RIGA TECHNICAL UNIVERSITY
Faculty of Civil Engineering
Institute of Structural Engineering and Reconstruction

Līga GAILE
PhD student of doctoral program „Construction”

DYNAMIC RESPONSE OF LIGHT-WEIGHT LATTICE TOWERS TO HUMAN INDUCED LOADS

Civil Engineering, Structural Engineering (RTU P-06)

Doctoral Thesis

Scientific supervisor
Dr.sc.ing., Professor
Ivars RADIŅŠ

Rīga 2014

UDK 624.97(043)
Ga 165 r

This work has been supported by the European Social Fund within the project «Support for the implementation of doctoral studies at Riga Technical University».

ANNOTATION

Nowadays the serviceability criteria often govern contemporary structural design. Structures such as lightweight pedestrian bridges, slender floors, grandstands and long span stairs are prone to vibrations caused by human activities. Also lattice tower type structures are remarkably flexible, low in damping and light in weight that results in structures that are susceptible to human induced vibrations. Traditionally for such type of structures dynamic analysis is performed to evaluate only wind induced vibrations and effects on the structure. But there is a lack of understanding and inadequate design information of the building codes, regarding the slender tower dynamic response to human induced loads.

The Thesis consists of three main parts: literature review on which the objective of the study and the tasks of the theses are based (section 1); part two deals with the experimental identification and approximation of human induced time varying forces (section 2); part three deals with the application of those loads to the lattice self-supporting tower structure and dynamic response to it (section 3).

During the research numerous experimental investigations that can be divided in to two main groups were performed: 1) experimentally obtained continuous walking histories of individuals; 2) modal tests and measurements of response to the walking excitation of 19 full scale observation towers as well as the subjective assessment of vibration amplitudes of test persons. The theoretical part deals with the parameters that mostly influence the structure response to human induced loading, the effect of different walking force component on the total vibration of the structure, phenomenon and response of the structure to loads induced by a group of people.

As a result the thesis presents a calculation methodology of assessing the maximum vibration level of light-weight lattice towers with different dynamic and geometric parameters due to human movement initiated dynamic loads of stochastic nature. It includes recommendations about the range of structures that requires considering the human dynamic loads, applicable loads itself, its dispositions, necessary parameters to adopt for calculations, the analytical solution for preliminary design calculations and criteria to limit vibrations due to comfort of visitors. Comparisons of experimentally obtained tower response and predictions were used to determine the accuracy of the proposed methodology and it is found to be sufficiently accurate to be used in the design process.

The thesis contains 135 pages, 82 figures, 28 tables and a reference list of 136 sources.

ANOTĀCIJA

Mūsdienās konstruktīvo būves risinājumu bieži vien nosaka ekspluatējamības (lietojamības) kritēriju izpilde. Vispārzināms, ka cilvēku aktivitāte (iešana, lēkšana, skriešana, vandālisms) var izraisīt gājēju tiltu, lokanu kāpņu, tribīņu un salīdzinoši vieglu pārsegumu manāmas svārstības. Arī režģoti skatu torņi ir pret gājēju slodzi jūtīga būve, kas iepriekš cilvēku – konstrukcijas dinamiskās mijiedarbības kontekstā nav pētīta. Tradicionāli šādām torņveida konstrukcijām dinamiskie aprēķini tiek veikti tikai, lai novērtētu vēja radīto iespaidu uz torņu svārstību amplitūdām vai izliecēm, jo šobrīd būvnormatīvos nav pieejami norādījumi vai rekomendācijas projektēšanai, kas attiecas uz lokanu torņu dinamiskās uzvedības paredzēšanu cilvēku radīto dinamisko iedarbju rezultātā.

Promocijas darbs sastāv no trim galvenām daļām: literatūras apskata, uz kura pamata noformulēts promocijas darba mērķis un uzdevumi tā īstenošanai (1. Nodaļa); nodaļa, kas saistīta ar cilvēka pārvietošanās rezultātā radītu laikā mainīgu slodžu eksperimentālu noteikšanu un to aproksimāciju (2. Nodaļa); nodaļa, kas saistīta ar šo laikā mainīgo spēku pielikšanu režģotām skatu torņu konstrukcijām un to iespaida noteikšanu uz torņu svārstību amplitūdu (3. Nodaļa).

Darba izstrādes laikā veikti vairāki eksperimentāli pētījumi, kas var tikt iedalīti divās grupās: 1) nepārtrauktas gājēju slodzes izmaiņas laikā noteikšana *(continuous walking force history)*; 2) 19 skatu torņu dinamisko parametru noteikšana un svārstību rakstura pētīšana gājēju slodzes iespaidā, kā arī reālo torņa svārstību amplitūdu subjektīva novērtēšana. Teorētiskā daļā tiek analizēti parametri, kas ietekmē gājēju izraisītās torņu svārstību amplitūdas, dažādo laikā mainīgās slodzes komponenšu (harmoniku) ietekmi uz kopējām torņa svārstību amplitūdām, kā arī torņa svārstību amplitūdu atrašana no cilvēku grupas, kas pārvietojas pa torņa augstumu.

Rezultātā tiek piedāvāta aprēķinu metodoloģija vieglas režģotas konstrukcijas torņu maksimālās svārstību amplitūdas noteikšanai apmeklētāju pārvietošanās iespaidā. Tā sevī ietver ieteikumus kādām konstrukcijām būtu jāņem vērā cilvēka radītās dinamiskās iedarbes, pieliktās slodzes un to novietojums, par aprēķinos izmantojamajiem parametriem, analītisku risinājumu sākotnējo projekta aprēķinu veikšanai, kā arī svārstību ierobežošanas kritērijus apmeklētāju labsajūtas uzlabošanai. Rezultātu atbilstība eksperimentāli izmērītiem svārstību paātrinājumiem apliecina piedāvātās aprēķinu metodikas pamatotību.

Darbs satur 135 lappuses, 82 attēlus, 28 tabulas un literatūras sarakstu ar 136 nosaukumiem. Promocijas darbs uzrakstīts angļu valodā.

PREFACE

The doctoral thesis was developed at the Department of Structural Mechanics, Faculty of Civil Engineering, Riga Technical University from 2011 to 2013.

It was financially supported by the European Social Fund, Contract No: 2009/0144/1DP/1.1.2.1.2/09/IPIA/VIAA/005. Provided material support of the university is gratefully acknowledged.

I would like to express my very great appreciation to the supervisor of this work prof. Ivars Radinsh. His willingness to give his time so generously has been very much appreciated. Assistance provided by the head of the department of Structural Engineering prof. Leonids Pakrastinsh was greatly appreciated.

I furthermore thank all the people of the department, among them special thanks to prof. Feliks Bulavs and Ilze Shreibere.

I am thankful to prof. Raimonds Ozolinsh for his useful and constructive recommendations on this work.

I wish to acknowledge the help provided by all the people who participated in the experimental investigations as test subjects. Without them it would be impossible to develop the thesis on the subject.

I am grateful to my husband Ervins and daughters Lauma and Nora for their understanding, patience, and the support I received from them.

Līga Gaile,

September 2013, Riga

CONTENTS

INTRODUCTION

Subject Actuality and Formulation of the Problem

Vibration of the light–weight structures caused by the movement of pedestrians has been a particularly topical research subject in the world for more than 10 years now. Many researchers found this topic attractive when previously unforeseen lateral vibration experienced more than 18 million British pounds worth London Millennium Footbridge on its opening day in 2000. The research of human-structure interaction topicality and complexity also confirm researchers' ongoing discussions at international conferences and forums.

The best-known structures that are sensitive to vibrations caused by human activity (walking, jumping, running, vandalism, etc.) are pedestrian bridges, slender stairs, grandstands or slender slabs. In the case of the pedestrian bridges, vibrations are mainly induced in a transverse direction and are basically caused by the pedestrian lateral component of load. Vibrations of the pedestrian bridges are relatively well studied, consequently the design recommendations have been developed to ensure an adequate pedestrian comfort.

In Latvia, other pedestrian load sensitive structures are found more often than light-weight pedestrian bridges – the observation towers and these structures have not been studied from the human-structure dynamic interaction perspective. Unlike pedestrian bridges, the observation towers are subjected to both a pedestrian load transverse and a longitudinal component.

Historically, the free standing towers were primarily used by the military to provide a good observation of the surrounding area. The era of observation towers as a sightseeing symbol probably started in Paris during 1889 with the rise of the Eiffel at the World's Fair. It was designed using graphical methods to construct a tower of sufficient strength to support its weight. Empirical results from past experience were used to account for wind loading [117]. Observation towers like Eiffel that are located in the cities are usually tall structures and serve as an architectural symbol. Towers located in the countryside are designed to allow viewers an unobstructed view of the landscape and tend to have a design mostly driven by economic aspects.

Latvia has numerous observation towers mostly located in the regions of Latgale and Kurzeme. The map of light-weight lattice public observation towers that was inspected by the authors presented in Fig. 1. It was established that 18 observation towers of the 19 inspected are sensitive to human induced dynamic loads and vibrations cause uncomfortable feeling of visitors in certain circumstances.

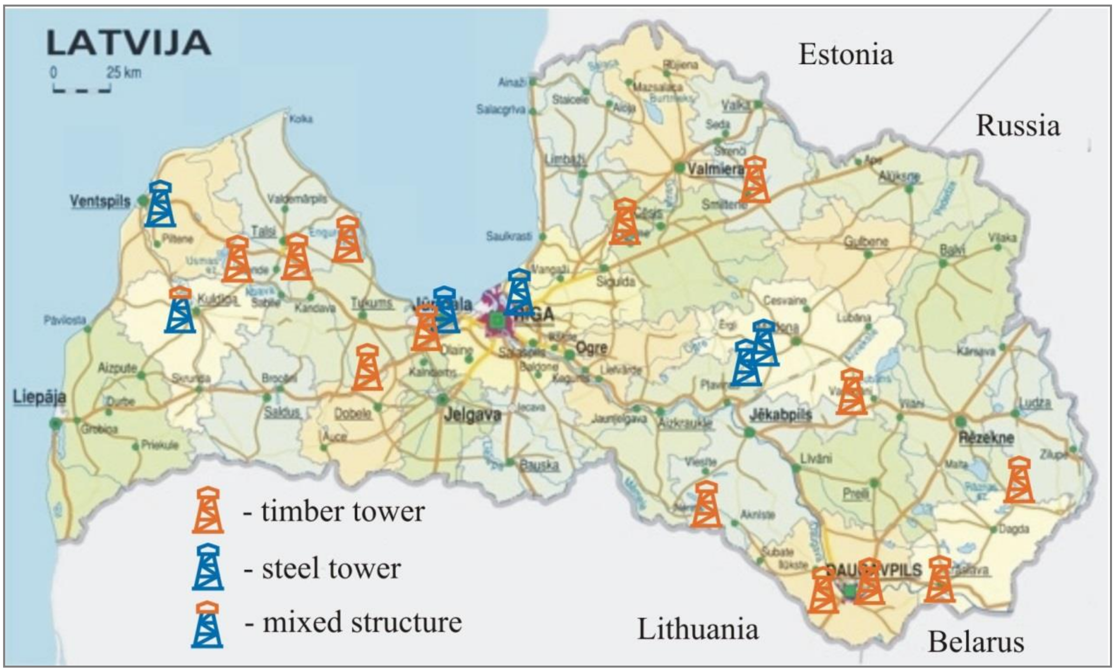


Figure 1. Location of observation towers in Latvia

Almost half of the observation towers are the responsibility of the state company JSC "Latvia's State Forests" that continuously develop the environmental infrastructure objects. The recently opened for public (October, 2012) 28,5m high timber observation tower "Ančupānu skatu tornis" in Rezekne serves as an example. Although construction of such towers is rather expensive, it is a great way to increase tourist attraction to the area otherwise unpopular.

There are some examples of mixed structure e.g. timber structure (columns, beams, and cladding) with a steel rod lateral resisting system but mostly observation towers can be divided in timber (70% of the inspected towers) and steel structures. An example of a typical steel and timber observation tower is presented in Figure 2.

Most of the towers in Latvia have a set of rules to limit the number of visitors from 5 to 10 people, however this limit is not based on any research information and the construction is based purely on previous experience, especially for timber observation towers.

In 2010 a light-weight eccentric steel structure observation tower was opened for public in Jurmala, Dzintari and experienced an unexpectedly high level of vibration amplitudes that caused uncomfortable feelings to the visitors of the tower. This structure highlighted the lack of understanding and inadequate design information of the building codes, regarding the slender tower dynamic response to the human induced loads. It demonstrates that in areas with a low seismicity and relatively low wind loads the human induced dynamic loads could be determinative in a slender and light-weight observation tower design as well as in checking the serviceability criteria.

Figure 2. a) An example of the steel observation tower (in Kalsnava); b) An example of the timber observation tower in the region of Latgale (Priedaine)

The research and better understanding of human induced dynamic loads and their correct application to the observation tower structure at the design stage is a necessary requirement to be able to develop aesthetically pleasing and economically justified light-weight structures in the future.

The formulated objective of the study is based on the literature review done in Section 1 of the thesis.

Objective of the Study

The objective of this study is to develop the method of analytical approximation of human movement induced dynamic loads based on the experimental investigation and to develop the calculation methodology for assessment of light-weight lattice self-supporting tower type structure dynamic response to typical human induced dynamic loads as well as to set a limit on the observation tower vibration acceleration amplitudes due to the comfort criteria of tower visitors.

Tasks that have to be resolved can be subdivided in two main groups in order to achieve the formulated objective of the thesis.

1) tasks associated with the experimental identification and approximation of human induced time varying forces;
2) tasks associated with the application of those loads to the lattice self-supporting tower structure and finding tower dynamic response to them.

The formulated tasks of the thesis are presented in detail in section 1.3 after the conclusion section that is drawn from the performed literature review.

The Scientific Novelty of the Work

It is experimentally and theoretically proven that vertical light-weight cantilever type structures like public observation towers with fundamental frequency less than 3.3 Hz may undergo vibrations induced by human activities that do not satisfy the serviceability limit criteria - required comfort criteria during the structure exploitation.

The scientific novelties of the work as well as tasks to be resolved can be subdivided in to two groups. The first group is associated with the experimental identification and approximation of human induced time varying forces but the second group is associated with the application of those loads to the lattice self-supporting tower structure and the dynamic response to them.

A new method from the branch of progressive inverse dynamic methods is developed that allows estimating dynamic forces induced by human activities (walking, running, jumping and body swaying) under a wide range of conditions (no limitations of laboratory environment) for civil engineering applications. Comparing to the traditional direct measurement methods several advantages can be highlighted such as required instruments cost relatively low, there is a possibility to obtain records over longer periods of time (continuous walking force histories) and test setup does not have a strong influence on human ability to behave or move naturally.

The experimental data processing method of obtaining the analytical expression of the mean continuous walking force histories is proposed. The approach preserves an important parameter such as the phase shift of relevant walking harmonic and obtained analytical expression of the mean continuous walking history can be further used in analytical calculations of the structure under consideration.

The mean dynamic load factors, the corresponding phase shifts and their dependence from the pacing rate of dynamic force longitudinal and lateral component for the actions of stair ascent and descent have been obtained for the first time.

As a result of experimental investigations the dynamic parameter data set (damping ratios, fundamental and natural frequencies of the structure) has been obtained for the first time of most of the lattice observation towers opened for public in Latvia.

Methodology of light-weight lattice tower maximum dynamic response calculation due to towers visitors' movement is given for the first time. It is based on the performed studies about the range of parameters of structures that require considering the human dynamic loads, applicable loads and its dispositions as well as the analytical solution for preliminary design calculations and the criteria to limit vibrations due to comfort of visitors.

Practical Application of the Thesis

The main practical gain as a result of reaching the doctoral thesis objective is that the methodology and recommendations of light-weight lattice tower type structure maximum dynamic response calculation to the typical human induced loads under structure serviceability conditions is given for the first time. This is useful material for the structural engineers working in the industry and undertaking the design of public observation towers as any other design information regarding this subject is not available.

The proposed calculation method provides possibility to justify, correct the set of rules that limit the number of visitors at a time on most of the public observation towers in Latvia.

The obtained mean dynamic load factors, corresponding phase shifts and their dependence from pacing rate of a person stair ascending and descending dynamic force of all three components is supplementary information to Table A.4. of International Standard ISO 10137:2007 [44] were presented examples of design parameters due to one person ascending or descending stairs only for first two harmonics of vertical direction.

During the research computer program of experimental data visualization was developed and can be successfully implemented as a quick tool of structure vibration level assessment during the dynamic testing.

Therefore the developments in the thesis are the necessary base to be able to develop more economically justified and aesthetically pleasing light-weight lattice observation towers for public use in the future.

