive characteristics of her survivability.

At this investigation phase the size of Local Zones, its interposition on test Design, nature of dependence of fatigue cracks emergence, preliminary analyses of durability, survivability and other characteristics of reliability of the Local Zone that may define reliability of total Design, is defined. Visual control of Local Zones (“special attention areas”) is exercised with the increased frequency and with adapting, if necessary, other methods of NDT: thermo vision, ultrasonic, eddy-current.

Fig.5.7 illustrates the total distribution of fatigue cracks on the tail beam of helicopter Mil-26T. The picture is concerned the right side of the beam (the review from inside).

There are shown, that the fatigue cracks (in stringers, gussets, skin, coverings etc.) are concentrated in some areas (Local Zones). It is seen that mainly the direction of the Zones is directed at the angle approximately 45° to the front-back body axis. Such disposition of fatigue damages is formed due to acting of torque about the beam center line. This torque is originated by side load of steering (tail) rotor propulsion which twists the beam anti clockwise if to look to from nose of helicopter’s fuselage (Fig.5.8).



Fig.5.7 The general view of tail beam's cracking after fatigue test loading.

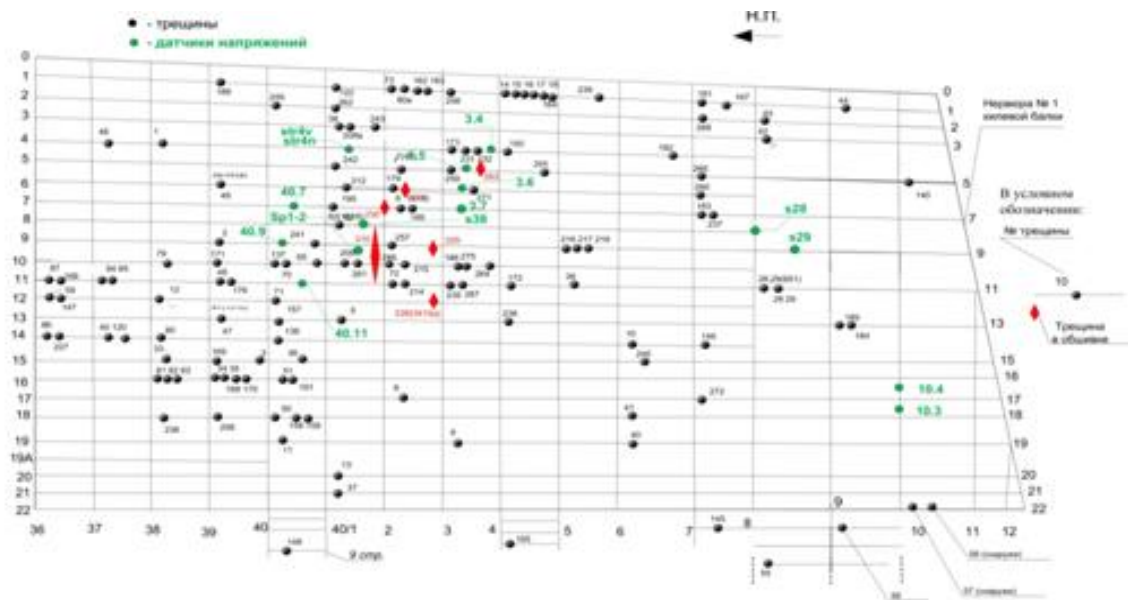


Fig.5.8 The distribution of all fatigue damages on Mil-26 helicopter's Tail Beam received in Bench Tests for the moment of their stop (green color – an arrangement of strain gauges, red – the skin cracks). Inside view to right board side of tail beam.

It is known that the more dangerous cracks are the break stringers and the cracks in skin.

The Fig.5.9 illustrates the dangerous cracks (break stringers and skin), selected from all cracks.

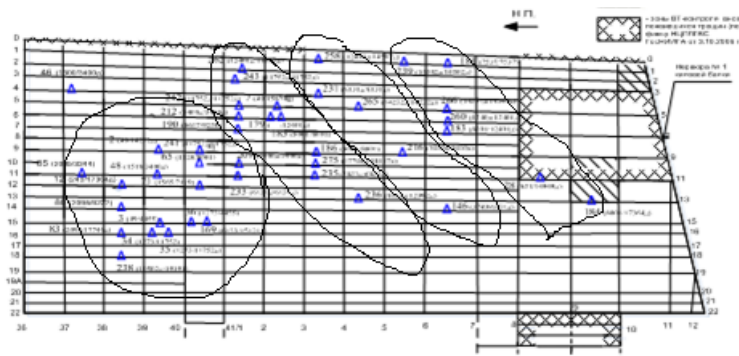


Fig.5.9 The disposition of dangerous cracks on the right side of tail beam.

It is also seen the concentration of dangerous cracks in some Local Zones directed to $\sim 45^\circ$ to the beam center line. Closer to a joint tail beam with the keel the four zones are visible. These zones are formed under action of torque of Z keel force, but the fourth zone (located between the 37 and 41 framing) have been additionally subjected with increased bending moment, caused by lateral force of P_z (Fig.5.6)

Before fatigue tests just this zone was considered as critical (a “weak” place), however the subsequent researches showed that the lateral force of P_z strongly overloads also a keel beam where subsequently critical destructions were found.

To check the localization of zones we will use Order Criteria for the designated four groups (zones) where under a rank of an observed crack there will be a serial number of its detection.

The results are listed in Table 5.1

Table 5.1

The distribution of through-cracks on Mil-26T helicopter’s tail beam.

Zone N	The disposition of Zone	Number of cracks	Order Rank Summa	R_i^2/n_i
Tail Beam				
1	6 – 10 framing	7	189	5103,0
2	3 - 6 framing	5	128	3276,8
3	1(41) – 4 framing	12	352	10325,3
4	36 – 41 framing	18	234	3042,0
Keel Beam				
5	7-10 rib	3	9	
6	5 – 7 rib	5	27	

For check of uniformity and shift existence (different values of parameters of shift in samples) let's use the Kraskell-Wallice's criterion of H [68] to experimental data depending on accessory to this or that (Zone N1 to Zone N4) Local Zone:

$$H = \frac{12}{N(N+1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(N+1) = 15,05 \quad (5.1)$$

where

$$N = \sum k_i = 7+5+12+18 = 42,$$

$\kappa = 4$ quantity of Local Zones,

$R_i = \sum_1^k R_i^j$ – the order rank summa for κ_i -th Zone

n_i – the sample volume in i-th Zone.

In spite of the fact that there is no exact decision for evaluation of the statistics critical value for our sample data, available estimates of critical values the statistician of Kraskell-Wallace criterion at approximately similar input parameters (less than 5 groups, no more than 8 supervisions in one separate group), is significant less than received on (5.1) value equal 15,05 [79].

In other words, the received results the moments of fatigue cracks emergences, are NON-UNIFORM through separate Zones, i.e. observed cracking in the described zones has local parameters of fatigue strength. Thereby the Hypothesis of Local Zones is confirmed in application to a tail beam. So we have possibility of representation of a large-size full Design by association of several Local zones.

Similar let's consider the helicopter Mil-26 keel beam.

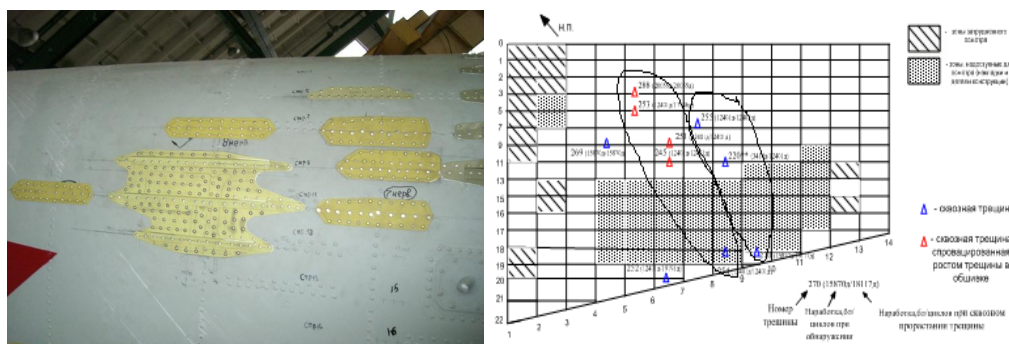


Fig.5.10 Fatigue through-cracks on right side of helicopter Mil-26 keel beam. Inside view to right board side of keel beam.

Let's compare two samples of two given-up Local Zones by Mann-Whitney's criterion Table 5.1 [80]:

The sum of ranks in samples:

$$U_4 = mn + 1/2m(m+1) - R_4 = 15 + 6 - 9 = 12 \quad (5.2)$$

$$U_5 = mn + 1/2n(n+1) - R_5 = 15 + 15 - 27 = 3 \quad (5.3)$$

Compared the minimal value of Test Statistics U_5 with critical $U_{kp} = 1$ (as usual the significant level $\alpha = 0,05$) we can see both samples are uniform. So we can assume that the moments of fatigue cracks emergences in given-up Local Zones belong to one sample and form ONE ZONE. It is confirmed also by Bench Tests and Operating experience.

Photos of the fatigue cracks which have arisen in this Zone despite frequent and thorough repairs during Bench Tests (Fig. 5.11, 5.12) and Operation (Fig. 5.13) are presented below.



Fig.5.11 The general view of repaired cracks on keel beam of helicopter Mil-26



Fig.5.12 Fatigue crack under splice plate of air-blow-off louver.