Results Presented for the Defense

1. The method of obtaining vertical, longitudinal and lateral components of human movement dynamic forces based on kinematics of the motion of human center of gravity (COG) (by utilizing accelerometery technology);

2. Experimental data processing method for obtaining the analytical expression of the mean continuous walking force history;
3. The mean dynamic load factors (DLF) and corresponding phase shifts of single person ascent and descent induced forces at different pacing frequencies;
4. The methodology of maximum vibration acceleration amplitude assessment of light-weight lattice self-supporting towers with different dynamic and geometric parameters due to human movement initiated dynamic loads of stochastic nature;
5. Preliminary recommendations of limiting the observation tower vibration acceleration amplitude to assure an acceptable comfort level of tower visitors.

Scope of the Study

The use of the developed methodology of light-weight lattice self-supporting tower maximum dynamic response calculation due to tower visitor movement is appropriate when the following requirements are fulfilled:

1) the maximum stress in the elements of the structure are less than limiting stresses of ultimate limit state;
2) the maximum displacements of the structure from appropriate wind loading are less than the limiting displacements of serviceability limit state.

The developed methodology is based on the experimental investigations of existing full scale timber and steel observation towers. The newly developed methodology is rational to use for the self-supporting towers with parameters in the following range:

– height *L* above the ground level: $15m \le L \le 50m$;
– stiffness *EI* and mass per meter *m*:

$$EI \le 2 \cdot 10^6 L^{2.968};$$

$$m \le 206241 \cdot L^{-1.032};$$

(the authors restrictions of the term „slender" and light-weight" in the thesis)

– the fundamental frequency of the structure *f*: $f \le 3.3Hz$.

The developed method that allows estimating dynamic forces induced by human activities is appropriate if the supporting structure on which activity is performed has a remarkably higher fundamental frequency than activity frequency. Also if activities (walking, running, jumping or body swaying) are performed with the frequency or speed, close to constant and have a rectilinear pattern.

The obtained mean dynamic load factors and the corresponding phase shifts of person dynamic force longitudinal and lateral component for the action of stair ascent and descent are appropriate to use if stair inclination β is in a range of: $24° \leq \beta \leq 42°$. The obtained mean dynamic load factors and corresponding phase shifts of a person while stair ascent or descent are for walking frequency range of $1Hz \leq f_p \leq 2.3$.

Theoretical and Methodological Bases of the Research

The bases of the new method of obtaining dynamic forces from human movement by utilizing accelerometery technology is Newton's Second Law of Motion and researches done in the field of biomechanics about the kinetics of human motion. To take into account the stochastic nature of the human loading, an algorithm has been developed that uses the random number generator integrated in the commercial software “Mathcad”.

Post processing of the experimental data to obtain the frequency spectrums is mostly done by the commercial software “ME’scopeVES” (version 5.1.2010.1215) from “Vibrant technologies”. The computer program of experimental data visualization to assist the dynamic testing was developed in Adobe Air environment.

The differential equation of the Euler – Bernoulli prismatic cantilever beam is the base of the analytical model used in the theoretical investigations of self-supporting lattice tower response to human induced load. Commercial finite element software STRAP (version 12.5) was used for carrying out the numerical experiments. The software uses the subspace iteration technique to extract the eigenvalues.

The performed researches, developed calculation models and methods are based on the following engineering science branches:

- Structural dynamics;
- Structural engineering;
- Structural mechanics;
- Biomechanics;
- Modal and experimental modal analysis;
- Probability theory;
- Signal analysis.

The Approbation of the Results - List of relevant International Conferences

1. RASD 2013 11th International Conference on Recent Advances in Structural Dynamics, Italy, Piza, 1st – 3rd July, 2013 (Gaile L., Radinsh I. Lattice Tower Dynamic Performance under Human Induced Loading).
2. The 9th International scientific conference „Environment. Technology. Resources”, Rezekne, June 20-22, 2013 (Gaile L. Analysis of Dynamic Parameters of Observation Towers in Latvia).
3. The International Scientific Conference „Civil Engineering’13” of Latvia University of Agriculture. Jelgava, 16-17 May, 2013 (Gaile L., Radinsh I. The Footfall Induced Forces on Stairs).
4. Riga Technical University 53rd International Scientific Conference to the 150th anniversary and The 1st Congress of World Engineers and Riga Polytechnical Institute / RTU Alumni, Riga, Latvia, 11-12 October, 2012 (Gaile. L., Radinsh. I. Dynamic loading and response of observation towers and Gaile. L. Analysis of Dynamic Parameters of Timber and Steel Observation Towers).
5. 25th International Conference on Noise and Vibration Engineering (ISMA2012/USD2012). Belgium, Leuven, 17-19 September, 2012 (Gaile L., Radinsh I. Steel Lattice Sightseeing Tower’s Horizontal Vibrations Induced by Human Movement).
6. 19th International Congress on Sound and Vibration Proceedings of Recent Developments in Acoustics, Noise and Vibration (ICSV19), Lithuania, Vilnius, 8-12 July, 2012 (Gaile L., Radinsh I. Eccentric Lattice Tower Response to Human Induced Dynamic Loads).
7. International Conference on Civil and Construction (ICSCE 2012). Sweden, Stockholm, 11-12 July, 2012 (Gaile L., Radinsh I. Human Induced Dynamic Loading on Stairs).
8. XVII International Conference on Mechanics of Composite Materials, 2012, May 28 - June 1, Jurmala (Gaile L., Radinsh I. Assessment of the Fatigue Life of a Tower by Using a Real-Time Loading History).
9. Rīgas Tehniskās universitātes 52. Starptautiskā zinātniskā konference. Rīga, 2011. gada 13.-16. oktobris. (L. Gaile, I. Radiņš. Cilvēku izraisīto svārstību ietekme uz konsoles veida konstrukcijām).
10. Apvienotais pasaules latviešu zinātnieku III kongress un Letonikas IV kongress "Zinātne, sabiedrība un nacionālā identitāte", Sekcija "Tehniskās Zinātnes". (Gaile L., Radiņš I. Ekscentriskas konstrukcijas darbība dinamisko slodžu ietekmē).

11. The International Scientific Conference „Civil Engineering'11" of Latvia University of Agriculture. Jelgava, May 12-13, 2011. (Gaile L., Radinsh I. „Time Depending Service Load Influence on Steel Tower Vibrations").
12. The 8th International scientific conference „Environment. Technology. Resources", Rezekne, June 20-22, 2011. (Gaile L., Radinsh I. Dynamic Response of Tower Structures).

List of the Authors Scientific Publications

1. Gaile L., Radinsh I. Lattice Tower Dynamic Response Calculation to Human Induced Loads: Case Study // 54th International Conference of Riga Technical University: „Innovative Materials, Structures and Technologies", Riga, Latvia, November 8, 2013, *(accepted for publication).*
2. Gaile L., Radinsh I. Lattice Tower Dynamic Performance under Human Induced Loading // RASD 2013 11th International Conference on Recent Advances in Structural Dynamics, Italy, Piza, 1st – 3rd July, 2013, pp.1-15.
3. Gaile L. Dynamic Parameters of Observation Towers in Latvia // Proceedings of the 9th International Scientific and Practical Conference "Environment. Technology. Resources", Latvia, Rezekne, 20-22 June, 2013, pp. 57-62.
4. Gaile L., Radinsh I. The Footfall Induced Forces on Stairs //„Civil Engineering'13": 4th International Scientific Conference Proceedings, Part I, Latvija, Jelgava, 16-17 May, 2013, pp. 60-68.
5. Gaile L., Radinsh I. Steel Lattice Sightseeing Tower's Horizontal Vibrations Induced by Human Movement // 25th International Conference on Noise and Vibration Engineering (ISMA2012/USD2012), (CD-ROM), Belgium, Leuven, 17-19 September, 2012, pp. 1211-1221.
6. Gaile L., Radinsh I. Eccentric Lattice Tower Response to Human Induced Dynamic Loads // 19th International Congress on Sound and Vibration Proceedings of Recent Developments in Acoustics, Noise and Vibration (ICSV19), Lithuania, Vilnius, 8-12 July, 2012, pp. 560-567, (SCOPUS).
7. Gaile L., Radinsh I. Human Induced Dynamic Loading on Stairs // Proceedings of International Conference on Civil and Construction (ICSCE 2012), Sweden, Stockholm, 11-12 July, 2012. Issue 67, pp. 626-632.

8. Gaile L., Radinsh I. Time Depending Service Load Influence on Steel Tower Vibrations // „Civil Engineering'11”: 3rd International Scientific Conference Proceedings, Latvija, Jelgava, 12-13 May, 2011, pp. 144-149, (SCOPUS).
9. Gaile L., Radinsh I. Dynamic Response of Tower Structures // Proceedings of the 8th International Scientific and Practical Conference "Environment. Technology. Resources", Latvia, Rezekne, 20-22 Jun, 2011, pp. 85-91.

1. LITERATURE REVIEW

1.1. Human Induced Vibrations on Light-weight Structures

Nowadays a contemporary engineer's goal of the structure design is to push strength to weight ratio to its limits. Together with the demand for aesthetically pleasing structures it results into slender, light weight structures with low damping ratios. Therefore contemporary structural design often governs serviceability criteria. Light weight slender structures such as pedestrian bridges [131], slender floors [41], grandstands [24,35] and flexible stairs [72] are often subject to vibrations caused by human activities and to assess the dynamic response and performance of the structure during the early stage of the designing process becomes more important. There are many different types of human activities such as walking, running, jumping and intentional swaying (vandal loading), that induce dynamic forces on structures. Except the vandal loading that is a provision of the accidental limit state according to the so-called limit state design code format [38], other activities are mostly associated with the comfort of light-weight structure users and therefore fall under the serviceability limit state.

One more type of structure that could be susceptible to human induced vibrations is a slender observation tower. In 2010 a light-weight eccentric steel structure observation tower was opened for public in Jurmala (Figure 1.1) and most of the visitors experienced vibration amplitudes causing uncomfortable feelings.

Figure 1.1. Eccentric sightseeing tower in Jurmala, Latvia

Traditionally for such type of structures dynamic analysis is performed and dynamic parameters such as fundamental frequencies, mode shapes and damping ratios are found to evaluate wind induced vibrations and effects on the structure. Even the most advanced and comprehensive codes concentrate mainly on these issues, including the Eurocodes.

In case of the lattice observation towers with low natural frequency of the structure human and structure interaction may play a role in the tower design. The tower in Jurmala highlights the lack of understanding and inadequate design information of the building codes, regarding the slender tower dynamic response to human induced loads [47] and demonstrates that in areas with low seismicity and relatively low wind loads the human induced dynamic loads could be determinative in a slender and light-weight observation tower design. Nevertheless no recommendations or studies about dynamic analysis of lattice tower type structures under human induced loads can be found.

To develop the methodology of tower response calculations to human induced loads it is important to understand the typical dynamic behavior of lattice towers itself and to analyze the existing design procedures regarding human induced vibrations for other light-weight structures.

1.1.1 Dynamic Parameters and Response of Lattice Towers

The prime use of light-weight lattice self-supporting towers is to support communication and broadcasting equipment therefore the available design information and studies about lattice towers mostly are in this context. Although an increasing number of communication structures are built, it is still noticed that dynamic analysis of lattice towers has lagged behind the state of art methods used in the design of large bridges and buildings. The dynamic effects on these towers, when incorporated at all, are now achieved through the substitution of equivalent static loads [81]. The standards and Codes of Practice that directly address the dynamic response of self-supporting lattice towers are [6, 40, 77, 99].

It is experimentally verified that dynamic behavior of self-supporting lattice tower may be assumed as linear [81]. The main parameters that denote the response level to dynamic loads are natural frequency, mode shapes, damping ratio and self-weight of the structure.

1.1.1.1 Natural frequencies and mode shapes of lattice towers

There is no closed-form solution for the evaluation of natural frequencies and mode shapes of self-supporting structures. The field of the research so called “modal analysis” is dealing with identification of those parameters. Using theoretical modal analysis where stiffness matrix, mass matrix and damping matrix of the structure is known by solving the eigenvalue problem the required dynamic parameters of the structure can be obtained (approach is used in FEM analysis software). Also the literature [58] provides a recommendation whenever fundamental frequencies are close to a critical range (from the point of view of the pedestrian excitation) to use a more precise numerical model, because hand formulas and simplified methods are not enough for assessment of fundamental frequencies. The finite element software is widely spread and accepted as a more precise numerical model.

The numerical values obtained from detailed tower 3-D modeling are usually upper bonds on frequency because of the effects of connection and foundation flexibility [81]. In terms of inertia the various light weight attachments to the tower are not significant if their weight does not exceed 10% of the tower self-weight [73]. Modes with natural frequencies that are matched by the frequency content of the input loads dominate the response. For most dynamic analysis under lateral loads the accurate prediction of five lowest modes is sufficient [81].

Slender lattice towers are line-like structures and for the purpose of response analysis it can be modeled as the cantilever with uniformly distributed mass along the height. This assumption corresponds well with work of Galvez [45] stating that self-supporting towers behave essentially as a cantilever beams and it is suggested to use expressions of prismatic cantilevers modified by geometrical taper coefficient and correction for shear deformation. Figure 1.3 illustrates the three lowest bending modes in one of the transverse directions indeed are similar to the prismatic cantilever bending modes presented in Table 1.1.

The natural frequencies (1) and transverse mode shapes (2) of the tower could be found from following equations of an ideal Euler-Bernoulli prismatic cantilever and Table 1.1:

$$\omega_n = c_n \sqrt{\frac{EI_0}{m_0 L^4}}; \tag{1}$$

$$\Phi_n = \cosh(a_n y) - \cos(a_n y) - \delta_n (\sinh(a_n y) - \sin(a_n y)); \tag{2}$$

$$\delta_n = \frac{\cos(a_n L) + \cosh(a_n L)}{\sin(a_n L) + \sinh(a_n L)}; \tag{3}$$

$$a_n = \sqrt{\frac{c_n}{L^2}}, \tag{4}$$

where ω_n - natural angular frequency of prismatic cantilever, rad/s;

EI_0 - flexural rigidity of the tower at the base, Nm^2;

m_0 - mass of the structure per unit length at the base, kg/m;

L - height of the tower, m;

To take into account the effects of taper ratio Galvez [45] proposes the correction coefficient F_t displayed in the Figure 1.2. The correction factor for self-supporting steel towers accounting for the shear deformations is proposed by Sackmann [105] where it is k_1=0.90 for the first flexural; second flexural mode k_2=0.78 but third flexural mode k_3=0.6. This is based on the study of ten lattice towers with height from 30m to 120m.

Table 1.1.

Natural frequencies and mode shapes of prismatic cantilever [112]

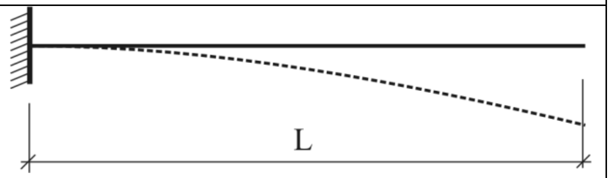
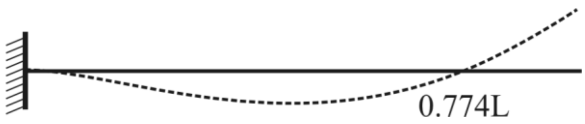
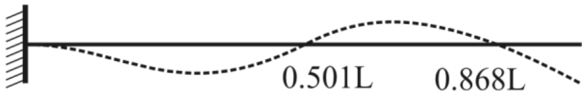
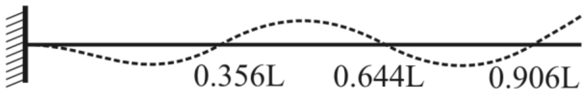
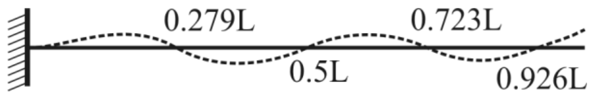
Mode n	c_n	δ_n	Shape
1	3.5160	0.734096	
2	22.0345	1.018466	
3	61.6972	0.999225	
4	120.0902	1.000033	
5	199.8600	1.000000	

Figure 1.2 reveals that by tapering the tower shape it is possible to increase the fundamental frequency of the structure by a maximum 1.4 times. At the same time there is a decrease in the following flexural natural frequencies.

Approximate natural frequency of the self-supporting steel tower can be estimated by the following equation according to work of the Sackmann [105]:

$$\omega_i = \omega_n \cdot F_t \cdot k_i \tag{5}$$

In case the tower shape is similar to the tower presented in Figure 1.3 fundamental frequency should be calculated based on the length:

$$L = 0.5 \cdot (L_{total} + L_{taper}), \tag{6}$$

where L_{total} - total height of the tower, m;

L_{taper} - height of the tapered part of the tower, m.

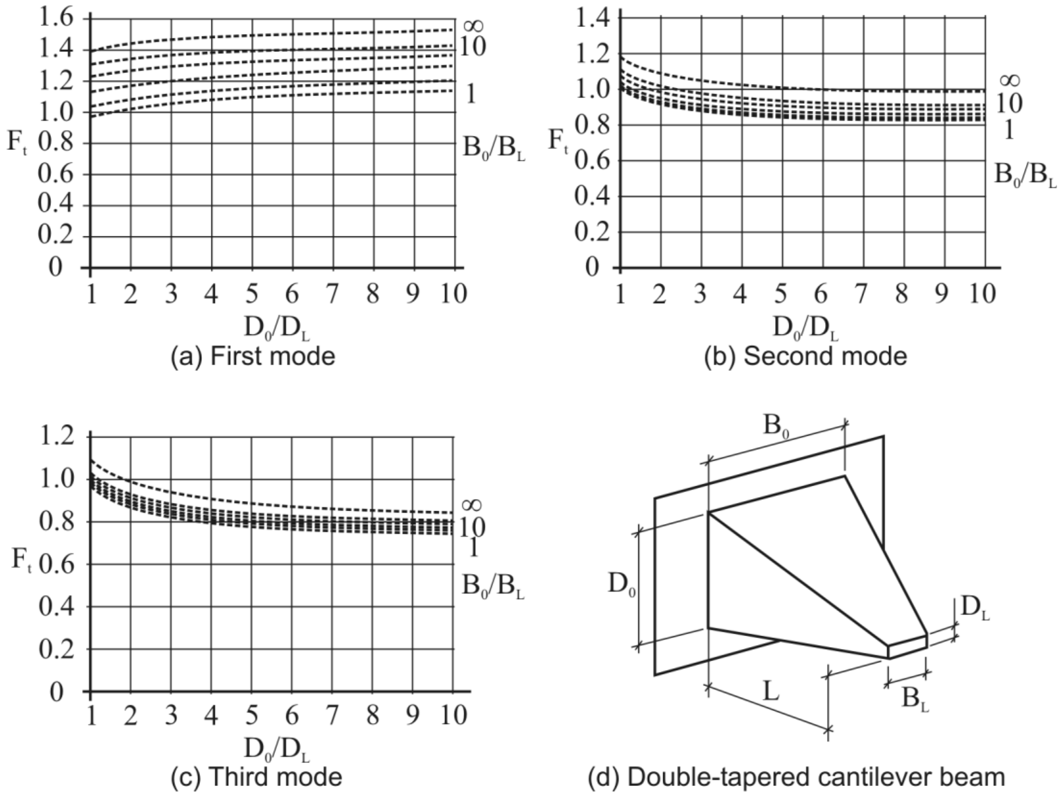


Figure 1.2. Taper correction Factor F_t after Galvez [45]

For the next two flexural mode shapes the total tower height L_{total} should be used. Sackmann also suggests the preliminary estimation of the two lowest torsional frequencies ω_{T1} and ω_{T2} based on the second and third flexural modes ω_2 and ω_3:

$$\omega_{T1} \approx 0.60 \textit{ to } 0.95\omega_2; \tag{7}$$

$$\omega_{T2} \approx 0.70 \textit{ to } 1.10\omega_3; \tag{8}$$

Almost all steel self – supporting towers studied by Sackmann [105], Galvez [45] and Mikus [87] showed the following pattern of modes:

a) the fundamental flexural mode is followed by the first torsional mode;

b) the second torsional mode is close to the third flexural mode;

c) the first axial mode occurs between modes 10 and 15;

d) in higher modes coupled flexural – torsional modes will occur.

From the Table 1.1 and studies of Sackmann, Galvanez and Mikus, it follows that flexural frequencies are usually well separated from each other but torsional and bending modes are sometimes nearly coupled.

The typical mode shapes of 67m high 4-legged self-supporting tower according to Amiri [2] presented in Figure 1.3 and Figure 1.4.

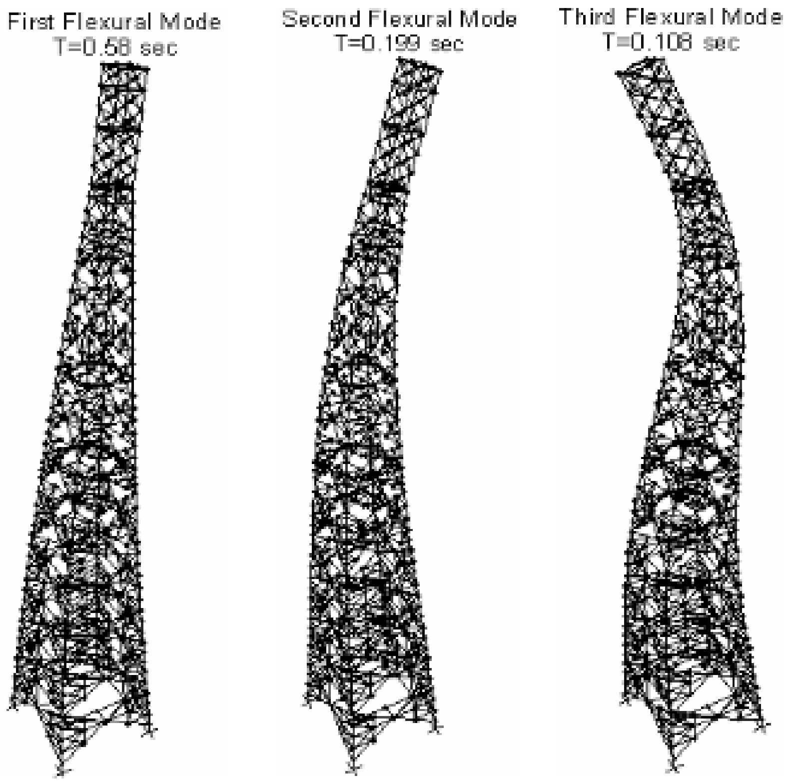


Figure 1.3. The lowest three flexural mode shapes of 67m tower [2]

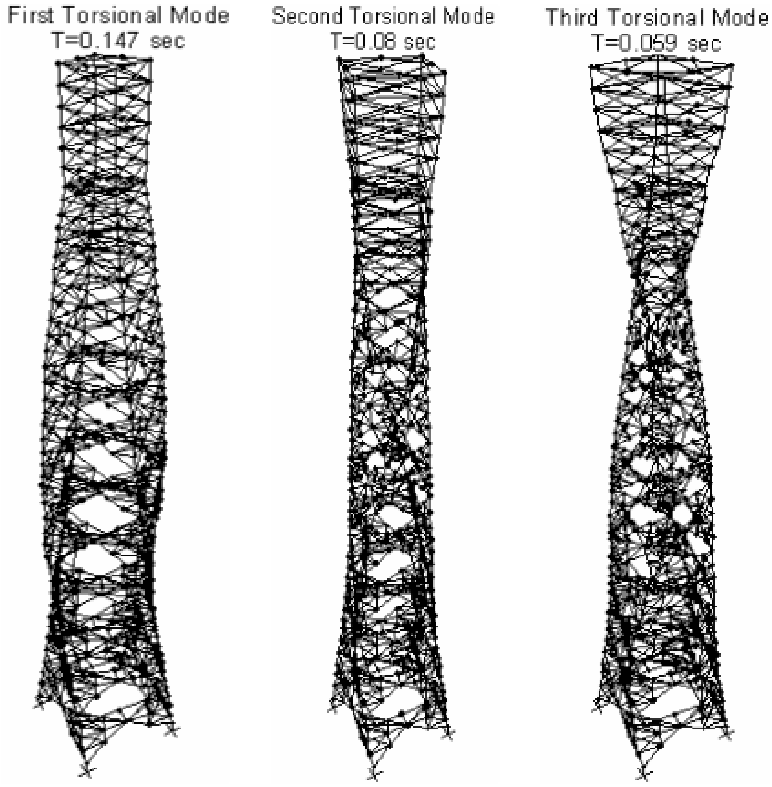


Figure 1.4. The lowest three torsional mode shapes of 67m tower [2]

In Latvia around 70% of all public observation towers are timber towers [46]. The Available design guidance information is only on the subject of timber communication towers [135 and 136]. Here a recommended height to depth ratio of timber tower is set to 1/8 to 1/10 and provided guidance to calculate the fundamental frequency of the tower for the wind effect evaluation on the tower response. Based on the case studies the preliminary equation for timber lattice tower self-weight calculation is given (9) [134]:

$$G_{\sigma} = g_{\sigma}^{H} \cdot V_{\sigma}; \tag{9}$$

$$V_{\sigma} = \frac{1}{3} H (S + S_1 + \sqrt{SS_1}); \tag{10}$$

where g_{σ}^{H} -density of structure (includes stair and platform elements), 40-50 kg/m^3;

H - total height of the tower, m;

S -tower area in plan at the ground level, m^2;

S_1 - tower area in plan at the top level, m^2;

1.1.1.2 Damping of the lattice towers

Some portion of energy is always dissipated in the real structures during vibration and the steady amplitude cannot be maintained without its continuous replacement. Although there are sources of damping like an aerodynamic damping when the resistance is provided by air and a soil damping when the vibrating energy of the tower is dissipated by the elastic waves generated in soil due to vibration of the structure, the main source is a structural damping that represents the inherent damping properties of the tower. Damping in bolted lattice steel structures is supplied predominantly by the friction at the joints [81]. This kind of structural damping is modeled usually as a viscous damper (within linear elastic limit) where the damping force is proportional to the velocity of the structure [21]. Damping ratios according to the Eurocode 1 Part 1-4 [39] steel lattice towers are:

a) fully welded steelwork ξ=0.31%;

b) high strength friction bolted steelwork: ξ=0.5%;

c) normal bolted steelwork: ξ=0.8%.

Damping ratios recommended in IASS [63] for steel lattice towers are:

d) fully welded steelwork ξ=1.2%;

e) high strength friction bolted steelwork: ξ=2%;

f) normal bolted steelwork: ξ=3%.

The damping is beneficial because it reduces the structural response to a dynamic excitation near resonance [131].

1.1.1.3 Dynamic response of lattice towers

Experimental full – scale measurements of lattice towers provide useful data for verification of analysis procedures and calibration of design ones. By using spectral analysis of digitized data dominant frequencies of recorded response are extracted (usually accelerations) from dynamic excitation [81]. To obtain mode shapes from the experimental data with ambient dynamic excitation is not so straightforward especially when mode shapes are not well separated.

To the best of the authors knowledge there is no available information concerning the experimental investigations of lattice self-supporting tower response to the human induced vibrations. Most researches analyses the self-supporting tower response to the wind loading [5, 61, 62, 115]. According to EC3 [40] lattice towers and masts should be examined for:

- gust induced vibrations (causing vibrations in the direction of the wind);
- vortex induced vibrations for towers or masts containing prismatic cylindrical or bluff elements or;
- shrouds (causing vibrations perpendicular to the direction of the wind);
- galloping instability (causing vibrations of the guyed masts);
- rain-wind induced vibrations.

More rarely there are researches on self-supporting tower response to the seismic loading [73, 74, 82]. Detailed linear dynamic analysis of six self-supportive microwave towers (with height from 20m to 90m) under seismic loading reviled the use of the lowest four lateral modes of vibration provides sufficient accuracy in the response calculations [87].

Generally two methods are used to obtain tower response to wind or earthquake loading:

- equivalent static load method;
- spectral analytical method.

The first one is practical for structural engineers and mostly incorporated in design codes because complex probabilistic and time – space dependent representation of wind loads are replaced by static wind loads allowing to combine them easily with other static loads such as self - weight or snow loads [15]. Basically the application of equivalent static wind load provides the same extreme value of a considered structural response.

The response of the structure obtained by the spectral analytical analysis method is more precise. Mode superposition produces the complete time history response of joint displacements and forces due to the applied dynamic forces. The seismic analysis method involves the calculation of maximum values of displacements in each mode, using the design spectrum that is the average of several earthquake records [81].

1.1.2 Footfall induced vibrations

Looking at slender structures such as bridges, floors, stairs and grandstands that can excessively vibrate under human activities, pedestrian bridge is probably the closest type of structure to the lattice observation tower from the point of view of dynamic analysis. Thus to develop the methodology of tower response calculations to human induced loads it is also crucial to review different methods dealing with human dynamic loads that are presently available for slender bridges.

From the extensive experimental and numerical researches in the last decade regarding the light-weight footbridge vibrations induced by human dynamic loads, it is known that slightly damped bridges become susceptible to vibrations when the natural frequencies of structures are in the range of human step frequencies [64, 104, 116, 128, 131]. In the case of bridge pedestrian density greatly influences the step frequency [22]. The mean step frequency for the low density (0.2-0.5 Persons/m2) pedestrian stream is 1.8 – 1.9Hz according to [22].

Willford and Young in the study [119] reviewed the presently available methods for the prediction of footfall induced vibrations in low frequency structures (transversal fundamental mode bellow 10Hz). Bridge calculation of resonant response on excitation by a single pedestrian who is walking at the most critical walking frequency is the main idea of the British methodology BS 5400 [4]. USA guidance AISC 1997 [90] proposes variation of harmonic pedestrian force with the frequency. Another method that was first developed in the company Arup is now recognized and applicable to broad range of structures. It is adopted by several organizations and completely described in CCIP – 016 [118]. The basic idea is to calculate the response to each harmonic load (four harmonics of pedestrian walking load that are derived statically from a large number of measured footfall force-time histories) and to obtain the total response of the structure by using “square root sum of the squares” (SRSS) calculation method (11):

$$a_i = \sqrt{\sum_{j=1}^{n} u_{ij}^2}\ , \tag{11}$$

where a_i - maximum response for the i^{th} component of the behavioutral response, m/s^2;

n - number of modes to be used in analysis;

a_{ij} - i^{th} component of the j^{th} modal behavioral response vector, m/s^2.

The majority of the current design procedures for serviceability checks assume that one or more walking harmonics (sinusoidal component of single pedestrian walking force) coincides with one of the natural frequencies of structures [130]. The amplitude of this force is expressed as a fracture of a person’s weight and is commonly called as dynamic load factor (DLF). This factor should be variably definable in a statistical sense to account for differences between each step of a single pedestrian and also among pedestrians [130]. The calculated total response of the structure (peak or root – mean – square (RMS) acceleration) then is compared to the limiting value due to serviceability criteria [131].

All above mentioned design procedures are so called time domain design procedures. Although there are studies developing frequency domain design procedures [20, 37, 88, 89, 130] they are not widely incorporated in the current design codes yet.

Until the beginning of 2000 scientists mainly concentrated on the pedestrian induced dynamic forces only from a vertical direction. But since the famous Millennium Bridge opening in London on 10^{th} of June 2000, when the newly built bridge experienced an unexpected sway in lateral direction, considerable public and professionals' attention has been attracted to the phenomenon called synchronous lateral excitation (SLE) [116].

Humans are noted to be much more sensitive to the lateral vibration than the vertical one. Even when the horizontal vibration is only 2-3 millimeters, the lateral motion affects the balance and pedestrians tend to walk with their feet further apart, which subsequently increases the lateral force imparted by individuals. In order to maintain balance, pedestrians tend to synchronize their footsteps with the motion of the structure. This instinctive behavior ensures that dynamic forces are applied at the resonant frequency of the structure and further increases the motion. With the increase of the motion, the synchronization between pedestrians increases as well. It does not go infinitely, but reaches a steady state when people stop walking, when the motion becomes too uncomfortable [44]. There is a research [28] stating that synchronous lateral excitation phenomena is not related to a specific structural type but it is possible on any bridge with lateral frequency bellow 1.3Hz loaded with a sufficient number of pedestrians. Tests performed on the Solferino footbridge [116] led to the conclusion that pedestrian-structure synchronization also known as lock-in started when reached critical value of structure's movement acceleration 0.1 m/s^2. The latest publications, for example [64], highlight the dispute about human structure interaction and the importance of the synchronization to reach excessive vibrations of the structure. Some of the researches question even the necessity of the phase synchronization [10, 19, and 80] with the structure.

In their experimental studies about lateral forces on vibrating structures Pizzimenti, Ricciardelli [93] and Ronnquist [102] concluded that the amplitude of horizontal walking force remained unchanged in case of small vibration amplitude which indicates the weak interaction between the pedestrian and the structure. In case of perceptibly moving structures there have been reports that, for example, vertical load is up to 10% lower than measured on a stiff ground [12, 92].

Recent extensive literature review is done by Venuti [116] on the subject of SLE.It highlighted the still uncompleted knowledge of the mechanisms that drive the synchronization phenomenon, the dependence of the force exerted by the pedestrians on the structural response, triggering of the lock – in and the force self-limitation.

Different design codes and studies suggest different ranges of fundamental frequencies of footbridges when they are susceptible to the human movement induced vibrations but generally it is <5Hz for bridge vertical direction and < 2.5Hz for bridge lateral direction according to the extensive literature review done by Zivanovic [131].

1.1.3 Human response to low frequency vibrations

After dynamic analysis of the structure calculated vibration amplitudes of the structure requires limitations to meet human comfort criteria. The limit values of the lateral acceleration in the international codes are directly related to the pedestrian comfort. Mainly values are given for the high-rise buildings of residential and office use or pedestrian bridges. To the best of the author knowledge there are no available recommendations regarding the observation towers.