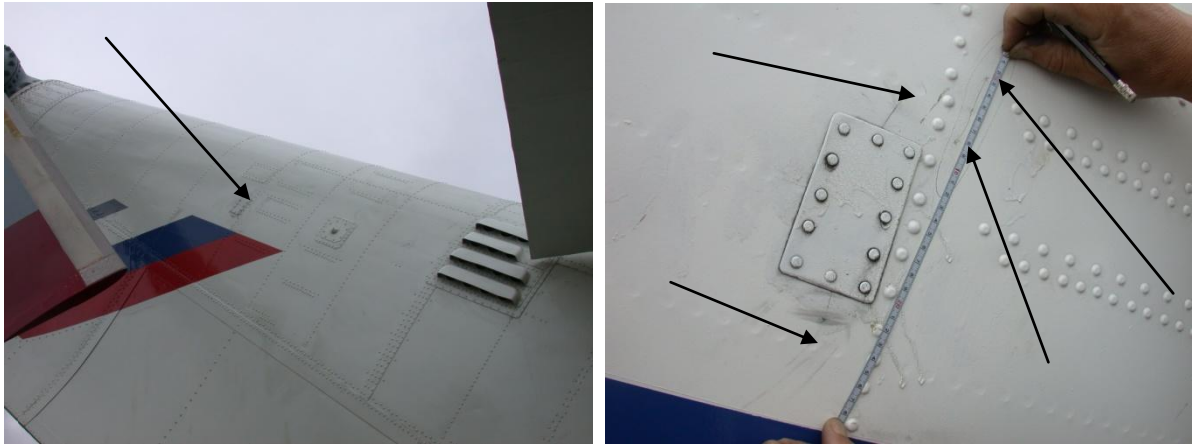


Fig.5.13 Fatigue cracks that was revealed on keel beam during Operation.

The same picture we can see in Local Zones given-up on Tail beam (Table 5.1). The Fig 5.14, 5.15 and 5.16 illustrate the fatigue cracks that emergence under repair pads and new ones in one Zone.

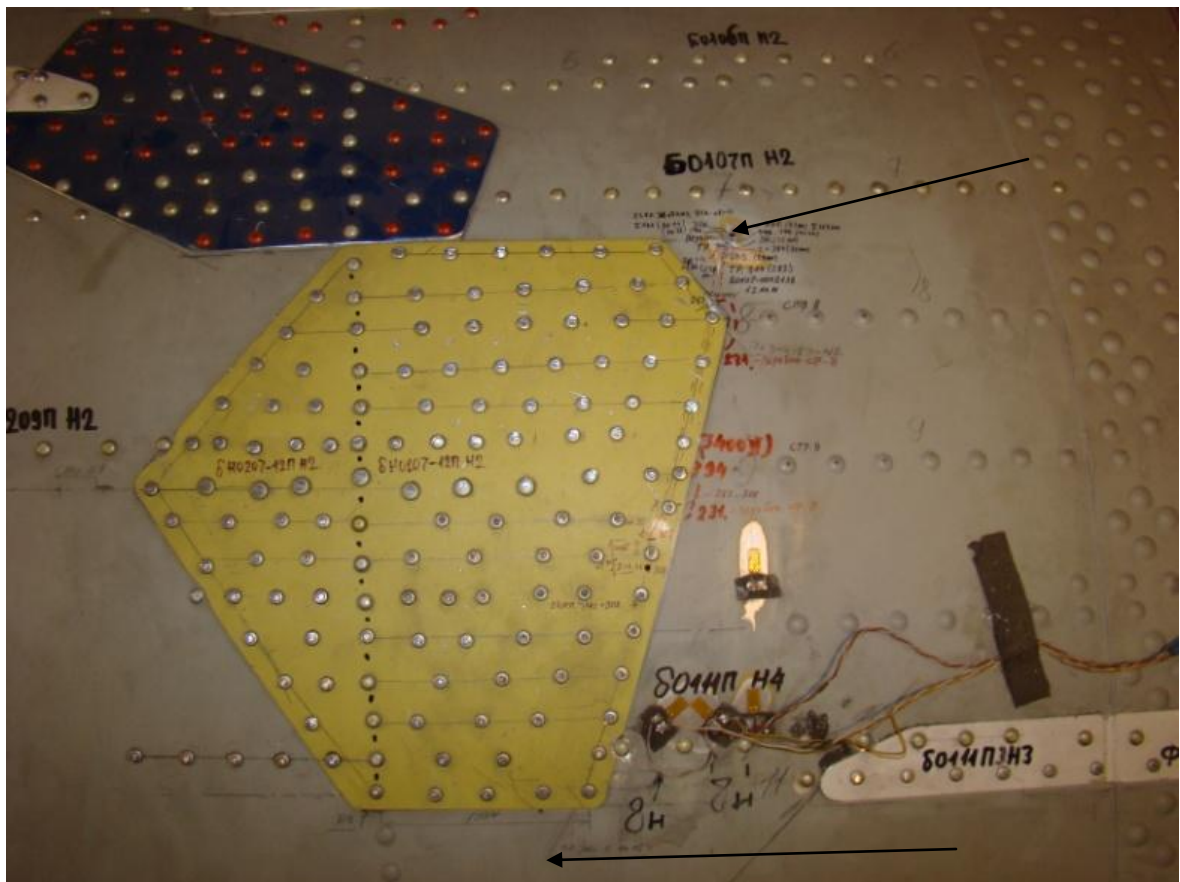


Fig.5.14 The fatigue crack that was revealed under repair pad (are shown by arrows)

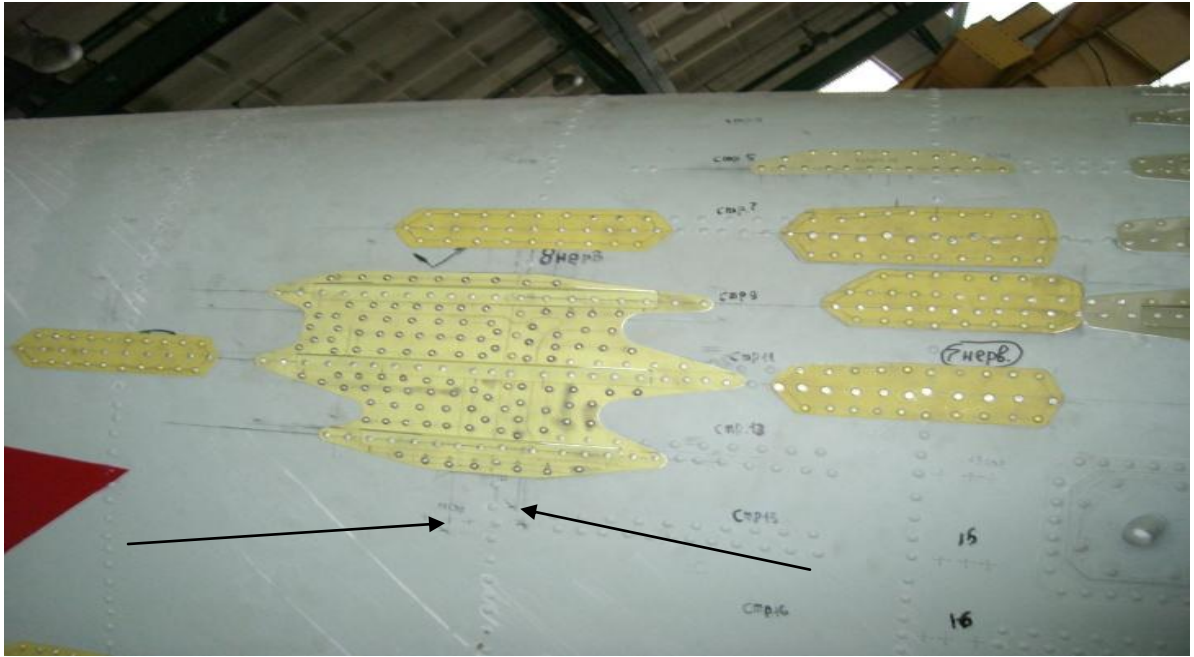


Fig.5.15 Two fatigue cracks (are shown by arrows) that were revealed under repair.



Fig.5.16 Three fatigue cracks that emergences under repair pads and new one. It is clear seen how three fatigue cracks have to coalesce to one mainline crack.

The analysis of the given results confirms existence of Local Zones of dependent FDA and therefore leads to the main conclusions of Hypothesis of Local Zones:

- the first cracks started being formed in a Zone between 36 and 41 frame. Before tests initially this Zone was considered as a “weak place” of a tail beam. The emergence of multiple cracks in this Zone redistributed loadings from the next zones, in particular – the third (Table 5.1) probably under action of negative stochastic relationships. As a result that cracks began to arise here at later stage, but it is already accelerated when their number began to grow under the influence of local positive stochastic relationships;

- the formed Local Zone became a new critical place where the main cracks already began to be formed by merge of intermediate cracks.

This Zone became as new object of “special attention”, therefore the numerous new strain gages were in addition pasted in this Zone (Fig. 5.7). The sticker of strain-gages was carried out at mouths of fatigue cracks in a skin of a beam:

- the keel beam is may be considered as lonely Local Zone, and if the fatigue crack are revealed in this Zone it seems obvious and necessity to repair the total area of keel beam. The emergence of fatigue cracks is a warning call that the Design is “fatigue overheated”;

- considering possibilities of early detection of fatigue damages to Local Zones it is possible to reduce considerably the volume and labor input of carried-out repairs, to reduce number of strain-gages channels and to raise an attention to research of development of fatigue cracks.

The life time moments than the fatigue cracks are revealed, propagated and repaired, is listed in Table 5.2 (attachment 9) for two Programs of loading: Block loading (1st program) + Repeated Static Load (2st program). It is necessary to reduce them to equivalent loads. This procedure is performing in Attachment 7 when the estimation of the Safe Life Time is carried out.

5.4.4 Stage 4. Decision to the test continuation

By results of processing of data on intensity of fatigue cracks emergence in a Local Zones, the analysis of measurements of deformations from strain gages, periodic visual and NDT check-up, character of AE-signals there is accepted and defined the decision on a continuation of tests and the list of further researches.