The Handbook of Human Vibration [54] presents comprehensive comparison of proposed limits of building vibration. The perception of vibration depends on the vibration frequency. Most researchers on the topic suggest that when the frequency is in a range from 1 to 2 Hz the perception of the vibration is at its peak (see Figure 1.5).

It corresponds with the comfort evaluation curves for wind-induced vibrations of buildings in the horizontal (x,y) direction for a period of one year given in the state-of-the-art design guidance in Europe: ISO 10137:2007 [67]. The variation of vibration sensitivity is practical to take into account by attenuating the calculated response for frequencies where perception is less sensitive. This is referred to as “frequency weighting” [109].

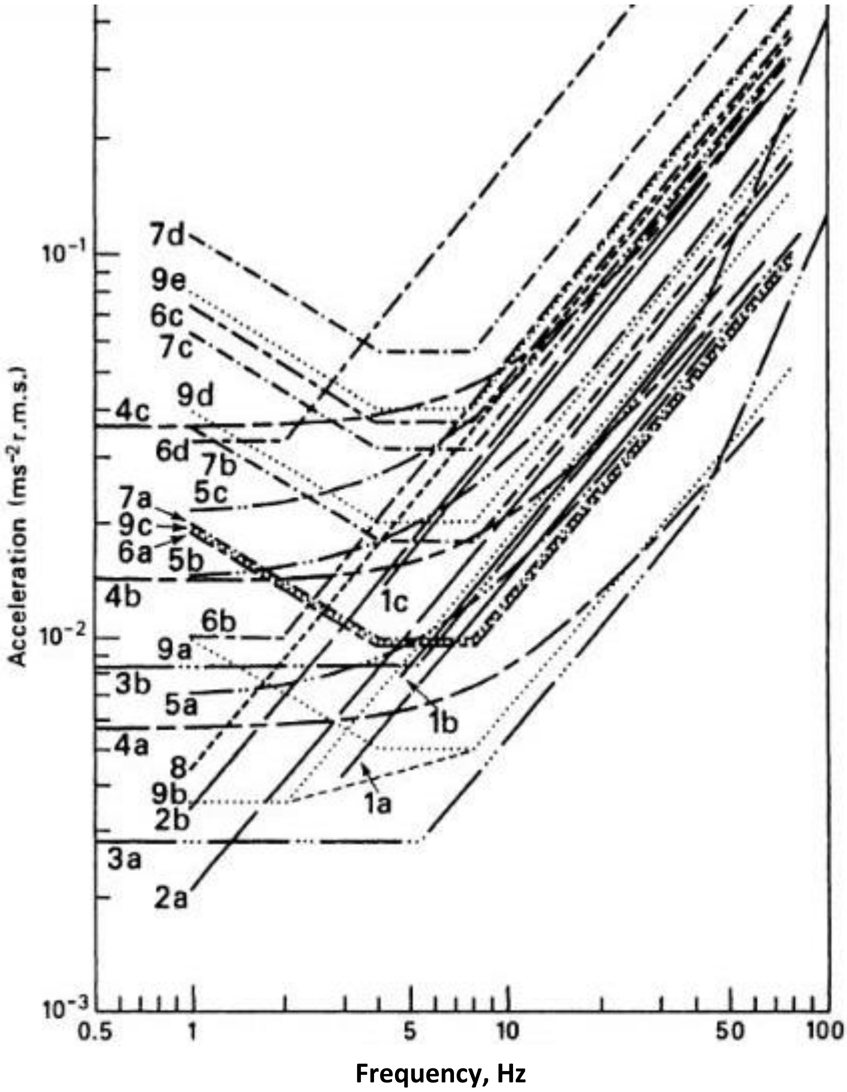


Figure 1.5. A historical comparison of proposed limits for building vibrations [54]

(1a) Reither and Meistre (1931) below "weakly perceptible" for vertical vibration of standing persons; (1b) Reither and Meister (1931) below "weakly perceptible" for lateral vibration of standing persons; (2a) DIN 4150 (1939) PAL=0; (2b) DIN 4150 (1939) PAL=5; (3a) DIN 4025 (1958) K=0,1; (3b) DIN 4025 (1958) K=0,3; (4a) VDI 2057 (1963) K=0,1; (4b) VDI 2057 (1963) K=0,25; (4c) VDI 2057 (1963) K=0,63; (5a) DIN 4150 (1975a-c) KB=0,2; (5b) DIN 4150 (1975a-c) KB=0,4; (5c) DIN 4150 (1975a-c) KB=0,6; (6a) ISO 2631 (1974) z-axis threshold; (6b) ISO 2631 (1974) x and y - axis threshold; (6c) ISO 2631 (1974) z-axis 24-h reduced comfort boundary; (6d) ISO 2631 (1974) x and y-axis 24-h reduced comfort boundary; (7a) Japanese (1976) 60dB; (7b) Japanese (1976) 65dB; (7b) Japanese (1976) 70dB; (7c) Japanese (1976) 75dB; (8) Greater London Council (Anon 1976b); (9a) ISO 2631 Part 2 (1989) z-axis base curve; (9b) ISO 2631 Part 2 (1989) x and y-axis base curve; (9c) ISO 2631 Part 2 (1989) z-axis, multiplying factor = 2; (9d) ISO 2631 Part 2 (1989) z-axis, multiplying factor = 4; (9c) ISO 2631 Part 2 (1989) z-axis, multiplying factor = 8.

The international standards and sources in literature propose different acceleration limit values for different reasons. However most of these values coincide within a certain bandwidth. The guidelines [58] give the recommended bandwidth for the different comfort levels for pedestrian bridges:

1) For maximum comfort level the acceleration limit is recommended to be < 0,1 m/s^2.

2) For medium comfort level the acceleration limit is recommended to be 0,1 m/s^2 - 0,3 m/s^2.
3) For minimum comfort level the acceleration limit is recommended to be 0,3 m/s^2 - 0,8 m/s^2.
4) Unacceptable discomfort if > 0,8 m/s^2.

General human perception levels for low frequency vibrations (0…1Hz) are summarized in Table 1.2.

Table 1.2.

Human perception levels in low frequency range after [86]

Acceleration, m/s^2	Effect
0.05 – 0.1	a) sensitive people can perceive motion b) hanging objects may move slightly
0.1 – 0.25	a) majority of people will perceive motion b) level of motion may affect desk work c) long – term exposure may produce motion sickness
0.26 – 0.4	a) desk work becomes difficult or almost impossible b) ambulation still possible
0.4 – 0.5	a) people strongly perceive motion b) difficult to walk naturally c) standing people may lose balance
0.5 – 0.6	Most people cannot tolerate motion and are unable to walk naturally
0.6 – 0.7	People cannot walk or tolerate motion
>0.85	Objects begin to fall and people may be injured

The subjective rating of perceiving vibrations depends on many factors such as previous experience in dealing with vibrations, degree of expectation of the structure vibration [66], mood, visual clues, noise, age, familiarity with the structure, height above the ground [128]. A level of vibration that causes one individual to complain might be unnoticed by another [52]. Gathering information on different person's subjective rating of felt vibrations is still a valuable instrument to identify the maximum accelerations of the structure when it is becomes disturbing for the majority of the public.

The acceleration of the system is often presented as peak acceleration a_{peak} or root – mean – square (RMS) a_{rms}. Peak acceleration is the largest value in the acceleration function *a(t)*. RMS (12) acceleration additionally gives the indication of the amount of time the system is subjected to this level of acceleration [109]:

$$a_{rms} = \sqrt{\frac{1}{T}\int_0^T a(t)^2\,dt} \tag{12}$$

where T - the period under consideration, s;

$a(t)$ - acceleration function;

t - time, s.

Although vibration annoyance is evaluated mostly by the calculation or measurement of RMS acceleration, the method affectivity depends on the type of vibration. It is more appropriate for continuous steady - state vibrations than transient vibrations where amplitude is not consistent [108].

1.2. Footfall Induced Forces

Several recent extensive literature reviews and new guidelines highlight researchers' interest in experimental identification and modeling human walking forces [22, 95, 116, 131].

Human walking induces dynamic and time varying forces which have components in vertical, lateral and longitudinal directions (Figure 1.6) that are due to accelerating and decelerating of the mass of its body.

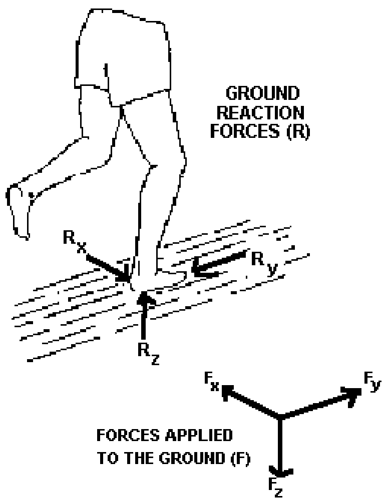


Figure 1.6. Schematic representation of ground reaction forces GRF [55]

The lateral forces are a consequence of the sideway oscillation of the gravity center of a human's body while stepping alternatively with the right or left foot forwards [42] (Figure 1.7) and it's walking frequency is found to be half of the vertical and longitudinal one [8].

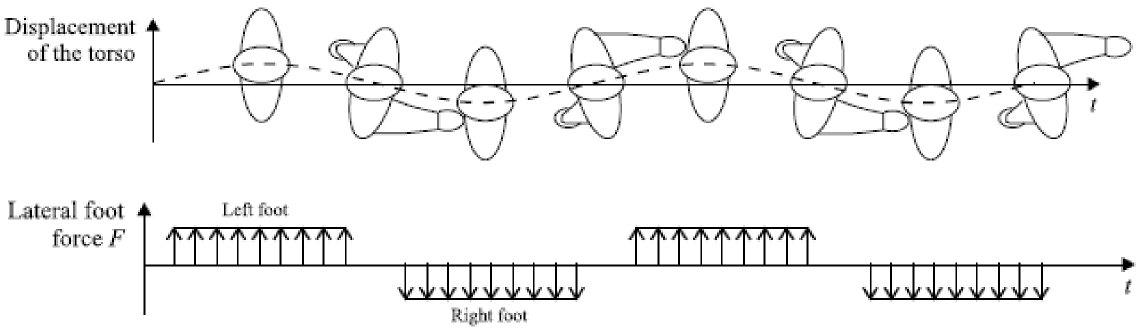


Figure 1.7. Schematic representation of lateral walking forces [22]

The Figure 1.8 presents characteristic frequencies of three walking modes on a flat surface. In a broad experiment [79] established pacing frequencies of 12239 individual pedestrians were found to be 1.825Hz with standard deviation of 0.221Hz.

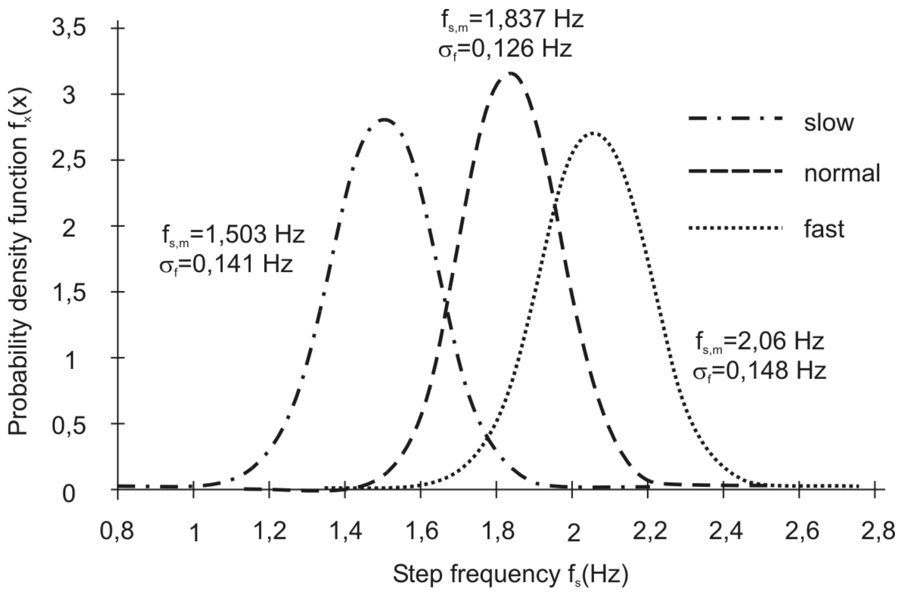


Figure.1.8. Probability density of step frequencies regarding the walking intention on a flat surface [79]

In the case of stairs there is a wide variation of walking speeds found in the literature therefore distribution of typical frequencies is different from those in Figure 1.8. Walking speed and thus frequencies very much depend of the details of the situation as the age, gender of the test persona, the motivation, the length and slope of the stairway [75].

The stairway of a sightseeing tower is a case of a long stairway. In the study [75] the measured walking speed of 485 individuals on long stairs is presented. Observed persons had climbed approximately 25 m high before their walking speeds were measured. The angle of stair inclination was 35.1^{o}. It is confirmed that the mean upward walking speed on the long stairway is roughly twice smaller than on a short one. It is found that the mean slope speed depends on the situation (Table 1.3).

Table 1.3.

Measured walking speeds on a “long stairway” [75]

	Individuals’ walking obviously influenced by anyone else	Small or no visible influence from one to another	High density situation, each person clearly influences the others in the surrounding
Mean slope speed, m/s	0.517	0.468	0.439
Standard deviation, m/s	0.159	0.091	0.048
Mean frequency, Hz	1.567	1.416	1.331
Standard deviation, Hz	0.481	0.277	0.147
Minimum frequency, Hz	0.820	0.480	1.300
Maximum frequency, Hz	4.690	4.250	1.590

Considering the tower type structures the GRF components that can cause the vibration are longitudinal and lateral force components (Figure 1.9).

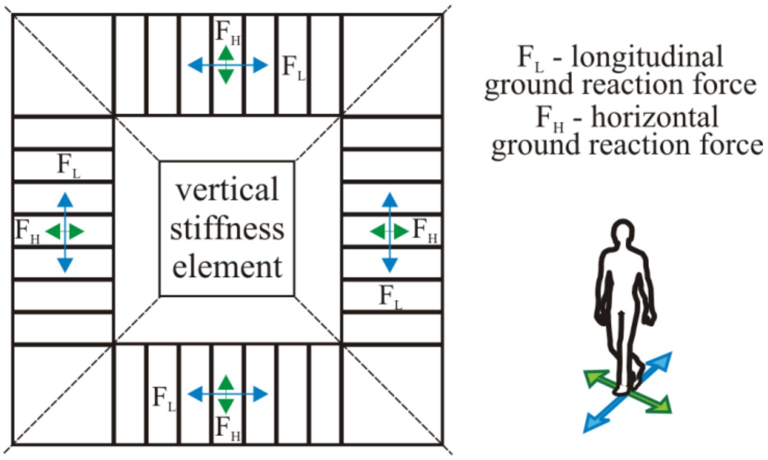


Figure.1.9. Critical human force directions for sightseeing towers with a vertical stiffness element at the center of the tower [48]

A lot of research is done on human ground reaction forces (GRF) in the field of biomechanics [7]. The interest mostly is GRF values for distinct points and their chronological occurrence on the single foot step force time history [53]. In the field of civil engineering dynamics there is an interest to simulate the continuous walking force histories that can be applied to the structure during design process.

1.2.1 Modeling of human walking forces

Although human induced forces are complex because of dependence on many parameters such as subject body mass, mechanical properties of surface, stiffness of footwear, gait style and other external factors [26, 31, 43, 83, 91] there is a necessity to model analytically human induced time varying forces to predict an accurate dynamic response of the structure during the design stage. Mainly two types of force models can be found in literature:

1) time domain force models that can be further subdivided in deterministic and probabilistic force models;
2) frequency domain force models.

The deterministic time domain force model does not take into account random variation of different gait parameters among individuals and presents uniform walking force model. The probabilistic model takes into account those diversities usually via probability density functions [95].

The basic idea behind frequency domain force models is assessment of vibration of structures by using the theory of stationary random processes [13]. Mean square value of response $E[y^2]$ is calculated from the auto spectral density (ASD) of the structure's response to human induced loads that are also defined as auto spectral density (ASD) [33, 95]:

$$E\left[y^2\right] = \int_{-\infty}^{+\infty} S_y(\omega) d(\omega); \tag{13}$$

$$S_y(\omega) = \left|H(\omega)^2\right| S_x(\omega), \tag{14}$$

where $S_y(\omega)$ - ASD of structure's response;

$H(\omega)$ - frequency response function;

$S_x(\omega)$ - ASD of the force

Use of frequency domain methods is not widely spread into the community of civil engineers and means for finding the frequency response function are not usually incorporated in the typical structural analysis software. In this sense time domain methods that usually dominate in the available design guides are more convenient.

The most common way of modeling the walking force of a single person in the time domain is based on the Fourier decomposition for perfectly repeatable footfalls. This way the walking force is represented as a sum of Fourier harmonic components – Fourier series [72].

The Fourier coefficient $\lambda_i$of the ith harmonic often referred as the dynamic loading factor (DLF_i) is the base of this model.

Theoretically the continuous walking force histories can be obtained by using kinematics of the motion of human center of gravity (COG) [120, 121]. Dynamics of different parts of the body translate the center of gravity from one point to another in the most energy efficient way [3]. Thus the vertical walking force function can be then obtained from a simple dynamic equilibrium based on the Newton's Second law (15):

$$F(t) = Mg + Ma(t), \tag{15}$$

where M is a body mass of the person, kg;

g – gravitational constant, m/s^2;

$a(t)$ – acceleration function of time of the human center of gravity (COG), m/s^2.

Then vertical walking force is [9]:

$$F(t)_{vert} = G + \sum_{i=1}^{n} G\lambda_i \sin(2i\pi f_1 t + \varphi_i). \tag{16}$$

For longitudinal or lateral walking force direction:

$$F(t)_{long,lat} = \sum_{i=1}^{n} G\lambda_i \sin(2i\pi f_1 t + \varphi_i), \tag{17}$$

where G – is a static weight of the subject body, N;

i – order number of the harmonic;

n – the total number of contributing harmonics;

λ_i – the Fourier coefficient of the i^{th} harmonic (DLF);

f_1 – pacing rate, Hz;

φ_i – the phase shift of the i^{th} harmonics.

Typical pacing rates under different human activities (walking, running, ascending or descending stairs, jumping or bouncing) and dynamic load factors (DLF) of different harmonics have been experimentally investigated widely by the research community [8, 16, 72, 97, 125, 126]. The description for some of those experimental investigations and relevant results are presented in the next section 1.2.2.

1.2.2 Experimental identification of human walking forces

In most studies on human walking forces the dynamic load factors (DLF) are extracted from experimentally obtained ground reaction forces (GRF) via Fourier analysis.

The common way to obtain the GRF is using the force platforms (Figure 1.10). It is an instrumented plate installed flush with the ground to register GRF [53]. Also the main results in the field of civil engineering dynamics regarding the GRF on stairs are obtained using this technology [68, 72] where one or few of the steps is replaced with the force plate.

Figure 1.10. Example of the force platform for GRF measurements

Figure 1.11. Example of instrumented treadmill for GRF measurements [36]

Another common way to obtain GRF is to use an instrumented force measuring treadmill (Figure 1.11). Comparing to the single force plate, the treadmill technology allows analysis of many consecutive cycles over a longer period of time [95] but it is suitable only for obtaining forces on flat or inclined surfaces. One of the most recent works to obtain experimental values of the walking force lateral component with the treadmill technology is done by Ingolfsson [65]. Both methods have a serious drawback because measurement devices have a strong influence on human ability to move naturally. Even more, Bocian [17] highlighted the possible inaccuracies of Ingolfsson work that mainly address shortcomings of the experimental setup in his study. He noted that pedestrian behavior and therefore loading is dependent on the quality of visual information available to the walker. Lack of compatibility between visual and non-visual stimuli can be considered as an important procedural shortcoming and any restrictions imposed on the ability to freely adjust gait can prevent natural behavior. The new literature review [96] about modern facilities for experimental measurement of dynamic loads induced by human showed that the state-of-the-art force measurements are usually limited to individuals in artificial laboratory environments. It is concluded that there is a serious need for group-and crowd-induced force data records on as-built structures, such as footbridges, grandstands and floors. This is still a remaining challenge due to the complexity of human actions and the lack of adequate equipment.

A relatively new concept to measure the GRF is using accelerometers that are capable of monitoring, storing, and downloading data of relatively small time intervals over a long period of time. Accelerometers are sensors that produce electrical signals proportional to the acceleration in particular frequency band and might be based on different working principles [27].

The benefits of using accelerometers compared to more traditional gait analysis instruments include low cost, testing is not restricted to a laboratory environment, accelerometers are small in size, therefore walking is relatively unrestricted and with an option of direct measurement of 3D accelerations [70]. The main categories of accelerometers used in Civil Engineering are:

a) piezoelectric;
b) piezoresistive and capacitive;
c) force – balanced.

Piezoelectric accelerometers advantages over other types are [27]:

a) does not require external power source;
b) stable in the long term;
c) relatively insensitive to the temperature;
d) linear over a wide frequency range.

Common specification for the interested frequency range of 0.5 – 20Hz are following:

a) Frequency range (with 5% linearity): 0.1 – 50Hz;
b) Minimum sensitivity: 10mV/g;
c) Range: $\pm$ 0.5g.

Another way with a great potential to obtain GRF is to combine the visual motion tracking data recorded using cameras or sensors during the analysis [23] with known body mass distribution [53]. Both of those methods are the so called inverse dynamic methods based on the kinematics of the motion of human center of gravity (COG) and could be valuable in civil engineering applications to estimate the continuous human induced forces applied to the structure under a wide range of conditions.

COG also known as a body center of mass (BCOM) represents the mean position of the total mass of human body as a multi-segment system [95] (Figure 1.12). The segmental masses and their centers can be found from different authors [34, 114, 122]. This approach is usually used in the field of inverse dynamics (Figure1.13) where most modern motion capturing systems use video-based optoelectronic technology to quantify the position and orientation of bodies in real time [23].

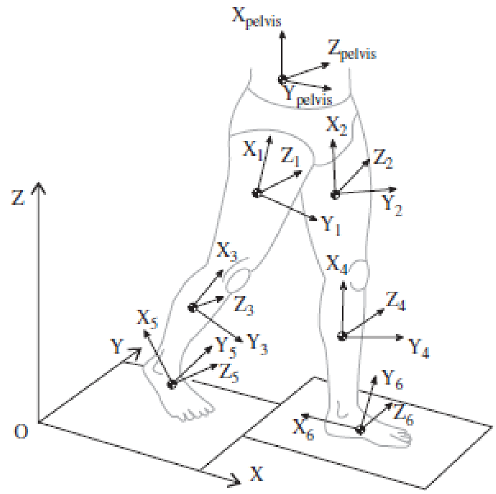


Figure 1.12. Segments of lower extremities [114]

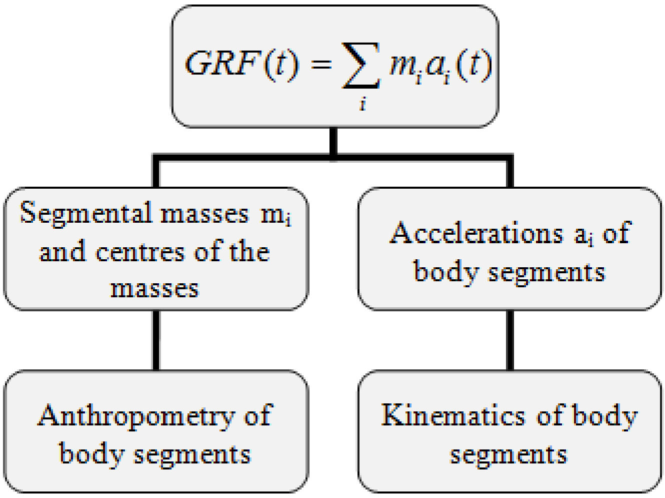


Figure 1.13. Flow chart of indirect measurement interpretation of human – induced loading

The drawbacks of the method are (creating) an incorrect assumption that body segments are rigid and placing markers or sensors accurately on the relevant segment of the body is problematic. Also the huge amounts of data due to the number of body segments under consideration are subject to errors. In the field of biomechanics it is known that the approximate location of COG for women is 55% of height from the floor and 57% for men [11].

The major part of researches about footfall induced forces is done for human walking forces on flat surfaces.

The overview of DLF for a single person's vertical force reported by different authors is summarized in Table 1.4. Fewer results are obtained for walking force lateral or longitudinal directions (Table 1.5).

Table 1.4.

DLF values for single person force models after [30] for vertical direction

Authors	DLF for considered harmonic	Phase shift	Activity	Comment
Blanchard [16]	$\lambda_1 = 0.257$		walking	DLF is lessen from 4 to 5 Hz
Bachmann & Ammann [8]	$\lambda_1 = 0.4 - 0.5$		walking	Between 2.0 Hz and 2.4 Hz
Bachmann et. al. [9]	$\lambda_1 = 0.4/0.5$ $\lambda_2 = \lambda_3 = 0.1$	$\varphi_2 = \varphi_3 = \frac{\pi}{2}$	walking	2.0 Hz / 2.4 Hz
Bachmann et. al. [9]	$\lambda_1 = 1.6$; $\lambda_2 = 0.7$ $\lambda_3 = 0.3$		running	Between 2.0 Hz and 3.0 Hz
Kerr [72]	$\lambda_1 = 0.07$ $\lambda_3 = 0.2$		walking	λ_1 is frequency dependant
Young [126]	$\lambda_1 = 0.37(f_p - 0.95) \le 0.5$ $\lambda_2 = 0.054 + 0.0088 f_p$ $\lambda_3 = 0.026 + 0.015 f_p$ $\lambda_4 = 0.01 + 0.0204 f_p$		walking	Mean values for Fourier coefficients
Eurocode 5 and DIN 1074	$\lambda_1 = 0.4$; $\lambda_2 = 0.2$		walking	
Eurocode 5 and DIN1074	$\lambda_1 = 1.2$		jogging	

SYNPEX findings [22]	$\lambda_1 = 0.0115 f_p{}^2 + 0.2803 f_s - 0.2902$; $\lambda_2 = 0.0669 f_p{}^2 + 0.1067 f_s - 0.0417$;	$\varphi_1 = 0$ $\varphi_2 = -99.76 f_p + 478.92 f_p - 387.8[°]$	walking	DLF for obtaining mean human ground reaction forces

Table 1.5.

DLF values for single person force models for lateral or longitudinal directions

Authors	DLF for considered harmonic	Phase shift	Activity	Comment
Bachmann et. al. [9]	$\lambda_1 = \lambda_2 = \lambda_3 = 0.1$	$\varphi_2 = \varphi_3 = \dfrac{\pi}{2}$	walking (lateral)	at 2.0 Hz
Bachmann et. al. [9]	$\lambda_1 = 0.2$; $\lambda_2 = 0.1$		walking (longitudinal)	at 2.0 Hz
Charles [25]	$\lambda_1 = 0.05$		walking (lateral)	
Charles [25]	$\lambda_1 = 0.2$		walking (longitudinal)	
Eurocode 5 and DIN 1074	$\lambda_1 = \lambda_2 = 0.1$		walking (lateral)	

The fundamental element of any lattice observation tower without a lift is stairs. Thus the main activities of tower visitors will be conducted whether on stairs or sightseeing platforms. Still there is little work done for studying the walking forces on stairs. The most relevant and recent study on this subject is done by S.C. Kerr, N.W.M. Bishop [72] and M. Kasperski [68, 69].

S.C. Kerr presented more than 500 individual footstep measurements of 25 subjects ascending and descending stairs with inclination 22^0 - 28^0 and walking on a flat surface. Comparing the results of measurements, he concluded that footstep forces significantly differ whether walking on a flat surface or on stairs. The staircase loads are generally much higher. Figures 1.14 to 1.15 are the results obtained by Kerr for stair ascending and descending cases for different pacing frequencies. The results reveal a very significant scatter of obtained DLF values of vertical force especially for the second harmonic. In the case of stair descend for the second harmonic it is almost impossible to trace the relationship between the walking pace and DLF value.

These results are not easily applicable to the dynamic analysis of structure under human walking loads nevertheless Kerr suggests the typical harmonic values for stair ascend and descend cases in his research (Figure 1.18 and 1.19). Figure reveals that the first four harmonics of vertical walking force would be enough to approximate the walking force history but Kerr does not give the phase values from (16) of relevant walking harmonics. Therefore it is impossible to obtain the maximum amplitude of human walking forces ascending or descending the stairs.

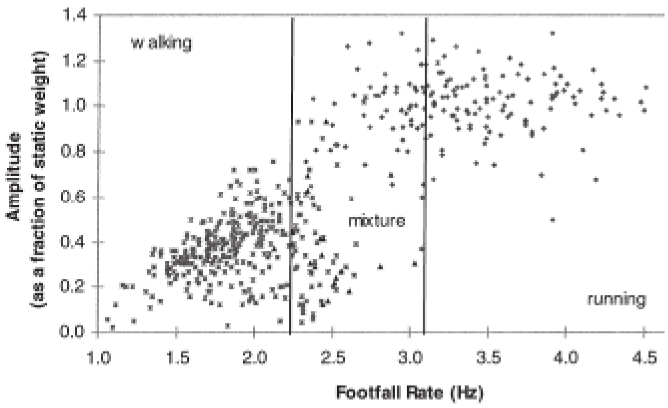


Figure 1.14. First harmonic values for ascending stairs by Kerr [72]

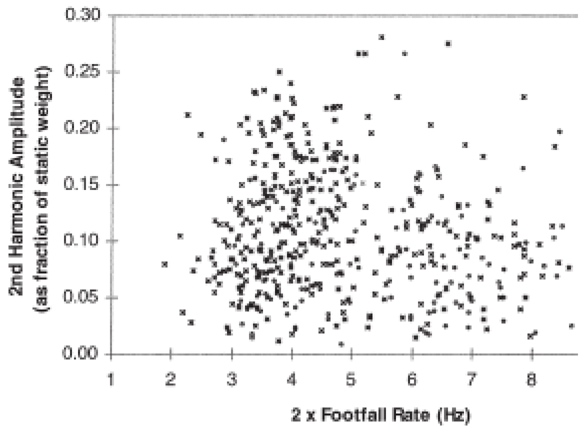


Figure 1.15. Second harmonic values for ascending stairs by Kerr [72]

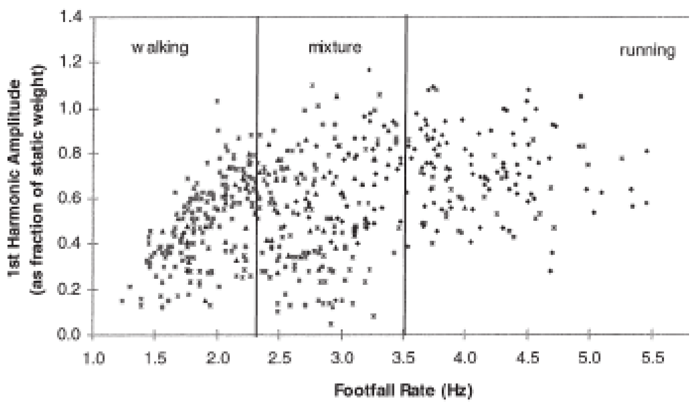


Figure 1.16. First harmonic values for descending stairs by Kerr [72]

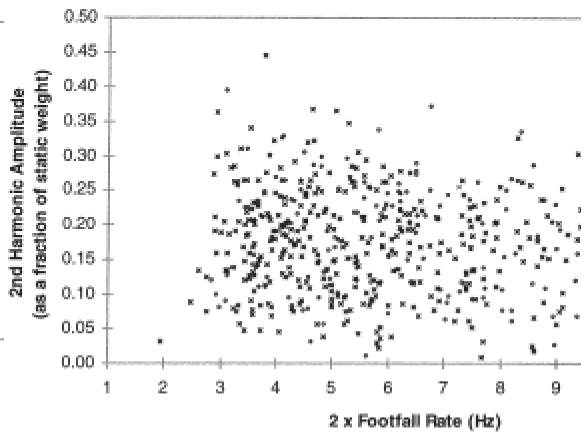


Figure 1.17. Second harmonic values for descending stairs by Kerr [72]

There have been reports that a single step force record suggested to be unreliable [56] because of potential inability of the single step force record to present continuous walking force.

To get more than one single step force record M. Kasperski [69] obtained load time histories of 105 people. His test stair case had four active steps and four passive steps which served as the lead-in or lead-out section. Four active steps were mounted on one multi component force plate through a wooden frame. He concluded that about 50% of the tested people showed a relative difference larger than 4% between both legs. Additionally he obtained results for the loads induced by taking two steps at once and they are about 35% larger than those for taking single steps. However in M. Kasperski's experiment it is questionable how the wooden structure's above force plate dynamic properties influence precision of obtained load time histories.