Usually it is necessary:

- to carry out the repair of the damaged Local Zone and to continue tests according to the same load program. The purpose is to evaluate the efficiency of repair and maintainability of a studied zone;
- to carry out the repair of the damaged Zone, to estimate its durability (Safe Life Time) and to change conditions of loading for an evaluation of Fatigue Strength characteristics of other Local Zones,
- as it is mentioned earlier, these repair needs to be done for TOTAL ZONE, instead of separate cracks and damages, otherwise, there will be continuation of checking of the same Local Zone about which the necessary experimental information is already received. Besides further cracking of a Design, (if it isn't a definite purpose of Tests investigation Program), will sharply reduce reliability of carrying out the further researches,
- to carry out repair of the out of interest Local Zones and change to research of survivability of a Design: to bring artificial damages (concentrators) and develop from them natural fatigue cracks or to investigate the growth rate of already existing fatigue cracks,
- at achievement of assumed critical length of a studied fatigue crack – to carry out tests for residual strength.

5.4.5 Stage 5. Development of recommendations about results of tests

At this stage fatigue tests of a large-size Design come to an end.

After that it is begin to develop recommendations for monitoring of a technical condition for further Bench Tests and Operation. If necessary essential completions of a Design, fatigue tests of the separate units, fragments-specimens cut out from a Design, static tests of a Design are carried out. Special attention gives to a complete repair of the found Local Zones.

5.5 Phenomenological description of process of fatigue damages accumulation

Analyzing the process of FDA on a tail beam as some description of fatigue failure similar to process in large-size Design it can be allocated the following stages, obviously fair and for others large-dimensional Designs:

- at the beginning of FDA, her uniform distribution on a surface of tested Design is observed. It may be a consequence of equal-strain principle of design;
- concentration of fatigue cracks on separate zones in which goes stochastic dependent (with positive relationship) emergence of cracks, and which is temporary relax emergence of cracks (negative stochastic relationship) in other zones is observed. Thus various Local Zones with his own the characteristics of Fatigue Strength are formed;
- considering an FDA independently for each Zone separately, it is possible to allocate the Zone with being accelerated cracking. In further this Zone will define durability (Safe Life Time) of total Design. The existence of negative stochastic relation localize this Zone;
- monitoring of this dangerous critical Zone allows to carry out one of the principles of operation on a technical condition, and at a certain designing of this Zone and the principle of safe destruction (Fail Safe Concept).

The evaluation of reliability needs to be performed taking into account what of destruction models (on technical condition or on safe destruction) is accepts for critical.

Application of the principles of a Hypothesis of Local Zones gives the chance to estimate characteristics of a studied large-size Design and to make the effective decision directed on increase of her reliability.

Who is allocated with knowledge, bears on itself responsibility. Who has no knowledge, lives only with his assumptions.

Sufy Selim Abdali

CONCLUSION.

1 The Hypothesis of Local Zones where fatigue damages dependent accumulation is developed and experimentally confirmed.

Phenomenological and mathematical models of fatigue damages' accumulation in Local Zones, the sizes of a Local Zone and characteristics of fatigue strength are developed.

2 Due to experimental data received by the Author, and also these of other Authors main assumption of the Hypothesis of Local Zones are proved:

- the full (large) Design can be presented as association of several Local Zones;
- in Local Zones accumulation of fatigue damages happens stochastic dependent, and beyond its limits – is independent;
- with increase in distance from initial damage, stochastic relationship between the moments of emergence of damages decreases;
- some Design units may be represented as one Local Zona;
- characteristics of durability, survivability of large-size Design are defined by characteristics of Local Zones.

3 It is experimentally shown that fatigue damage's accumulation (Design cracking) passes from evenly casual damages to concentrated in several „weak places” (Local Zone) of a Design.

Often this Local Zone defines the durability of total Design.

4 In Local Zones stochastic relationships (both positive and negative) localize a disposition of initial damages which can be considered subsequently as one damage. Negative relationship leads to formation of other Local Zones. The distance of negative relationship influence is more, than action of the positive one. The later the initial fatigue damage is arising; the early fatigue damages are arising in other Local Zones. That leads to “fatigue

overheats” of Design as accelerated flow of damages. Early emergence fatigue damages allow managing the NDT and other monitoring of technical condition of Local Zone and employ the Fail Safe Concept.

5 The durability of a Local Zone is defined by function of double exponential distribution with logarithm of life time (cycles).

6 The mathematical models of the sizes of Local Zones and characteristics of their Fatigue Strength are provided and experimentally confirmed. It is shown that the main characteristics of Fatigue Strength of a Local Zone (parameters of fatigue durability distribution function, fatigue limits of endurance distribution function, S-N fatigue curve and sensitivity threshold on cycles may be described by only four parameters.

As example the evaluation of Limited Safe Life Time of helicopter Mill-26 keel beam is presented.

7 One of the suitable methods for monitoring of Local Zones of Design at Tests and in Operation is the Acoustic Emission method. The AE-method allows determinatig the beginnig of degradation (fatigue damages accumulation) process even BEFORE the emergence of visible damages, both at static, variable loads and loading of other nature. In this case the Local Zone can serve as the indicator of FDA for all total Design, that allows to develop the Fail Safe Concept of the Design.

8 When required to carrying out the repair of a Local Zone it is necessary to repair TOTAL LOCAL ZONE, instead of separate damages in it.

9 The Hypothesis of Local Zones together with application of a method of acoustic emission allows to manage the monitoring of a large-(full)-size Designs with minimization of temporary, material and human expenses and to provide most to an early detection of damages at Branch Tests and Operation.

10 The methodology for fatigue Bench Tests of large-size Designs’ carrying out with application of NDT methods on the basis of acoustic emission is developed.

“Skill of the Designer is not to develop a Design there are not any cracks, but to design such One, that will work at any quantity of cracks”.

The Dream of Fail Safe Concept Designer

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ATTACHMENTS

Table 3.1

The Result of Fatigue Test of “V”a) -type specimens

NN	Number of cycles till crack revelation N	Concentrator's durability $\tau = \lg N$
1	145500	5,1628629933
2	150400	5,1772478363
3	153200	5,1852587653
4	154000	5,1875207208
5	162500	5,2108533653
6	168300	5,226084116
7	172300	5,2362852774
8	172700	5,2372923376
9	173300	5,2387985627
10	181200	5,2581581933
11	185000	5,2671717284
12	191900	5,2830749747
13	193000	5,285557309
14	194500	5,2889196057
15	202700	5,3068537487
16	213500	5,3293978794
17	221800	5,3459615418
18	243300	5,3861421089
19	256400	5,4089180208
20	265300	5,4237372500

Sample Size	$n_{in/a} = 20$
Location Parameter (sample mean)	$\hat{a}_{in/a} = 5,27230$
Sample Variance	$\hat{b}_{in/a}^2 = 0,00582$
Scale Parameter (Root-mean-square deviation)	$\hat{b}_{in/a} = 0,07628$
Coefficient of Variation	$\hat{v}_{in/a} = 1,45\%$
Kramer-Mizes-Smirnov Test:	$W_n^2 = 0,0450$

Table A1

Goodness-of-Fit Tests for Fatigue life
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0963925	0,119756	0,172697
DMINUS	0,0752841	0,0821893	0,135786
DN	0,0963925	0,119756	0,172697
P-Value	0,992391	0,936569	0,589752

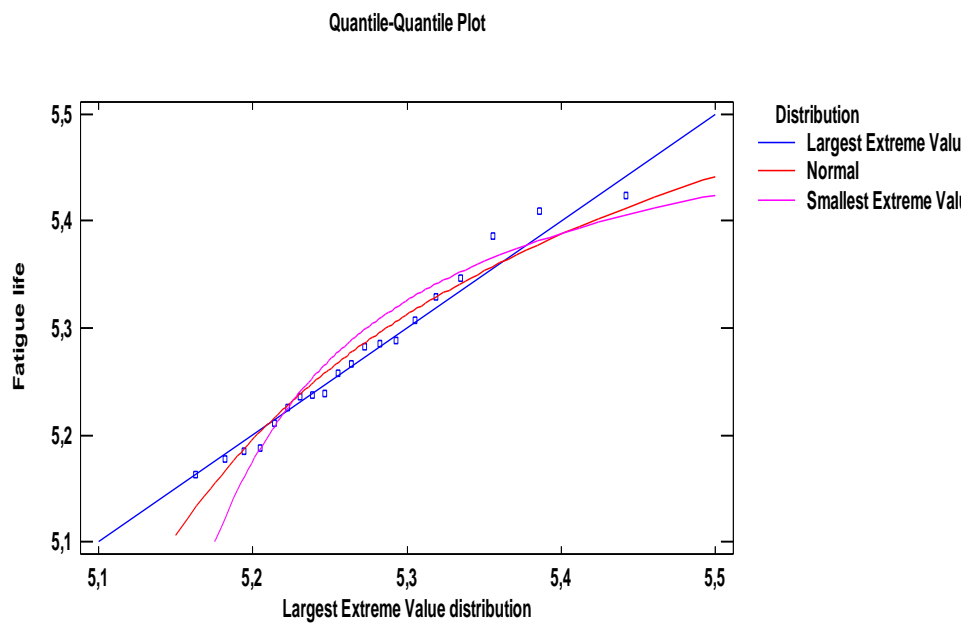


Fig.A1 The comparison of three main CDF for experimental data.