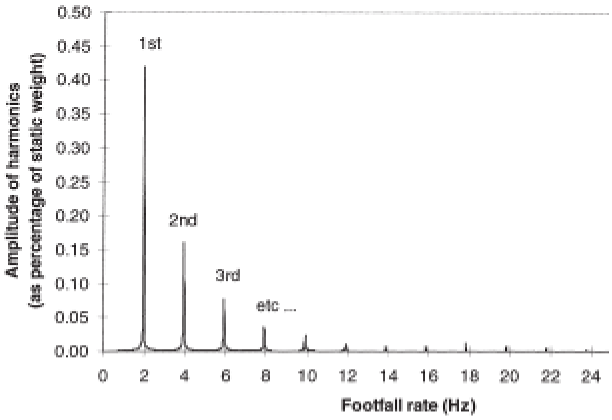


Figure 1.18. Typical harmonic values - ascending at 2.0 Hz by Kerr [72]

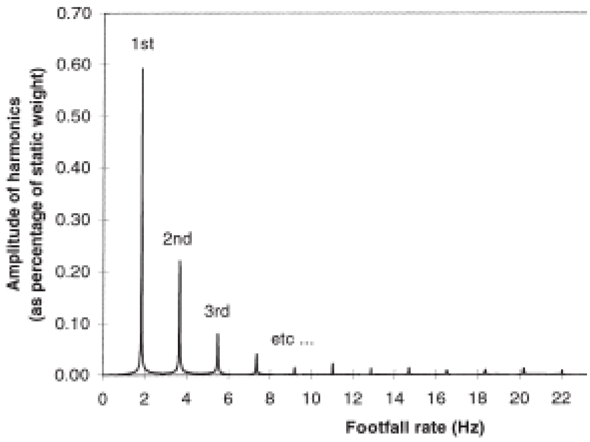


Figure 1.19. Typical harmonic values - descending at 1.85 Hz by Kerr [72]

There are recent suggestions [124] that widely accepted equations of DLFs based on the statistical gait results of Europeans are somewhat conservative if used for dynamic assessment of structures in Asia due to differences in typical human induced loading.

From the research field of biomechanics in his experimental investigation Riener [100, 101] found that ground reaction forces (GRF) were not significantly affected by the staircase inclination (tests were performed on the following stair inclinations: 24^0, 30^0, 42^0) but differed from level walking. Tests were done on a staircase that was composed of four steps and a platform at the upper end that was adjustable in height. The lower three steps were instrumented with six strain-gauge force transducers each. This allowed the collection of kinetic data for three steps in a row. The results of a single footstep are presented in Figure 1.20.

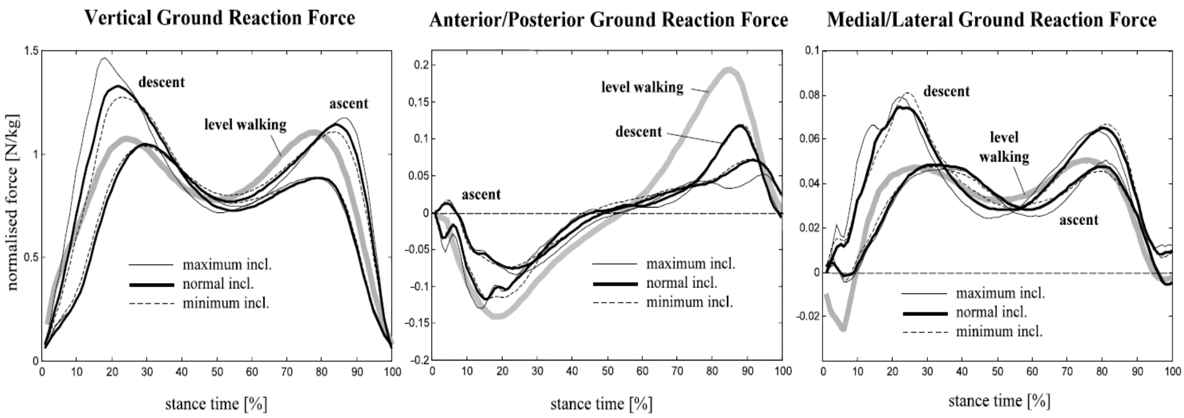


Figure 1.20.Averaged ground reaction forces of one step during ascent (~1.4Hz) and descent (~1.65Hz) at minimum, normal and maximum inclinations and during level walking [101]

Later in the research Stacoff [111] measured ground reaction forces not only for a vertical direction but also at different inclinations. His findings were that vertical GRF force pattern changes slightly from level walking to stair ascent (largest during the steep stair) and considerably during stair descent. The steep stair descent condition was found to be the most demanding test showing the largest variability and asymmetry and thus, the least stable gait pattern. While descending the stairs, the typical double waveform that can be seen in the work of Riener was no longer present (Figure 1.20).

Other researchers evaluate human loading on staircases by measuring an actual stair structure dynamic response either to a single person or a group loading. By measuring dynamic response of a steel fire escape staircase due to different activities of individuals, small groups and uncontrolled crowds, A. Bougard [18] obtained ratio P_{dyn}/P_{stat} which is applied dynamic load to applied static load (DAF). This approach of experimental investigation would not be suitable in the case of a tower because there would be difficulties to apply statically human force in a horizontal direction.

The above mentioned researches about DLF values on stairs do not give recommendations about DLF mean values and relevant phase shifts to be able to correctly apply human walking ground forces to civil engineering structures. The suggested DLF values in state-of-art standards are not complete. For example the published design parameters in ISO 10137:2007 [67] are based on the work of Bishop in 1995 [14] for only vertical walking force two harmonics (Table 1.6).

Table 1.6.

Design parameters for forces due to one person ascending or descending stairs published in ISO 10137:2007 [67] Table A.4.

Activity	Harmonic number	Common range of forcing frequency, Hz	DLF for vertical direction	DLF for horizontal direction
Ascending or descending stairs	1	1.2 to 4.5	1.1	**Not available**
	2	2.4 to 9	0.22	

Researchers who applied existing load models for stair ascending or descending cases reported noticeable differences between predicted and measured accelerations due to climbing. B. Davis and M. Murray [29] tested a slender monumental stair to obtain a response to human walking. For analytical comparison of results finite element techniques were used. To predict a peak steady-state acceleration response, a footstep forcing function reported by S.C. Kerr, N.W.M. Bishop [72] was applied. B. Davis and M. Murray reported a considerable difference between predicted and measured accelerations due to walking (Table 1.7).

Bin Zhou [127] also obtained slender indoor spiral steel stair accelerations experimentally and numerically by measuring human walking and running activities. For numerical analysis he used only the first two harmonic dynamic components of walking force to load the stairs. The predicted stair response of numerical analysis in some reference points was even 27% less than experimentally measured ones.

Table 1.7.

Measured and predicted accelerations due to climbing the stairs [29]

Description	Measured peak acceleration (%g)	Predicted peak acceleration (%g)	Difference, %
Ascending, 2nd harmonic	4.1	1.9	-53.6%
Ascending, 3rd harmonic	1.1	1.3	+18.2%
Descending, 2nd harmonic	4.7	4.2	-10.6%
Descending, 3nd harmonic	1.7	1.9	11.8%

In those works as a dynamic load for predictions only the vertical force component seems to be applied. This indicates that loading models are still not complete and tuned properly. This correlates with the conclusion in the recent literature review done by

V.Racic [95] that disregarding the activity investigated, only the vertical ground reaction forces (GRF) on rigid surfaces for a single person are tested with a modern non direct measurement technologies. There is a clear necessity to improve the existing load model of pedestrian induced forces to obtain a better agreement between numerically calculated and experimentally measured structure's response to human activities.

1.3. The Objective of the Study and the Tasks of the Thesis

The following conclusions can be drawn from the performed literature review:

- Contemporary light weight slender structures such as pedestrian bridges, slender floors, grandstands and flexible stairs are often subject to vibrations caused by human activities. There are different types of those activities that induce dynamic forces on structures. Mostly human induced vibrations of structures are associated with the comfort of structure users and therefore fall under the serviceability limit state.
- One more type of structure that is susceptible to human induced vibrations and has not received attention of researchers before is a slender lattice observation tower. Traditionally for such type of structures full dynamic analysis is not performed and to evaluate wind induced vibrations on the structure equivalent static load methods are used. The case studies highlight the lack of understanding and inadequate design information of the building codes, regarding the slender tower dynamic response to human induced loads.
- Usually the prime use of light-weight lattice self-supporting tower is to support communication and broadcasting equipment therefore available theoretical and experimental studies about lattice tower dynamic performance are mostly in this context. There is no available information about typical dynamic parameters (natural frequencies, damping ratios and modal masses) regarding light-weight lattice observation towers required for full dynamic analysis of such structure.
- The calculated expected vibration of the structure requires limitations to meet human comfort criteria. The limit values of vibration acceleration in the international codes are directly related to the pedestrian comfort. Values are mainly given for the high-rise buildings of residential and office use or

pedestrian bridges. To the best of the authors knowledge there are no available recommendations regarding observation towers.

- There is a necessity to analytically model human induced time varying forces to predict an accurate dynamic response of the structure during the design stage. There is still a requirement for reliable experimentally obtained data of human ground reaction forces under different activities. Especially scarce data is available on human induced time varying forces for the stair ascending and descending cases. Reliable data on stair ascend and descend walking force longitudinal and lateral components that can be a cause of tower type structure excessive vibrations under human loading is missing.
- The state-of-the-art human walking force measurements are usually limited to individuals in artificial laboratory environment therefore there is a necessity to develop methods for experimental identification of the footfall induced forces that is not restricted to the laboratory environment and does not have strong influence on person move naturally during the tests.
- Currently approaches and methodologies to assess the dynamic response and performance of the structures under human induced dynamic loads for footbridges and slender floors are still developing. But those methods are not straightforward to apply for vertical structures like observation towers so therefore this type of structure methodology should be developed as well.

Therefore to be able to assess the dynamic response and performance of lattice self – supporting towers during the early stage of the designing process objective of the study is developed:

- To develop the method of analytical approximation of human movement induced dynamic loads based on the experimental investigation and to develop the calculation methodology for assessment of light-weight lattice self-supporting tower type structure dynamic response to typical human induced dynamic loads as well as to set a limit on the observation tower vibration acceleration amplitudes due to the comfort criteria of tower visitors.

Tasks that have to be resolved can be subdivided in two main groups in order to achieve the formulated objective of the thesis.

The first group of tasks associated with the experimental identification and approximation of human induced time varying forces:

1. To develop the new method of experimental identification of the footfall induced forces that is not restricted to the laboratory environment and therefore does not have strong influence on a person during the tests move naturally and is suitable for the civil engineering applications;
2. To develop an experimental data processing method of obtaining the mean continuous walking force history of a person that could be analytically approximated and therefore used in analytical calculations of observation tower response to human induced loading;
3. To obtain dynamic load factors and corresponding phase shifts of footfall induced forces on stairs by the newly developed method and compare to other researcher work data that is available for justification of the newly developed method.

The second group of tasks have to be resolved is associated with the application of those loads to the lattice self-supporting tower structure and finding its response to it.

4. To obtain the dynamic and geometric parameters of most of the public observation towers in Latvia and to develop the criteria for tower type structures that are sensitive to the human movement as well as experimentally identify the loading events from human movement under actual serviceability conditions that cause the highest response levels of observation tower vibration;
5. To develop preliminary recommendations of criteria that would ensure the acceptable comfort level for the observation tower visitors based on the observations made during the experimental researches (subjective assessment);
6. To assess the different parameters (structural damping, ratio between pacing rate and natural frequencies of the structure, stiffness of the structure, separate walking harmonic importance, number of the visitors, mode of vibration etc.) importance and contribution to the dynamic response level of observation tower and based on that develop calculation methodology and recommendations for assessment of light-weight lattice tower type structure dynamic response to typical human induced dynamic loads.

2. FOOTFALL INDUCED FORCES ON STAIRS

There are three main shortcomings of the available state-of-art experimental investigation methods that are used to obtained human footfall induced dynamic forces:

- strong influence on person move naturally during the tests;
- restriction to the laboratory environment;
- complex experimental set up.

Especially influence on person move naturally during the tests lately is addressed as serious shortcoming [17] due to the fact that locomotion is adaptive in its nature and optimizes according to circumstances [1]. Therefore to get reliable results of GRF (ground reaction forces) it is not enough to record a few steps where the test subject is concentrated on performing them. The laboratory environment restriction does not allow to easily record GRF measurements for numerous steps in the row in the case of stairs.

More suitable are indirect measurement methods widely used in the field of biomechanics and discussed in the section 1.2.2. These methods are based on kinematics of the motion of human center of gravity (COG) and walking force function can be obtained from a simple dynamic equilibrium based on the Newton's Second law (15). The most common experimental setup used up to date in the field of biomechanics: motion capture systems gathering information of markers movement attached to the test body segments and afterwards proceeded via appropriate software to obtain continuous time histories of GRF.

For the civil engineering applications it would be more convenient if the placement of the sensor close to the actual COG of the whole body and not to the separate segments. The developed new experimental method to obtain continuous walking force histories is described in the next subsection 2.1. The verification of a method is performed by comparing the GRF results for stairs obtained with the new method and traditional force plate method done by other authors and presented in the section 2.3. The developed method allows to estimate continuous human – induced forces of different actions applied to the civil structures under a wide range of conditions due to the non-laboratory restrictions and not only for the stairs.

2.1. Experimental investigation of footfall induced forces on stairs

The design of experimental set-up and the choice of the measuring devices were performed based on the following criteria:

- the measuring device (sensor) should be able to record simultaneously direct measurements of 3D accelerations eliminating errors associated with differentiating displacement or velocity data;
- the captured and recorded continuous motion data should be stored in the measurement device to avoid the presence of wires that could influence the test person natural movement and also eliminating errors associated with data transfer through distance;
- the sensor should be tightly attached on the test subjects body through the detail that reduces effect of the "soft tissue artifact" [78];
- the required sensor should be light-weight and with small dimensions to reduce its inertia during movement;
- the selection of the required sensor sensitivity is based on the maximum range of accelerations associated with normal walking and running measured close to the foot:
    a) walking: $\pm 2-5g$ [71, 103, 123];
    b) running: $\pm 10g$.
- The majority of gait-related movements are dominated by relatively low frequencies [70].

Considering the criteria above to record the accelerations chosen two 3-axis light weight USB accelerometers (Model X6-1A (Figure 2.1 and 2.2) with following features:

- 3-axis accelerometer - acceleration is collected in X, Y and Z axes and stored at a user selectable rate;
- User selectable ±2 or ±6g range;
- User selectable sample rate of 10, 20,40, 80, and 160 Hertz;
- 12-bit and 16-bit resolution;
- User selectable dead band and trigger;
- Accurate time stamped data using Real;
- Time Clock (RTC) with power back-up;
- Convenient on/off button;
- Data recorded to a removable micro SD card;
- Easily readable, comma separated text data files;
- Data transfer compatible with Windows or Linux via Universal Serial Bus;
- (USB) interface (no special software);

- System appears as USB Mass Storage;
- Device to Windows and Linux OS's;
- Standard replaceable "AA" type battery;
- LED indicator lights for system status;
- Weighs 55g with alkaline battery.

Figure 2.1. USB accelerometers (Model X6-1A)

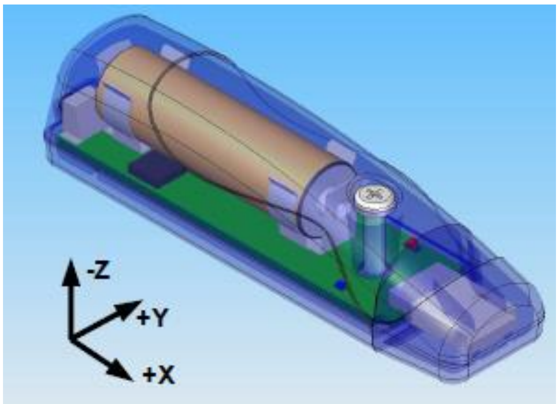


Figure 2.2. Accelerometer Sensor Orientation [110]

Accelerometers were fixed to the foam plastic light weight boards that were tightly attached with straps to a subjects COG horizontal axis in front and back of the body. In the field of biomechanics it is known that the approximate location of COG for women is 55% of height from the floor and 57% for men [11]. That is a region of the lower trunk of body and it has low transverse plane rotation relative to axial rotation of pelvis or thorax [70]. It is known that during normal walking upper body rotation in sagittal plane (Figure 2.3) can be in magnitude of 1-2 degrees [113] and in coronal plane 4 degrees [106].

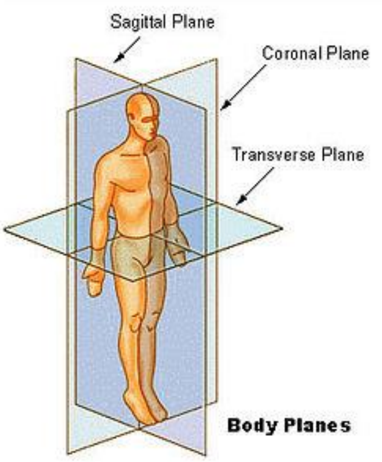


Figure 2.3.Body planes. Retrieved from http://en.wikipedia.org/wiki/Sagittal_plane

The measurement sample rate was chosen 160 Hz. Other sensor parameters selected for experiments are summarized in Table 2.1 and the experimental set-up is presented in Figure 2.4.

Table 2.1.

Selected accelerometer sensor parameters

Parameter	Condition	Min	Typical	Units
Acceleration range	Low Gain	$\pm 5.6g$	$\pm 6.0g$	g
16-bit Resolution	Low Gain (±6g)		0.00020	g/count
Linearity	X,Y axis		±2	%FS
Linearity	Z axis		±3	%FS

Static calibration method that is the most convenient method for an accelerometer calibration and validation is used. It involves comparing the output of a stationary accelerometer to a known constant-gravity. Basically the output of an accelerometer that is kept stationary and aligned with the global vertical direction must be 1g and -1g when inverted. This is verified for every local direction of an accelerometer by turning it 90 degrees.

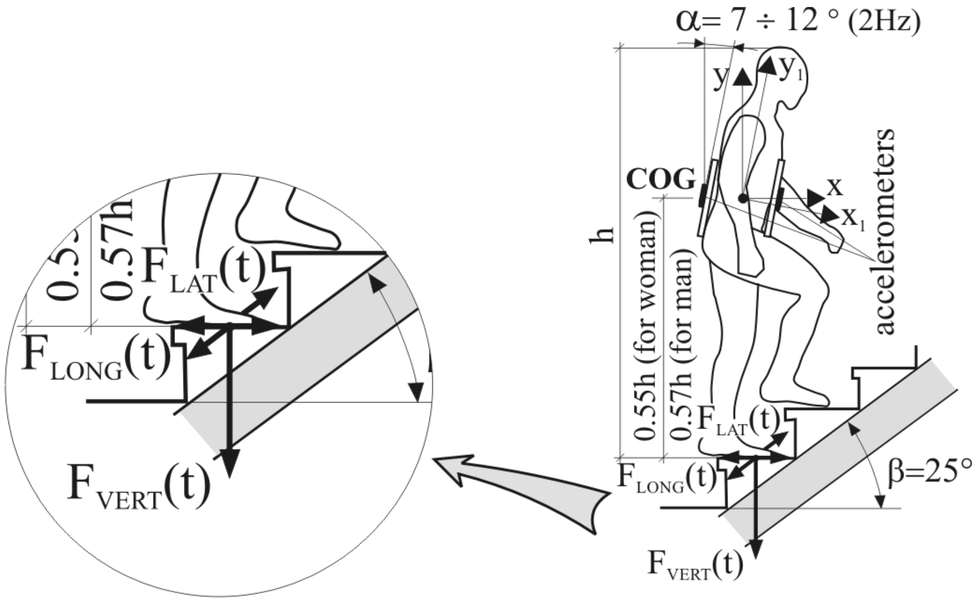


Figure 2.4. Illustration of the experimental set up

The acceleration of the person’s center of gravity (COG) in vertical, lateral and longitudinal directions during stair ascent and descent were recorded to obtain individual continuous walking force time histories. The experiment took place in Riga Technical

University's staircases A and B (Figure 2.5 to 2.8). Inclination of the test stairs is 25^0 which is a common inclination. Two flights of the stair were used to perform the test.

11 men and 7 women with a normal walking pattern took part in the experiment. The mean weight of tests subjects was 71 kN.

During the experiment there were more than 216 continuous walking acceleration time histories recorded which resulted in more than 2160 individual footfall traces.

Figure 2.5. Test stairs A

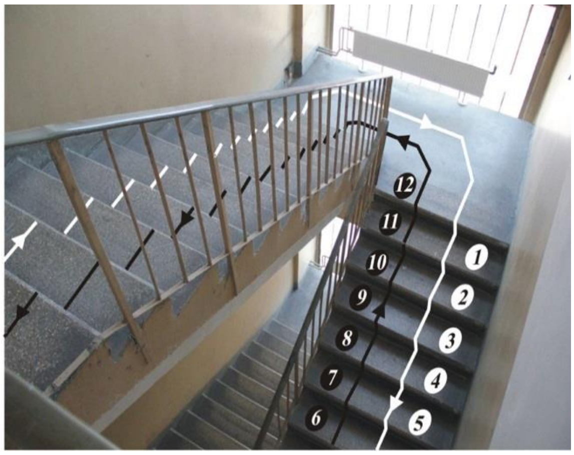


Figure 2.6. Test stairs B

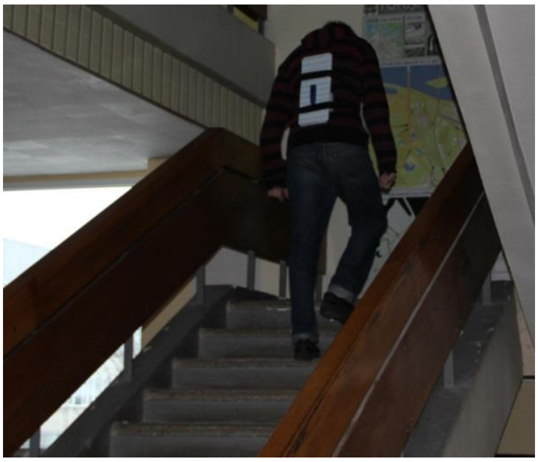


Figure 2.7. Test subject during the test (test stairs A)

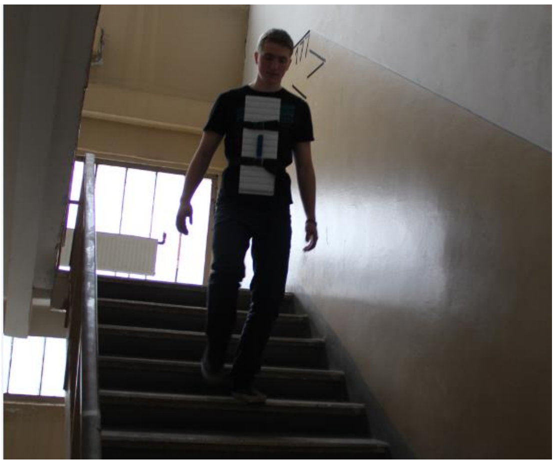


Figure 2.8. Test subject during the test (test stairs B)

Example of raw data from the accelerometer is presented in Figure 2.9. For analysis altogether 60 continuous walking acceleration time histories were used which contained 540 individual footfall traces. The recorded acceleration data in one file is discrete data in time domain with following variables:

1) N=3200 – total number of discrete data points taken;
2) T=20s - total sampling time;

3) $\Delta t = T/N = 0{,}00625s$ - time between data points;

4) f_{samp}=160Hz – sampling frequency.

Each participant of the experiment had several attempts. Individuals admitted that the equipment did not restrict their natural movement on the stair and chose their own convenient constant walking rate and path during the two flight stair ascent and descent. Measurements only of the second flight during stair ascent at a pacing rate close to 2.0 Hz and measurements of the first flight during stair descent at pacing rate close to 2.15 Hz were taken further for the data processing to minimize any psychological impact on the test subjects thus avoiding natural gait pattern alternation. Steps taken for further analysis are numbered in the Figures 2.5 and 2.6. The rest of continuous walking histories at other pacing rates are used to obtain the relationship between maximum walking force amplitude and pacing rate later in the thesis.

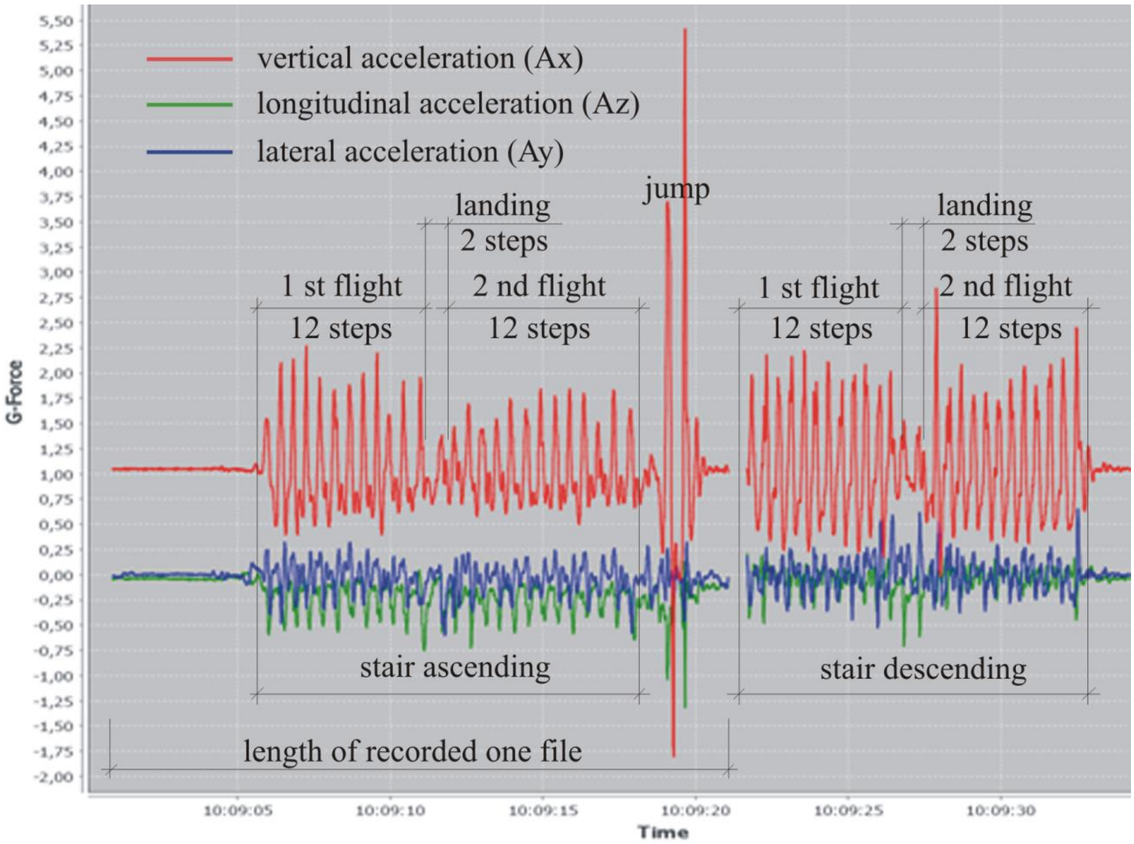


Figure 2.9. Test subject during the test (test stairs B)

The raw acceleration data already reveals that each step of the test subject has its own amplitude and is not perfectly periodic. There are also noticeable differences between each flight of the steps. Generally the pattern of maximum amplitude acceleration stabilizes on the second flight when the test subject made a turn on the stair landing and cannot be seen by test supervisors thus feeling more relaxed about the process of experiment and climbing more naturally. Especially the differences in acceleration amplitudes and the patterns between first and second stair flight emerges during the stair ascent. While starting movement the test person is concentrated on the task and tends to hit their foot harder on the stairs. This indicates that walking force histories based on measurements of only a few steps does not correctly reflect the forces in natural environment.

Additionally, during some of the tests a laser streamer was mounted on the front of the board that lased on the staircase wall while the video camera recorded the sagittal plane angle α changes shown in from Figure 2.4. Body's upper part forward inclination during stair ascends was found to be in a range of 7 degrees to 12 degrees that is more than walking on flat surface. Recoded experimental data was adjusted to take into account this angle and deviation from global direction due to misalignment by applying basic trigonometry and known constant acceleration of gravity.

Experimental data processing methods of continuous walking force histories are described in the next section of the thesis. The obtained results and their analyses are presented in the section 2.3.

2.2.New experimental data processing method for obtaining the analytical expression of the equivalent continuous walking history

Although the probabilistic force models (described in section 1.2.1) would be more suitable when simulating the walking forces as it is a stochastic narrow band process and depends on many parameters, more convenient from the designing point of view would be a deterministic force model that takes into account a non-periodicity of the force. The most common way of walking force modeling is based on the Fourier decomposition. For perfectly repeatable footfalls walking force can be represented in the time domain as a sum of Fourier harmonic components (16), (17). Here the Fourier coefficient of the i^{th} harmonic referred to as the dynamic loading factor (DLF_i) is the base of this model. The example of walking force longitudinal component decomposition is presented in Figure 2.10.

Most researchers try to quantify walking force through the DLF values of harmonics. The example in Figure 2.11 shows a very significant scatter of obtained DLF values of vertical force second harmonic (according to different authors) thus the average of these data are questionable and it is not clear which value of DLF_2 should be used further in dynamic analysis of the structure under consideration.

Mostly researchers experimentally obtain the GRF function from one or few steps using force plates and then decompose into harmonics. But in each walking force history harmonics and their amplitudes (DLF values) are connected between themselves as they are members of the series. So, one of the reasons of DLF data scatter is that these connections have not been taken into account. Furthermore there is a loss of information about the phase shift values φ_i that is crucial to know to be able to replicate the walking force time history back from separate harmonics.

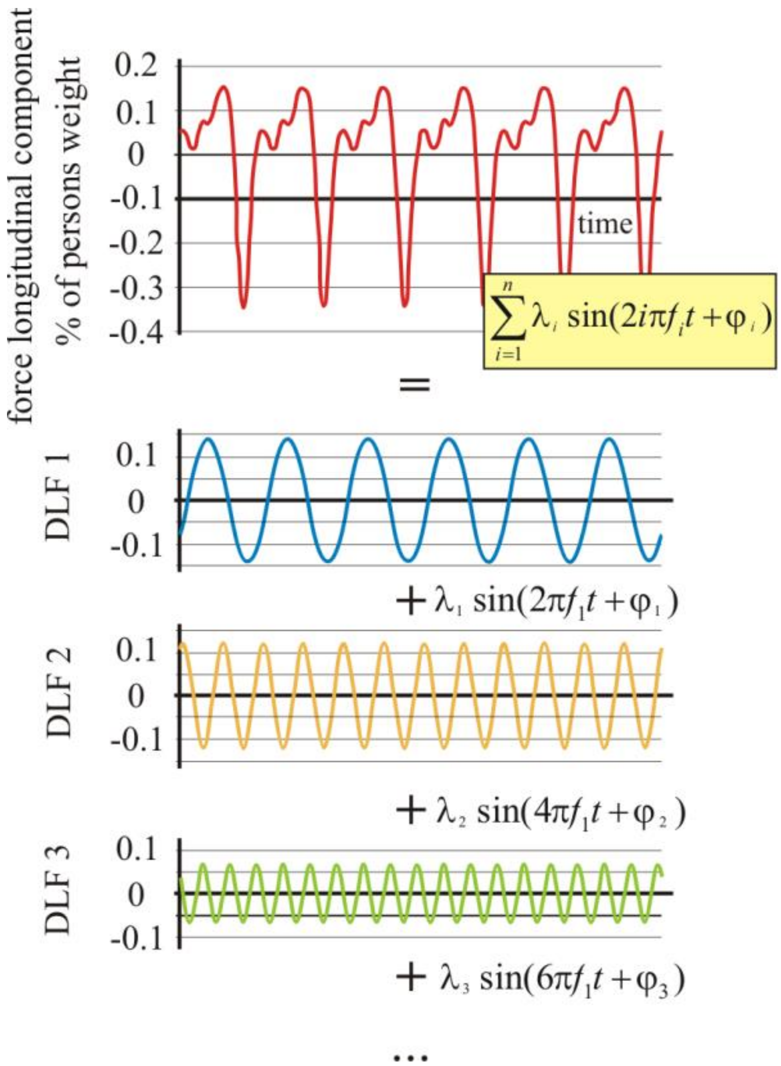


Figure 2.10.Fourier decomposition of walking force longitudinal component

(where i - order number of the harmonic; n - the total number of contributing harmonics; λi– the Fourier coefficient of the i[th] harmonic (DLF); f_1– pacing rate, Hz; φ_i– the phase shift of the i[th] harmonics)

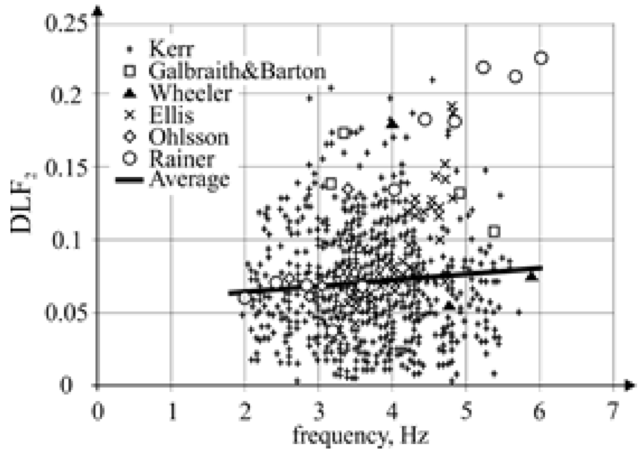


Figure 2.11. DLF values of the second walking harmonic vertical direction for flat surface [131]

The example of obtained longitudinal walking force history that was presented in the previous figure when phase shifts of the ith harmonics have not been taken into account is presented in Figure 2.12

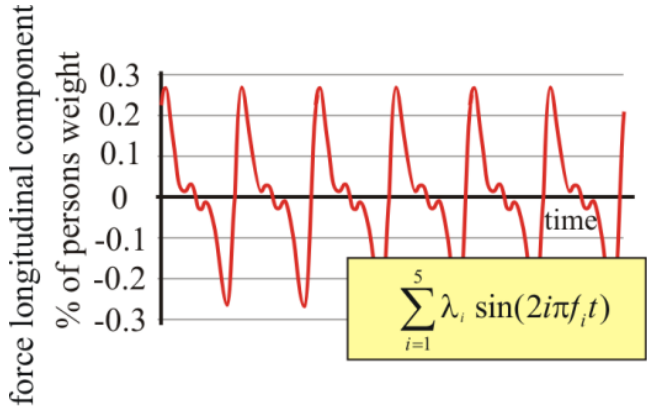


Figure 2.12.Walking force time history when all phase shifts of the ith harmonics are taken as zero

It has been found that phase angles of individuals also vary considerably between measurements [94, 129].