ATTACHMENT 2

Table 3.2

The Results of fatigue tests of “V”b)-type specimens

The Cycles Number (N) till Cracks revelation and Concentrator durability (τ)				
NN	The 1 st Crack		The 2 nd Crack	
	$N_{(1)}$	$\tau_{(1)} = \lg N_{(1)}$	$N_{(2)}$	$\tau_{(2)} = \lg N_{(2)}$
1	94 600	4,97589	165 100	5,21775
2	100 300	5,0013	144 600	5,16017
3	112 100	5,04961	129 400	5,11193
4	112 800	5,05231	136 100	5,13386
5	116 200	5,06521	195 500	5,29115
6	127 600	5,10585	221 600	5,34557
7	128 500	5,1089	145 300	5,16227
8	131 100	5,1176	194 500	5,28892
9	131 300	5,11826	218 600	5,33965
10	132 800	5,1232	161 200	5,20737
11	159 500	5,20276	169 500	5,22917
12	164 800	5,21696	167 300	5,2235
13	189 600	5,27784	208 600	5,31931
14	193800	5,28735	204 100	5,30984

$n_{ot/b(1)} = n_{ot/b(2)} = 14:$	The 1 st Crack:		The 2 nd Crack:	
5,23860	$\hat{a}_{ot/b(1)} = 5,12165$		$\hat{a}_{ot/b(2)} =$	
0,00610	$\hat{b}^2_{ot/b(1)} = 0,00899$		$\hat{b}^2_{ot/b(2)} =$	
0,07808	$\hat{b}_{ot/b(1)} = 0,09482$		$\hat{b}_{o/tb(2)} =$	
1,49%	$\hat{v}_{ot/b(1)} = 1,85\%$		$\hat{v}_{ot/b(2)} =$	
The Pooled Sample:				
$n_{ot/b} = 28:$	$\hat{a}_{ot/b} = 5,18012$	$\hat{b}^2_{ot/b} = 0,01081$	$\hat{b}_{ot/b} = 0,10397$	$\hat{v}_{ot/b} = 2,0\%$
Kramer-Mizes-Smirnov Test				$\hat{W}^2_n = 0,058$

Table A2.1

Goodness-of-Fit Tests for Fatigue life OUTER
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,102194	0,10084	0,136161
DMINUS	0,126569	0,112057	0,0998214
DN	0,126569	0,112057	0,136161
P-Value	0,760834	0,873478	0,676903

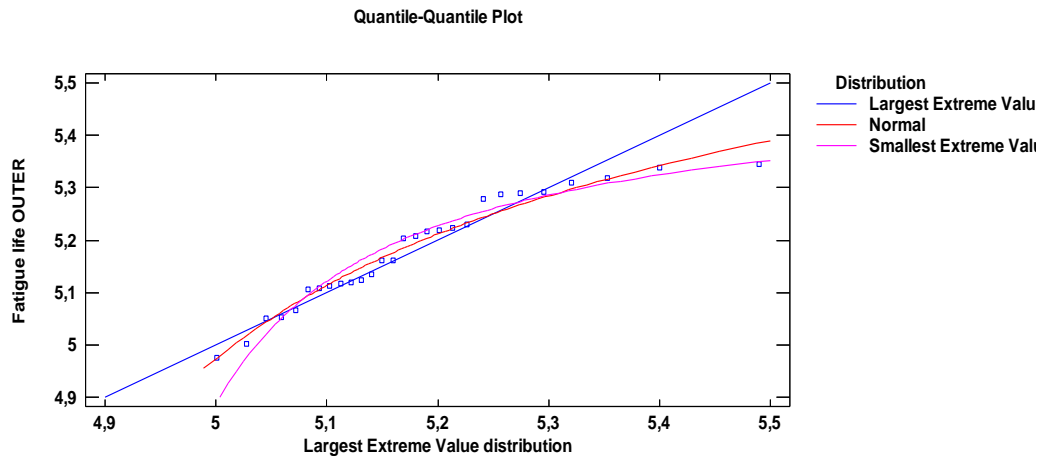


Fig.A2.1 The Comparison of three main CDF (n=28)

Table A2.2

Goodness-of-Fit Tests for fatigue life INNER total
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0554592	0,0911952	0,158127
DMINUS	0,0548811	0,0522328	0,124563
DN	0,0554592	0,0911952	0,158127
P-Value	0,99271	0,700564	0,0995295

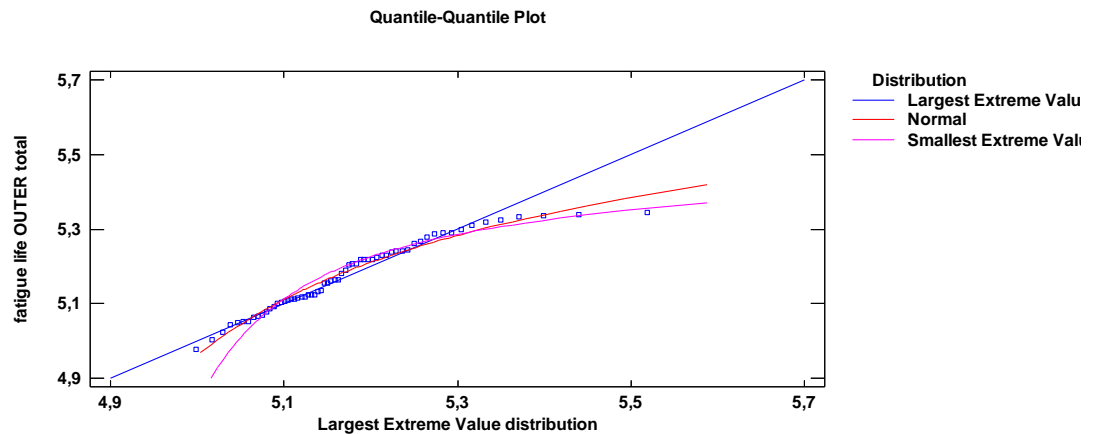


Fig.A2.2 The Comparison of three main CDF (n=62)

ATTACHMENT 3

Table 3.3.

The fatigue Test results of “V”c)-type specimens (two “inner” concentrators)

The Cycles Number (N) till Cracks revelation and Concentrator durability (τ)				
№№	The 1 st Crack		The 2 nd Crack	
	$N_{(1)}$	$Lg N_{(1)}$	$N_{(2)}$	$Lg N_{(2)}$
1	151 300	5,179838928	165 000	5,219846386
2	196 800	5,2940250941	211 200	5,3246939139
3	198 600	5,2979792442	221 500	5,3453737306
4	252 800	5,4027770696	260 300	5,4154741681
5	181 700	5,2593549273	200 300	5,3016809493
6	138 900	5,1427022457	170 400	5,2314695904
7	155 300	5,1911714557	197 600	5,2957869403
8	155 600	5,1920095927	166 400	5,221153322
9	186 700	5,2711443179	206 000	5,3138672204
10	195 400	5,2909245594	250 000	5,3979400087
11	206 200	5,3142886609	235 000	5,3710678623
12	149 000	5,1731862684	165 200	5,218010043
13	307 100	5,4872798164	316 100	5,4998244958
14	176 500	5,2467447097	189 100	5,2766915288
15	190 100	5,2789821169	203 400	5,3083509486
16	128 700	5,1095785469	163 900	5,2145789536
17	133 700	5,1261314073	163 500	5,213517757
18	156 100	5,1934029031	173 700	5,2397998184
19	183 500	5,2636360686	206 100	5,3140779918
20	164 900	5,2172206556	169 800	5,2299376859

$n_{in/c(1)} = n_{in/c(2)} = 20:$

$\hat{a}_{in/c(1)} = 5,24662$

$\hat{a}_{in/c(2)} = 5,29766$

$\hat{b}_{in/c(1)}^2 = 0,00840$

$\hat{b}_{in/c(2)}^2 = 0,00626$

$\hat{b}_{in/c(1)} = 0,09165$

$\hat{b}_{in/c(2)} = 0,07912$

$\hat{v}_{in/c(1)} = 1,75\%$

$\hat{v}_{in/c(2)} = 1,49\%$

$n_{inc} = 40:$

$\hat{a}_{in/c} = 5,27214 ;$

$\hat{b}_{in/c}^2 = 0,00781 ;$

$\hat{b}_{in/c} = 0,08837 ;$

$\hat{v}_{in/c} = 1,68\%$

Table A3.1

Goodness-of-Fit Tests for fatigue life INNER
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0794737	0,116653	0,182816
DMINUS	0,0822091	0,0563634	0,126849
DN	0,0822091	0,116653	0,182816
P-Value	0,949745	0,647765	0,138011

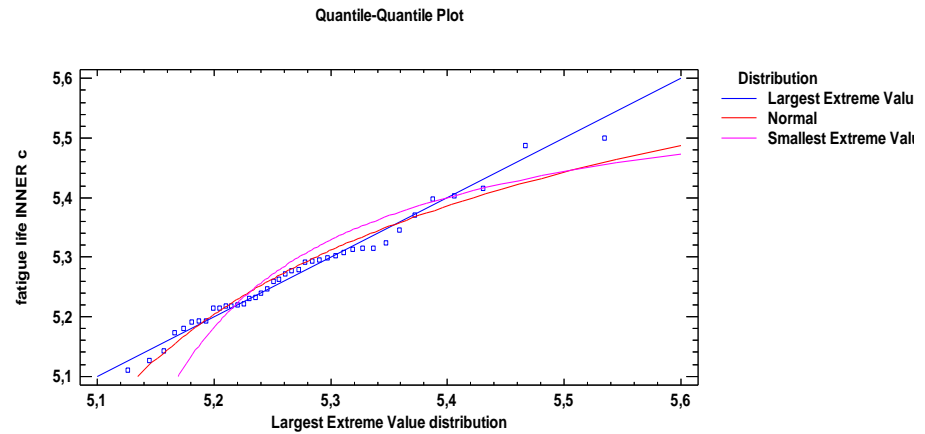


Fig.A3.1 The Comparison the main CDF for total data of INNER concentrators durability (n=40)

Table A3.2

Goodness-of-Fit Tests for fatigue life INNER total
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0554592	0,0911952	0,158127
DMINUS	0,0548811	0,0522328	0,124563
DN	0,0554592	0,0911952	0,158127
P-Value	0,99271	0,700564	0,0995295

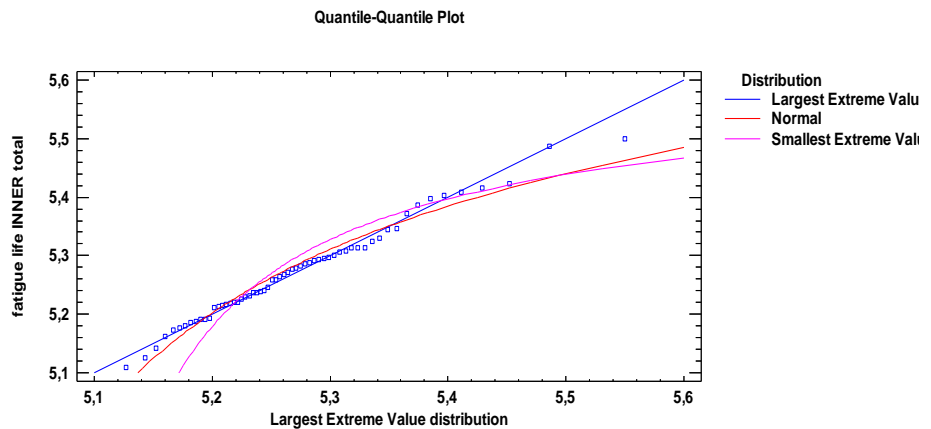


Fig.A3.2. The Comparison the main CDF for total data of INNER concentrators durability (n=60)

ATTACHMENT 4.

Table 3.4

The fatigue test result of “V”d)-type specimens

№	The Cycles Number (N) till Cracks revelation and Concentrator durability (τ)								
	The 1 st Crack			The 2 nd Crack			The 3 rd Crack		
	N	LgN	Mark	N	LgN	mark	N	LgN	mark
1	105600	5,0236639182	outer	173600	5,2395497208	outer	185300	5,2678754193	inner
2	110100	5,041787319	outer	165400	5,2185355052	outer	204400	5,3104808915	inner
3	112400	5,0507663112	outer	129300	5,1115985249	outer	193300	5,286231854	inner
4	115800	5,0637085594	outer	127200	5,1044871113	outer	188800	5,27600199	inner
5	117400	5,0696680969	outer	170400	5,2314695904	inner	210500	5,3232521002	outer
6	119200	5,0762762554	outer	143000	5,1553360375	outer	289700	5,4619484952	inner
7	121700	5,0852905782	outer	130600	5,1159431769	outer	298100	5,474361976	inner
8	123400	5,0913151597	outer	165300	5,2182728536	outer	230600	5,362859303	inner
9	126200	5,1010593549	outer	154600	5,1892094896	outer	235200	5,3714373174	inner
10	132700	5,1228709229	outer	207300	5,3165993021	inner	216800	5,3360592779	outer
11	133100	5,1241780555	outer	168000	5,2253092817	inner	174300	5,2412973871	outer
12	135800	5,1328997699	outer	181500	5,2588766294	inner	215700	5,3338501451	outer
13	143100	5,1556396338	outer	165800	5,2195845262	inner	182500	5,2612628688	outer
14	145800	5,163757524	outer	174900	5,2427898095	outer	235900	5,3727279409	inner
15	151900	5,1815577739	outer	159000	5,2013971243	inner	184300	5,2655253352	outer
16	160600	5,2057455409	outer	169700	5,2296818423	outer	205500	5,3128118262	inner
17	174400	5,2415464806	outer	199400	5,299725154	outer	203700	5,308991029	inner

Mark: “inner” crack on “inner” concentrator

“outer”: crack on “outer” concentrator

Table 3.9

The fatigue test result of "M.N.Stepnov [30,fig.1,b] specimens (load cycles $N \times 10^{-6}$)

Alloy	B95		AB											
	30		20		18		16,5		15		14		13	
σ_a (kg/mm ²)	N	P%	N	P%	N	P%	N	P%	N	P%	N	P%	N	P%
1	0,460	0,10	1,15	0,07	1,60	0,10	3,85	0,20	8,05	0,50	16,3	1,00	29,5	1,00
2	0,470	0,22	1,20	0,20	1,75	0,30	4,30	0,50	8,96	1,50	22,0	3,00	43,0	3,00
3	0,500	0,50	1,30	0,40	2,00	0,50	4,60	1,00	11,0	2,10	26,0	5,00	49,0	5,00
4	0,505	0,60	1,35	0,50	2,20	0,60	4,95	1,30	12,0	3,50	27,0	7,00	60,0	7,00
5	0,507	0,90	1,40	0,60	2,30	0,90	5,10	2,00	12,5	4,90	28,2	9,0	63,0	9,00
6	0,520	1,00	1,45	0,90	2,60	1,00	5,45	2,50	13,3	6,00	30,5	11,0	69,0	11,0
7	0,560	1,30	1,50	1,10	2,65	1,30	5,50	4,90	14,5	10,0	34,5	14,5	79,0	14,5
8	0,599	2,00	1,55	1,50	2,70	1,90	6,10	6,00	15,3	15,0	40,0	15,0	88,0	15,0
9	0,610	2,50	1,60	2,00	3,06	2,50	6,80	10,0	16,5	20,0	44,0	19,5	90,0	19,5
10	0,690	4,90	1,85	2,50	3,60	4,90	8,05	15,0	26,0	30,0	49,5	20,0	99,0	20,0
11	0,760	7,00	2,20	4,90	3,95	6,00	9,05	20,0	31,0	40,0	49,9	21,0	130	21,5
12	0,810	10,0	2,40	6,00	4,50	10,0	13,5	30,0	35,0	50,0	57,0	30,0	150	30,0
13	0,920	13,0	2,60	10,0	5,10	13,0	14,5	40,0	42,0	60,0	65,0	35,0	165	35,0
14	1,10	20,0	2,80	13,0	5,95	20,0	16,0	50,0	52,0	70,0	79,0	42,0	175	42,0
15	1,30	30,0	3,05	20,0	7,05	30,0	22,0	60,0	65,0	90,0	80,5	45,0	203	45,0
16	1,50	40,0	3,70	30,0	8,90	40,0	27,5	70,0	89,0	92,5	92,0	50,0	230	50,0
17	1,56	50,0	4,10	40,0	9,80	50,0	37,5	90,0	145	95,0	110	55,0	255	55,0
18	1,70	60,0	4,70	50,0	13,0	60,0	39,5	92,5	190	97,0	140	65,0	270	65,0
19	2,00	70,0	5,30	60,0	14,5	80,0	52,0	95,0	300	99,5	160	70,0	330	60,0
20	2,55	90,0	6,10	70,0	18,0	90,0	77,0	97,0			180	90,0	370	65,0
21	3,20	92,5	7,10	80,0	25,0	95,0	91,0	98,0			220	91,5	450	70,0
22	3,80	95,0	9,20	90,0	31,0	98,0	110	98,5			280	92,5	630	85,0
23	4,60	97,0	11,0	92,5	38,0	99,0	170	99,95			320	95,0	735	90,0
24	5,20	98,0	13,0	95,0	49,5	99,5					430	92,5	860	91,0
25	6,00	99,0	16,0	97,0	61,0	99,6					605	95,0	895	92,5
26	7,40	99,9	20,0	99,5	86,0	99,95					960	97,5	970	95,0
27	10,0	99,99	30,0	99,8										

ATTACHMENT 5

Table 3.5 (to be continued)

The Results of Fatigue Test of “W”-type specimens

Concentrator durability $\tau = LgN$																
Diameter	$\varnothing = 3 \text{ mm}$				$\varnothing = 8 \text{ mm}$				$\varnothing = 10 \text{ mm}$				$\varnothing = 20 \text{ mm}$			
Specimen No	The Concentrator No				The Concentrator No				The Concentrator No				The Concentrator No			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	5,170	5,267	5,396	5,262	4,756	4,914	5,176	4,924	4,708	4,720	4,623	4,628	5,236	5,433	5,470	5,045
	5,107	5,281	5,410	5,201	4,690	4,964	5,104	4,929	4,602	4,638	4,544	4,672	4,958	5,305	5,456	5,230
	5	6	7	8	5	6	7	8	5	6	7	8				
	4,973	5,068	5,045	5,061	4,748	5,041	4,663	4,653	4,724	4,699	4,732	4,681				
	4,987	5,093	5,033	5,083	4,857	5,072	4,771	4,763	4,663	4,648	4,740	4,695				
2	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	5,386	5,233	5,140	5,182	5,033	4,914	4,820	4,748	4,778	4,628	4,695	4,763	5,393	5,265	5,428	5,000
	5,377	5,204	5,158	5,196	4,978	4,690	4,853	4,875	4,771	4,602	4,658	4,806	5,173	5,185	5,410	5,281
	5	6	7	8	5	6	7	8	5	6	7	8				
	5,210	5,322	5,243	5,121	4,756	4,732	4,949	4,771	4,703	4,860	4,699	4,712				
5,220	5,258	5,305	5,243	4,778	4,806	4,940	4,785	4,748	4,820	4,663	4,756					

continued Table 3.5

3	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	5,253	5,021	5,444	5,265	4,785	4,908	4,833	4,863	4,672	4,813	4,686	4,628	5,389	5,505	5,407	5,079
	5,326	5,176	5,401	5,241	4,771	4,929	4,892	4,763	4,690	4,633	4,591	4,613	5,114	5,470	5,332	5,190
	5	6	7	8	5	6	7	8	5	6	7	8				
	5,086	5,155	5,217	5,146	4,813	4,740	4,875	4,845	4,813	4,602	4,653	4,491				
5,013	5,170	5,161	5,225	4,799	4,820	4,886	4,852	4,699	4,623	4,760	4,568					
4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
	5,207	5,544	5,371	5,301	4,875	4,806	4,991	4,740	4,544	4,643	4,580	4,462	5,455	5,500	5,277	5,164
	5,097	5,490	5,360	5,378	4,778	4,886	5,053	4,699	4,568	4,531	4,724	4,533	5,196	5,369	5,322	5,312
	5	6	7	8	5	6	7	8	5	6	7	8				
	5,297	5,305	5,312	5,332	4,763	4,903	4,959	4,839	4,572	4,740	4,716	4,690				
5,220	5,328	5,210	5,322	4,875	4,924	5,013	4,934	4,519	4,775	4,708	4,690					
5	1	2	3	4					1	2	3	4				
	5,267	5,146	5,512	5,241					4,695	4,708	4,591	4,544				
	5,272	5,121	5,571	5,217					4,677	4,756	4,580	4,623				
	5	6	7	8					5	6	7	8				
	5,407	5,334	5,4576	5,423					4,613	4,699	4,728	4,716				
5,377	5,190	5,450	5,439	4,633	4,653	4,744	4,643									

In Lower Part of Table Boxes for Concentrator No there are the durability of internal side of hole concentrator.

ORDER STATISTICS CUMULATIVE DISTRIBUTION FUNCTIONS (CDF) OF THE MIXTURE OF TWO DISTRIBUTIONS

Let's consider a static series, that included n_1 of observed data of random variable (RV) X with cumulative distribution function (CDF) $F_1(x)$ and n_2 observed data of RV with CDF $F_2(x)$, and $n_1 + n_2 = n$. We will consider the data as stochastically independent variables.

Let's find density of probability distribution $p_\zeta(x)$ for the ζ -th order statistics from such sequence of data (statistical series).

ζ -th plot may belong both to family of RV with CDF $F_1(x)$, so to RV with CDF $F_2(x)$ family. Then the probability of an event that " ζ -th plot belongs to family with CDF $F_1(x)$ and accepts value in the range of dx it is possible to define:

$$p_\zeta^{(1)}(x)dx = \sum_{i=1}^{\zeta-1} C_{n_1}^i C_{n_2}^{\zeta-1-i} F_1^i(x)(1-F_1(x))^{n_1-i-1} (n_1-i) dF_1(x) * F_2^{\zeta-1-i}(x)(1-F_2(x))^{n_2-\zeta+1+i}$$

and the probability of an event that " ζ -th plot belongs to family with CDF $F_2(x)$ and accepts value in the range of dx :

$$p_\zeta^{(2)}(x)dx = \sum_{i=1}^{\zeta-1} C_{n_2}^i C_{n_1}^{\zeta-1-i} F_2^i(x)(1-F_2(x))^{n_2-i-1} (n_2-i) dF_2(x) * F_1^{\zeta-1-i}(x)(1-F_1(x))^{n_1-\zeta+1+i}$$

where $C_n^i = n! / [(n-i)! i!]$ binomial coefficient.

As this two events are incompatibility, the probability that RV X take a value within the interval dx :

$$p_\zeta(x)dx = (p_\zeta^{(1)} + p_\zeta^{(2)})dx \tag{A6.1}$$

and CDF of the ζ -th Order Statistics is determined from:

$$F_{(\zeta)}(x) = \int_{-\infty}^{+\infty} p_\zeta(x)dx \tag{A6.2}$$

For the case of test results "V)d)-type specimens (Attachment 4, Table 2.4) we observed the data of two different NDF (2.1) - one with parameters (2.54) and two - with parameters (2.83), i.e. $n_1 = 1, n_2 = 2, n = 3$. Then the density of probability distribution of the minimal sample member $\zeta = 1$ (the 1st Order Statistic) and her CDF have an expression:

$$p_{(1)}(x)dx = (1 - F_2(x))^2 dF_1(x) + 2(1 - F_2(x))(1 - F_1(x))dF_2(x) = -d(1 - F_1(x))(1 - F_2(x))^2$$

or

$$F_{(1)}(x) = 1 - (1 - F_1(x))(1 - F_2(x))^2 \quad (\text{A6.3})$$

Similar, the density of probability distribution of the middle sample member $\zeta = 2$ (the 2nd order Statistics) and her CDF have the expression:

$$p_{(2)}(x)dx = 2F_2(x)(1 - F_2(x))dF_1(x) + 2F_1(x)(1 - F_2(x))dF_2(x) + 2F_2(x)(1 - F_1(x))dF_2(x) = \\ = 2d(F_1(x)F_2(x) - 2dF_1(x)F_2^2(x) + dF_2(x))$$

$$F_{(2)}(x) = F_2^2(x) + 2F_1(x)F_2(x)(1 - F_2(x)) \quad (\text{A6.4})$$

And the density of probability distribution of the 3rd Order Statistic($\zeta = 3$) and her CDF

$$p_{(3)}(x) = F_2^2(x)dF_1(x) + 2F_1(x)F_2(x)dF_2(x) = dF_1(x)F_2^2(x)$$

$$F_{(3)}(x) = F_1(x)F_2^2(x) \quad (\text{A6.5})$$

For the great values of n and κ (κ - the quantity of CDF observed) the CDF of minimal value of sample member (the 1st Order Statistic) and the maximal value of sample member:

$$F_{(1)n} = 1 - (1 - F_1(x))^{n_1} (1 - F_2(x))^{n_2} \dots (1 - F_k(x))^{n_k} \quad (\text{A6.6})$$

$$F_{(n)n} = F_1^{n_1}(x)F_2^{n_2}(x)\dots F_n^{n_k}(x) \quad (\text{A6.7})$$

where $n_1 + n_2 + \dots + n_k = n$

In general case of System with n components, the CDF of the 1st (1st emergences of failure) and CDF of n -th (all components failure) Order Statistics for the System of n component, are expressed as [33]:

$$F_{(1)}(\tau) = 1 - \int_{\tau}^{\infty} \int_{\tau}^{\infty} \dots \int_{\tau}^{\infty} p(\tau_1; \tau_2; \dots; \tau_n) d\tau_1 d\tau_2 \dots d\tau_n \quad (\text{A6.8})$$

$$F_{(n)}(\tau) = \int_{-\infty}^{\tau} \int_{-\infty}^{\tau} \dots \int_{-\infty}^{\tau} p(\tau_1; \tau_2; \dots; \tau_n) d\tau_1 d\tau_2 \dots d\tau_n \quad (\text{A6.9})$$

where $p(\tau_1; \tau_2; \dots; \tau_n)$ – the density of multidimensional (n - size) probability distribution.

A7.1 Stress Level 30kg/mm2, sample volume – 463 specimens

Table A7.1.1

Goodness-of-Fit Tests for LgN(30kg/mm2)¹¹
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,141846	0,149593	0,151134
DMINUS	0,105895	0,130341	0,178785
DN	0,141846	0,149593	0,178785
P-Value	0,648985	0,581462	0,356975

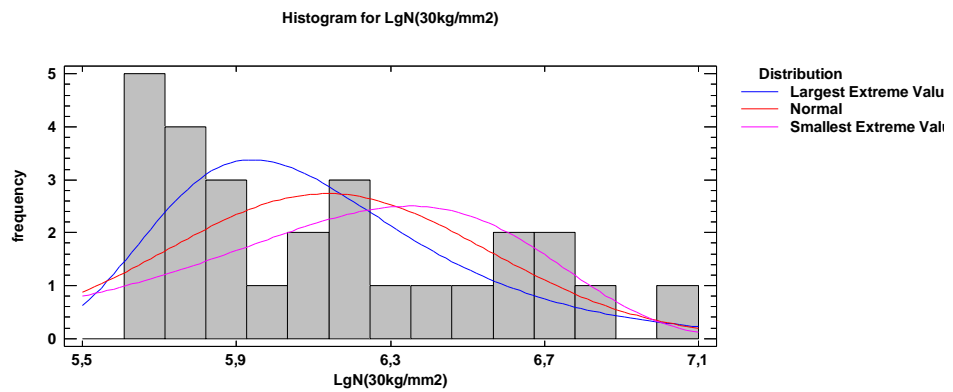


Fig.A7.1 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (30kg/mm2).

Table 7.1.2

Lower Tail Area (p if $X \leq x$) for LgN(30kg/mm2)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
4,90784	1,64594E-12	0,00173023	0,0325816
5,52132	0,021579	0,0718992	0,131545
6,1348	0,581386	0,5	0,45148
6,74828	0,92619	0,928101	0,922463
7,36176	0,989218	0,99827	0,999981

¹¹ In this attachment calculations had been made by use the Statistical Program "Statgraphics Centurion XXI.Version 16.1.17 (evaluation mode).

A7.2 Stress Level 20kg/mm2, sample volume – 600 specimens

Table 7.2.1

Goodness-of-Fit Tests for LgN(20kg/mm2)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,145635	0,131531	0,147069
DMINUS	0,0825054	0,118338	0,178423
DN	0,145635	0,131531	0,178423
P-Value	0,615832	0,738575	0,359496

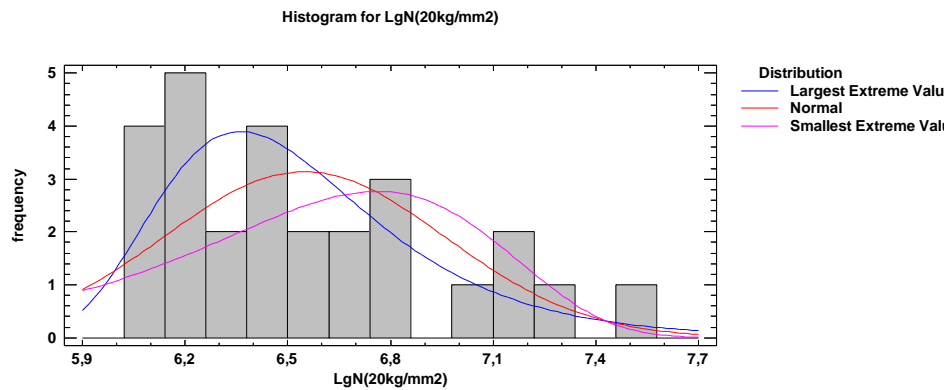


Fig.A7.2 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (20 kg/mm2).

Table 7.2.2

Lower Tail Area (p if $X \leq x$) for LgN(20kg/mm2)

X	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
5,23856	8,63762E-18	0,000739036	0,0287591
5,89337	0,00985306	0,0559747	0,124801
6,54819	0,580863	0,5	0,456088
7,20301	0,93812	0,944025	0,93808
7,85783	0,992517	0,999261	0,999997

A7.3 Stress Level 18kg/mm2, sample volume – 500 specimens

Table 7.3.1

Goodness-of-Fit Tests for LgN (18kg/mm2)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,117375	0,124591	0,146507
DMINUS	0,0944304	0,100363	0,154589
DN	0,117375	0,124591	0,154589
P-Value	0,866265	0,814398	0,56337

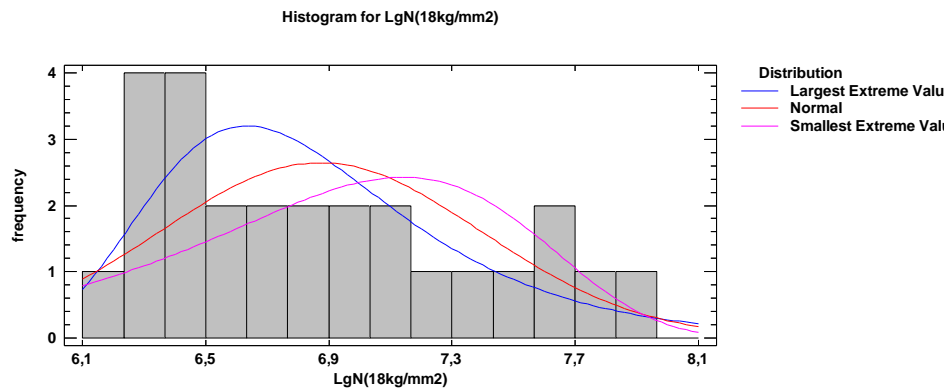


Fig.A7.3 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (18 kg/mm2).

Table 7.3.2

Lower Tail Area (p if $X \leq x$) for LgN (18kg/mm2)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
5,49842	3,19782E-8	0,00427761	0,0428658
6,18573	0,0462476	0,0943104	0,149689
6,87303	0,578426	0,5	0,45127
7,56033	0,907101	0,90569	0,891524
8,24763	0,982785	0,995722	0,999731

A7.4 Stress Level 16,5 kg/mm², sample volume – 250 specimens

Table 7.4.1

Goodness-of-Fit Tests for LgN (16,5kg/mm²)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,143434	0,157742	0,165285
DMINUS	0,119161	0,117534	0,166567
DN	0,143434	0,157742	0,166567
P-Value	0,731294	0,616228	0,546049

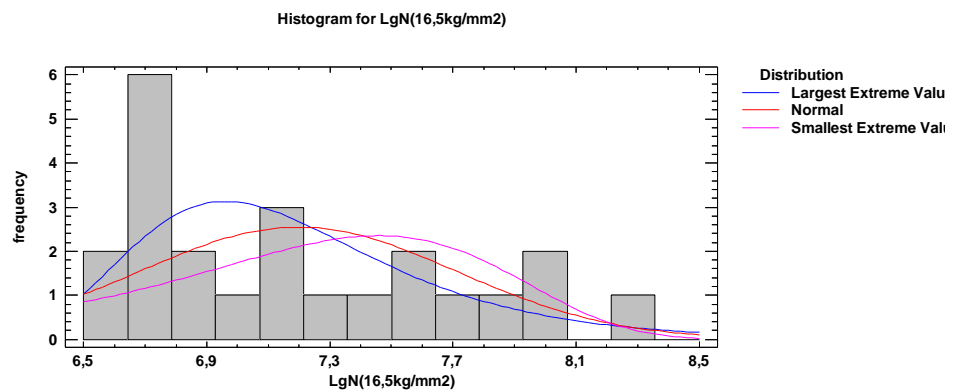


Fig.A7.4 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (16,5 kg/mm²).

Table 7.4.2

Lower Tail Area (p if $X \leq x$) for LgN (16,5kg/mm²)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
5,7573	1,64941E-10	0,00258404	0,0356693
6,47697	0,0302067	0,0810283	0,137139
7,19663	0,580576	0,5	0,450647
7,91629	0,918992	0,918972	0,912195
8,63596	0,986961	0,997416	0,999949

A7.5 Stress Level 15 kg/mm², sample volume – 100 specimens

Table 7.5.1

Goodness-of-Fit Tests for LgN (15kg/mm²)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,165368	0,189045	0,178596
DMINUS	0,0830857	0,109356	0,168274
DN	0,165368	0,189045	0,178596
P-Value	0,676348	0,518693	0,579517

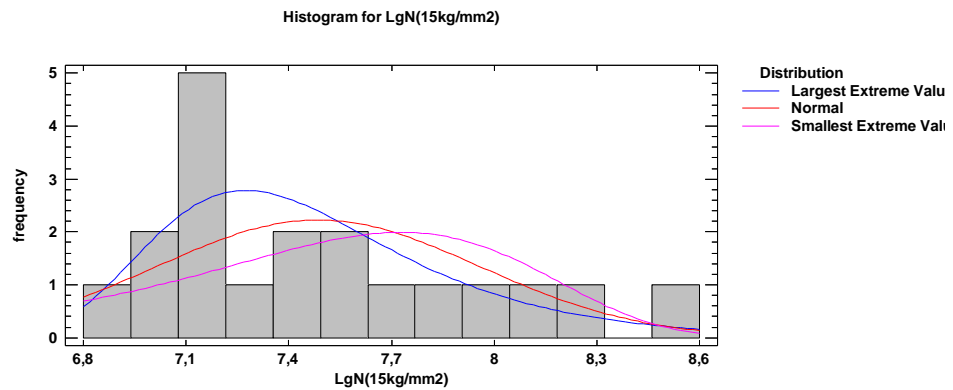


Fig.A7.5 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (15 kg/mm²).

Table 7.5.2

Lower Tail Area (p if $X \leq x$) for LgN (15kg/mm²)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
5,98872	2,77386E-18	0,000750911	0,0275825
6,73731	0,00921565	0,056236	0,122183
7,4859	0,580786	0,5	0,455107
8,23449	0,938947	0,943764	0,940921
8,98309	0,992723	0,999249	0,999998

A7.6 Stress Level 14kg/mm², sample volume – 50 specimens

Table 7.6.1

Goodness-of-Fit Tests for LgN (14kg/mm²)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0701722	0,110599	0,138767
DMINUS	0,0767329	0,0694411	0,129421
DN	0,0767329	0,110599	0,138767
P-Value	0,997974	0,908133	0,698586

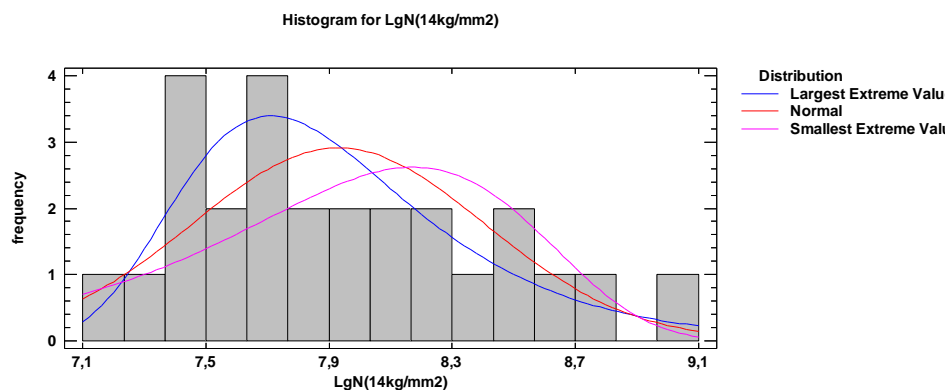


Fig.A7.6 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (14 kg/mm²).

Table 7.6.2

Lower Tail Area (p if $X \leq x$) for LgN(14kg/mm²)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
6,34421	2,91501E-17	0,000419507	0,0230147
7,13723	0,00996843	0,0474779	0,112024
7,93026	0,572482	0,5	0,45461
8,72328	0,934718	0,952522	0,954659
9,51631	0,991862	0,99958	1,0

A7.7 Stress Level 13 kg/mm², sample volume – 50 specimens

Table 7.7.1

Goodness-of-Fit Tests for LgN (13 kg/mm²)
Kolmogorov-Smirnov Test

	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
DPLUS	0,0818195	0,103213	0,11574
DMINUS	0,0734303	0,0846825	0,0852117
DN	0,0818195	0,103213	0,11574
P-Value	0,994983	0,944609	0,877036

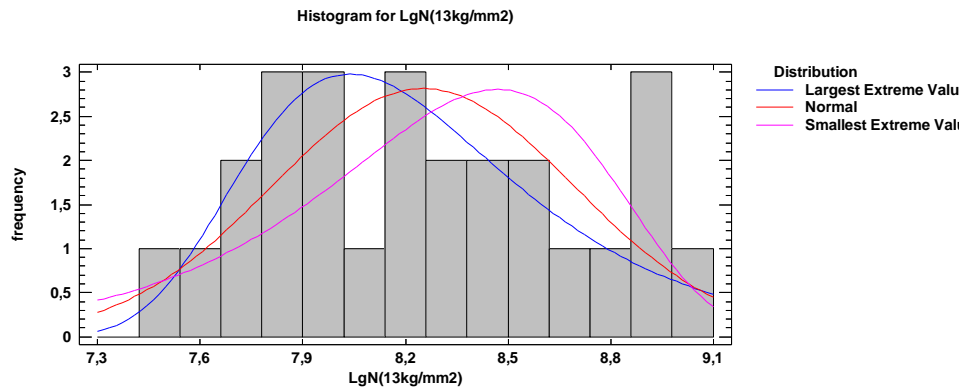


Fig.A7.7 The comparison of the main probability density distribution function of Life Time (Durability) of Specimens (13 kg/mm²).

Table 7.7.2

Lower Tail Area (p if $X \leq x$) for LgN (12 kg/mm²)

<i>X</i>	<i>Largest Extreme Value</i>	<i>Normal</i>	<i>Smallest Extreme Value</i>
6,60117	7,34736E-19	0,0000946868	0,0103042
7,42632	0,00742421	0,0309904	0,0749089
8,25147	0,562294	0,5	0,44308
9,07661	0,93463	0,96901	0,987725
9,90176	0,992093	0,999905	1,0

PRELIMINARY ESTIMATE OF THE MIL-26T KEEL BEAM SAFE LIFE BY RESULTS
OF BENCH TESTS

A8.1 The evaluation of reduces to loadings of two load programs of testing at branch test.

The helicopter Mil-26T was forced by two programs of loading:

- preliminary program of tail beam loading (P1);
- additional program of loading (P2).

Let's assume that the lateral loading on an axis Z (- P_z) introduces most fatigue damage.

A8.1.1 The loading program (P1) parameters (block loading)

“Ground-air-ground” (GAG) cycle: $P_z(P1) = 0 \dots 3\ 650\ \text{kg}$,

The Frequency of GAG cycles $N(P1) = 10\ 000\ \text{cycles}$,

or

cycle load mean: $P^{(GAG)}_{zm}(P1) = 1\ 825\ \text{kg}$

cycle load amplitude: $P^{(GAG)}_{za}(P1) = \pm 1\ 825\ \text{kg}$

Variable load:

Load mean: $P^{(var)}_{zm}(P1) = 3\ 650\ \text{kg}$

Load amplitude (2,2Hz): $P^{(2,2)}_{za} = \pm 630\ \text{kg}$

Load amplitude (17,6 Hz): $P^{(17,6)}_{za} = \pm 60\ \text{kg}$

The Time of cycles – $T = 400\ \text{sec}$,

For simplicity (and for Strength reserve) let's accepted only one variable load that is equal to sum of amplitudes:

Load AMPLITUDE: $P^{(var)}_{za}(P1) = \pm 690\ \text{kg}$

The frequency of load cycles in one loading block:

$N^{(var)}(\text{block}) = 400\ \text{sec} \cdot 2,2\ \text{Hz} \cdot 0,9\ (\text{the load regimes transit time}) = 800\ \text{cycles/block}$,

The Sum of variable cycles during Test Time:

$$N^{(\text{var})}(\text{P1}) = N^{(\text{var})}(\text{block}) \cdot N^{(\text{GAG})}(\text{P1}) = 800 \cdot 10\,000 = 8 \text{ ml cycles}$$

A8.1.2 The loading program (P2) parameters (repeated static load)

“Ground-air-ground” (GAG) cycle: $P_z(\text{P2}) = 0 \dots 5\,160 \text{ kg}$;

or

cycle load mean: $P_{zm}(\text{P2}) = 2\,580 \text{ kg}$;

cycle load amplitude: $P_{za}(\text{P2}) = \pm 2\,580 \text{ kg}$;

$N(2) = 16\,000$ cycles - frequency of cycles till keel beam destroyed.

A8.1.3 The loads have to reduced to “GAG” cycles of load program P1

To reduce the amplitude load of variable load P_{a1} with mean load P_{m1} to amplitude of variable load P_{a2} with other static load P_{m2} let's invokes the formula [27]:

$$P_{a2} = P_{a1} + \chi (P_{m1} - P_{m2})$$

where $\chi = 0.15 \dots 0.30$ – mean stress ratio,

for aluminum alloys: $\chi = 0.25$;

for steel: $\chi = 0.10$;

for 01420 alloys: $\chi = 0.15$.

Program (P2):

$$P_{a1}(\text{P2}) = P_{a2} + \chi (P_{m2} - P_{m1}) = 2\,580 + 0.15 \cdot (2\,580 - 1\,825) = 2\,693 \text{ kg}$$

Program (P1):

variable load:

$$P^{(\text{var})}_{a1}(\text{P1}) = P^{am}_{a1} + \chi (P^{(\text{var})}_{m1} - P_{m1}) = 690 + 0.15 \cdot (3\,650 - 1\,825) = 964 \text{ kg}$$

A8.2 The evaluation of two load programs equivalent cycles (life time) of testing at bench test.

As the parameter of S-N fatigue curve let's accept $m=6$ that used for aluminum alloys under high level of load [27] under test condition. That also gives the Strength reserve.

Equivalent cycles (live time) may be evaluated by known expression [8]:

$$P_{a1}^m(\text{P1}) N(\text{P1}) = P_{a2}^m(\text{P2}) N(\text{P2})$$

8.2.1 The life time “GAG”-cycle (P2) reduce to frequency of cycles of (P1)

$$N_1(P1) = P_{a2}^m(P2)N(P2)/P_{a1}^m(P1) = [2693/1825]^6 \cdot 16\,000 = 165\,200 \text{ cycles.}$$

8.2.2 The life time of variable load (P1) to frequency of cycles of (P1)

$$N_1^{\text{var1}}(P1) = P_{a1}^{\text{(var)}}(P1)N^{\text{(var)}}(P1)/P_{a1}^m(P1) = [964/1825]^6 \cdot 8 \cdot 10^6 = 173\,800 \text{ cycles.}$$

8.2.3 The Sum frequency of cycles reduce to frequency of cycles “GAG” of (P1)

$$N_{\Sigma} = N(P1) + N^{\text{var1}}(P1) + N_1(P1) = 10\,000 + 165\,200 + 173\,000 = 348\,900 \text{ cycles (flights).}$$

A8.3 The Estimation of Safe Life in Operation

The N_{Σ} is the minimal frequency of cycles observed at bench test

The number of tested Design may be accepted to two ($n = 2$) (3.52), as this Local Zone have been repaired and the Test were continued under others Test conditions.

So accepted $P_{norm} = 0,001$ – the value is recommended [8] and get (3.56) :

$$v = 999$$

So recommended the value of dispersion in operation [54]:

$$\frac{\sigma_{lgN}}{m} = 0.025$$

If $m = 12$ (the operation work area, Table 4.4) the $\sigma_{lgN} = 0,30$ and from (3.51) $\beta = 0,234$.

Then from (3.57)

$$\eta_N = 41,35$$

It is known that one reduce test cycle (paragraph 9.2.3) is equivalent to 30 min of flight, i.e. the minimum flight time in work condition is

$$N_{\text{min work}} = 175\,400 \text{ flight hours.}$$

Then the Safe Life (SL) reduced to Operation (3.58) is equal to:

$$SL = 175\,400/\eta_N = 4\,200 \text{ flight hours}^{12}.$$

So, taking into account the previous real flight hours (1000 f.h.), the total Limited Service Life will not exceed

$$R = 5\,200 \text{ flight hours.}$$

¹² This Calculation is given as an example and is used for evaluation's procedure example only.

Table 5.2

The List of Through-Cracks (break stringers, skin etc) that were revealed during
Branch Test under Fatigue Test

Nr	Crack's number	Crack's Seat		Life running	
		Frame	Stringer	Load Program 1 Block/cycles	Load Program 2 Block/cycles
1	46	37...38	4	1300	3400д
2	85	37...38	11	2095	3344
3	12	38...39	12	243	17364д
4	80	38...39	14	2095	9227
5	83	38...39	16	2095	17746д
6	238	38...39	18	10502д	19191д
7	2	39...40	9	49	14232д
8	48	39...40	11	1518	3400д
9	3	39...40	15	49	4455
10	34	39...40	16	1273	11752д
11	35	39...40	16	1273	11752д
12	241	40...41	9	11752д	11752д
13	65	40...41	10	1520	8001
14	71	40...41	12	1565	7119
15	36	40...41	15	1273	4455
16	169	40...41	15	6133	15121д
17	262	1...2	2	12401д	18117д
18	243	1...2	3	11752д	11752д
19	242	1...2	5	11752д	11752д
20	212	1...2	6	3400д	3400д
21	190	1...2	7	8617	9250
22	209	1...2	10	3400д	3400д
23	233	1...2	11	6131д	6131д
24	7	2...3	5	49	15870д
25	179	2...3	6		12401д
26	185	2...3	6	8001	8001
27	258	3...4	1	12401д	14597д
28	231	3...4	4	6131д	6131д
29	186	3...4	9	8001	8001
30	275	3...4	10	17746д	18117д
31	235	3...4	11	6823д	6823д
32	265	4...5	5	14232д	14232д
33	236	4...5	13	8113д	12992д
34	239	5...6	1	10502д	10502д
35	216	5...6	9	3410д	3410д
36	181	7...8	1	7537	7537
37	266	7...8	5	14597д	14597д
38	260	7...8	6	12401д	12401д
39	183	7...8	7	8001	12401д
40	146	7...8	14	3749	6131д
41	28	8...9	11	651	14949д
42	184	9...10	13	8001	17364д
43	269	4...5	9	15870д	15870д
44	288	5...6	3	20055д	20055д
45	253	5...6	5	12401д	17746д
46	251	6...7	9	12401д	12401д
47	245		11	12401д	12401д
48	252	6...7	20	12401д	19191д
49	255	7...8	7	12401д	12401д
50	220**	8...9	11	3410д	12401д
51	254	8...9	18	12401д	12401д
52	270	9...10	18	15870д	18117д
53	226	38...39	1	3410д	12401д