The experimentally obtained measurements of walking force histories of individuals by the newly developed method reveal that character of the walking time history of different people appear to be similar. Therefore it makes sense to perform averaging of continuous walking force histories instead of separate DLFs as it is usually done. This ensures the preservation of information about phase shift values and connection between the harmonics.

In the thesis a new experimental data processing method of obtaining the equivalent continuous walking histories that takes into account the imperfectness of the repeated footfall of the individual as well the differences between the individual walking force histories by averaging them has been developed and presented. Consequently it is possible to obtain the mean values of the DLF_i and the corresponding phase shifts φ_i that are the necessary parameters to obtain analytical walking force function based on the Fourier series.

Step by step description and schematic representation (Figures 2.13 to 2.17) of the new experimental data processing method for obtaining the equivalent (mean) continuous walking histories and their analytical expressions are the following:

1. Recorded desired number of individual walking acceleration histories at given pacing frequency (raw measurements) by the method described in section 2.1. and transformed from local axis of sensors due to their misalignment into global directions (vertical, lateral and longitudinal). This takes into account upper body inclination - angle α shown in from Figure 2.4. When individuals are walking with a constant speed and path in a straight line, positive and negative accelerations should be equal therefore it could be done easily by applying basic trigonometry and known constant acceleration of gravity;
2. Each obtained acceleration history of the test subject is then divided into periods $p_n(t)$ then averaging between the required number of subsequent steps n is performed (Figure 2.13):

$$p_{eq}(t) = \sum_{i=1}^{n} p_n(t)/n \tag{18}$$

The averaged steps are copied several times to obtain the equivalent continuous acceleration history of an individual which is a purely periodical signal and sampled data begins and ends at the same phase of the signal.

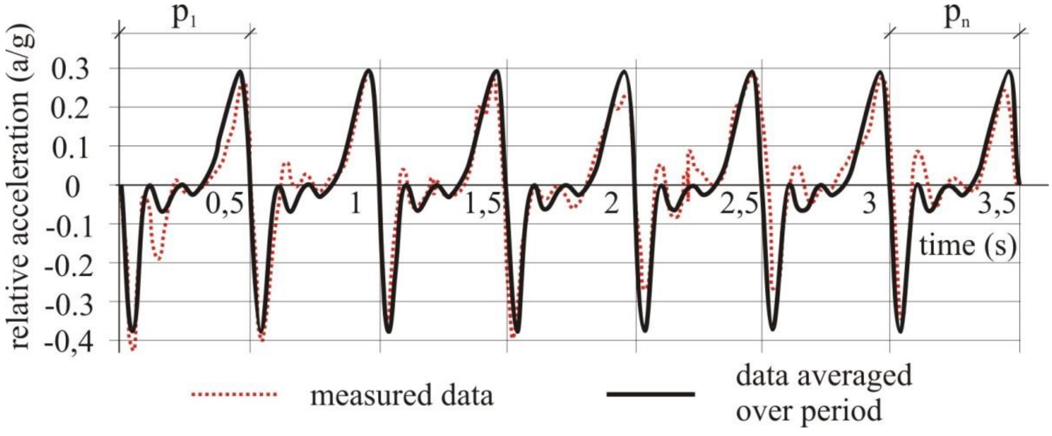


Figure 2.13. Averaging between walking history steps

3. Averaged acceleration history transformation from the time domain to the frequency domain via FFT (Fast Fourier Transform) allow to find the DLF values and relevant

phase shifts of harmonics for continuous walking force history of each test subject (Figure 2.14):

$$p_{eq}(\Delta t) \Rightarrow p_{eq}(\Delta f) \tag{19}$$

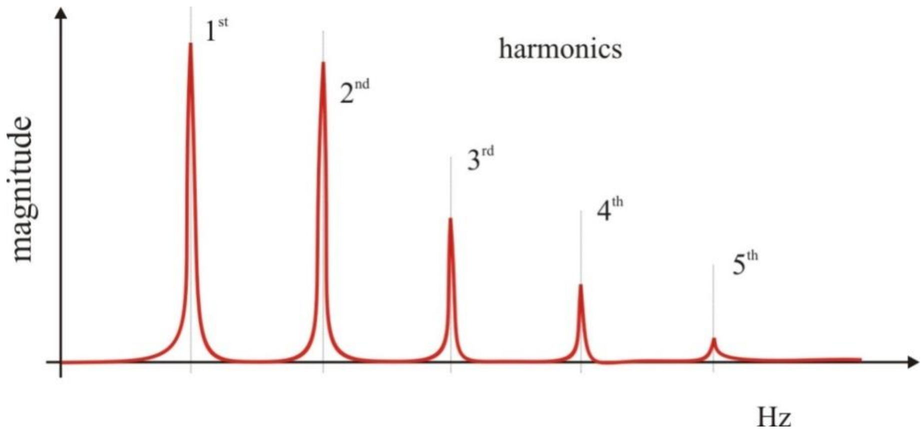


Figure 2.14.Frequency spectrum of averaged continuous walking force history for the one test subject

Fast Fourier transform is a Discrete Fourier transform algorithm for computer - digital tool that is used for analyzing the frequency content of discrete signal and defined as following [60]:

$$F(k\Delta f) \Rightarrow \sum_{n=0}^{N-1} f(n\Delta t) e^{-i(2\pi k\Delta f)(n\Delta t)} \text{ for k=0,1,2,...,N-1} \tag{20}$$

where N – total number of discrete data points taken;

Δf=1/T – frequency increment;

T - total sampling time;

Δt - time between data points;

$F(k\Delta f)$ - discrete Fourier transform output at the harmonic frequency *k*.

Here Δf is analogous to the fundamental frequency of a Fourier series because it provides information about the relative contribution of the harmonics as the Fourier series coefficients (in our case when analyzing walking force - DLF) provide information about relative contribution of the harmonics of the fundamental frequency. Therefore a frequency spectrum plot formed from an FFT is analogous to the harmonic amplitude plot formed from a Fourier series [60].

4. Use in the previous step found DLF value λ_i and relevant phase shift φ_i of harmonic i^{th} in order for each individual to obtain the analytical expression of individuals' walking force history $c_n(t)$ as follows:

$$c_n(t) = \sum \lambda^i \sin(2\pi f_i t + \varphi^i). \tag{21}$$

The representation of separate harmonics in the time domain presented in Figure 2.15.

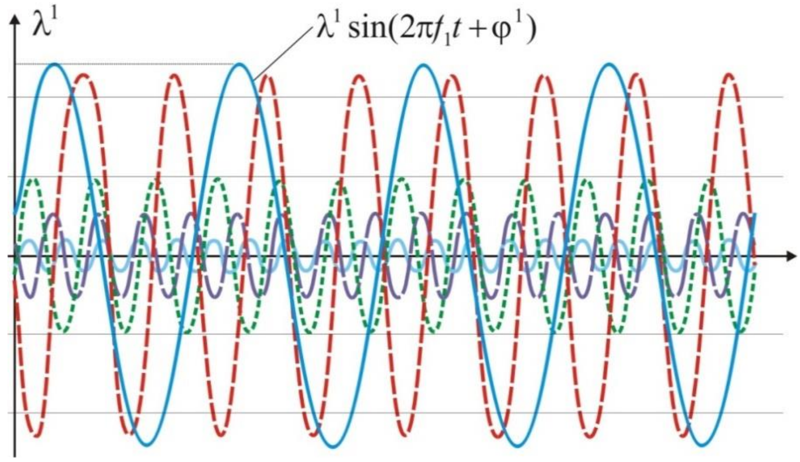


Figure 2.15. Walking harmonics of averaged continuous walking force history for the one test subject in the time domain

5. The obtained analytical expression of the walking force history of an individual is checked against the experimental data and, if necessary, the correction coefficient χ is used for force magnitude to maintain the same area under the function as the experimental data have:

$$\chi = \frac{\sum_{i=1}^{n} A_{\exp}}{\sum_{i=1}^{n} A_{ave}}. \tag{22}$$

where n – number of periods considered in the walking force history;

A_{ave} – area of the positive acceleration in period i of the averaged walking force history;

$A_{\exp}$ – area of the positive acceleration in period i of the experimental walking force history.

Then the analytical expression of individuals' walking force history $c_n(t)$ is:

$$c_n(t) = \chi \sum \lambda^i \sin(2\pi f_i t + \varphi^i). \tag{23}$$

6. Finally, to obtain the analytical expression of the equivalent (mean) walking force history, perform the averaging between functions $c_n(t)$ (between the averaged walking force history of individuals) priory adjusted to the exact same pacing frequency:

$$c_{eq}(t) = \sum_{i=1}^{n} c_n(t) / n \tag{24}$$

Averaged walking force histories $c_n(t)$ of individuals and obtained equivalent walking force history $c_{eq}(t)$ presented in Figure 2.16.

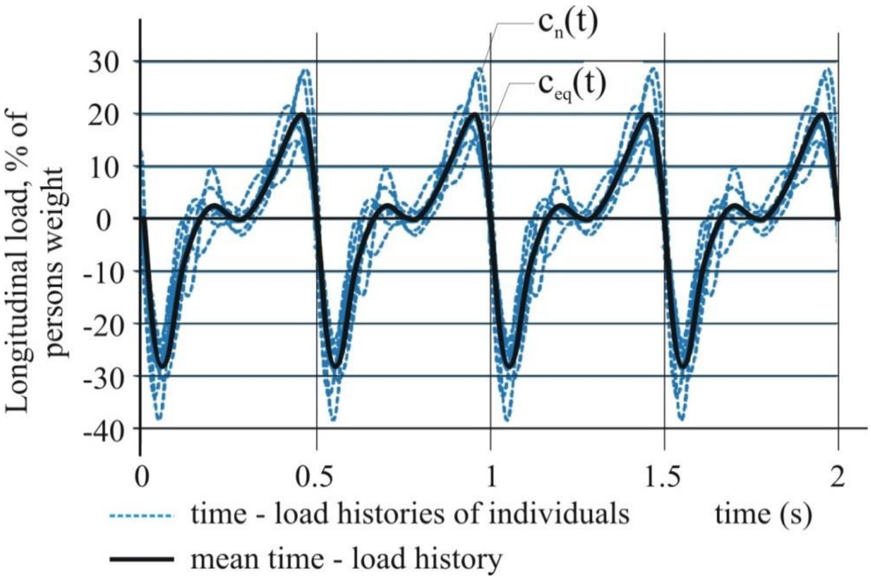


Figure 2.16. Equivalent (mean) walking force history and averaged walking force histories of individuals

7. Next step again is the transformation of the equivalent (mean) walking force history from the time domain to the frequency domain via FFT to be able to obtain the relevant DLF and phase shift values of this history:

$$c_{eq}(\Delta t) \Rightarrow c_{eq}(\Delta f) \tag{25}$$

8. Now it is possible to find the DLF value λ_{eq}^{i} (Figure 2.17) and relevant phase shift φ_{eq}^{i} of harmonic ith for the equivalent (mean) walking force history in order to obtain the analytical expression of the equivalent (mean) walking force history in vertical $F_{vert}(t)$ (26) and lateral or longitudinal directions $F_{long,lat}(t)$ (27):

$$F_{vert}(t) = G + \sum_{i=1}^{n} G\lambda_{eq,y}^{i} \sin(2i\pi f_1 t + \varphi_{vert}^{i}), \tag{26}$$

$$F_{long,lat}(t) = \sum_{i=1}^{n} G\lambda_{eq}^{i} \sin(2i\pi f_1 t + \varphi_{lat,long}^{i}). \tag{27}$$

where G – is a static weight of the subject's body (N).

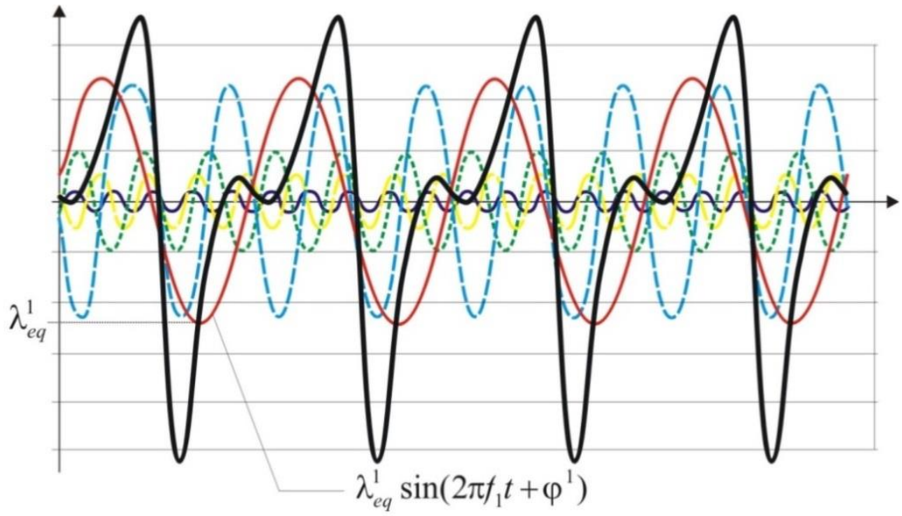


Figure 2.17. Equivalent (mean) walking force history and its walking harmonics

During the double averaging process the experimentally measured peaks smoothen and widen accordingly due to the lack of:

- perfect periodicity between each footstep of individual;
- differences in walking force histories of individuals.

Therefore this effect on the response of the structure was investigated by numerical calculations using the finite element software that is widely spread and accepted as a precise numerical model. To evaluate magnitude of vibration amplitude a model of a cantilever beam by structural analysis software STRAP 12.5 was created and applied at its tip in two cases of walking force history (Figure 2.18):

1) experimentally obtained period of walking force history;
2) averaged walking force history.

It is found that the error in vibration amplitude due to the double averaging process is less than 1% and can be regarded as negligible.

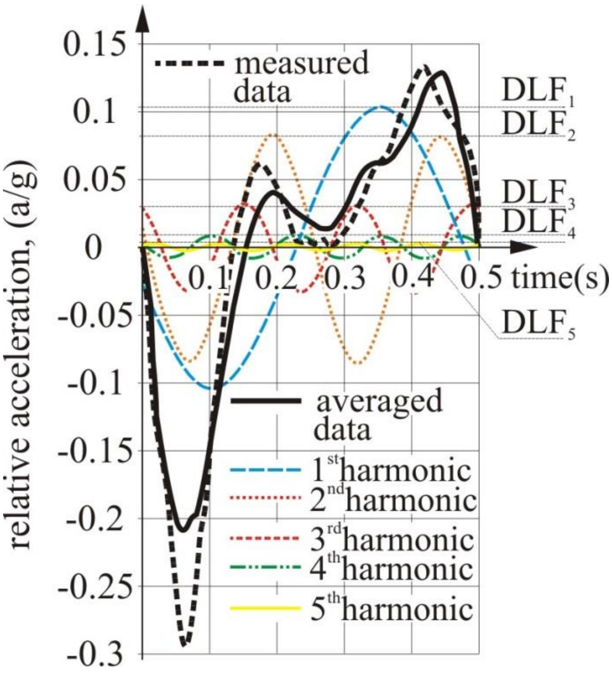


Figure 2.18.Effect of walking force history averaging.

The main advantage of the new experimental data processing method based on the averaging of the continuous walking histories and not the separate DLF values obtained from one or few steps is to be able to get a mean analytical function of human induced forces in three directions that can be easily used in response calculations of structures to human induced loads. It is not necessary for the structure to have a linear dynamic behavior because the analytical function of continuous walking force history can be applied to the nonlinear structures where modal superposition technique is not valid anymore. In comparison traditional methods based on the averaging of a particular Fourier coefficient between the different Fourier series will not necessarily give the average continuous forcing function due to neglected connection between the coefficients in each of the function.

Another advantage is that the obtained analytical function of equivalent walking force history contains information about imperfection of individual's footfalls and differences between the continuous walking histories of individuals but still it is a deterministic force model. Unlike the probabilistic force models it is more convenient to handle when performing analytical or numerical calculations of the structure under consideration.

The verification of the developed experimental data processing method is presented in the thesis in the section 2.3 where obtained mean DLF values for stair ascending and descending are compared with DLF values obtained by other authors with traditional methods.

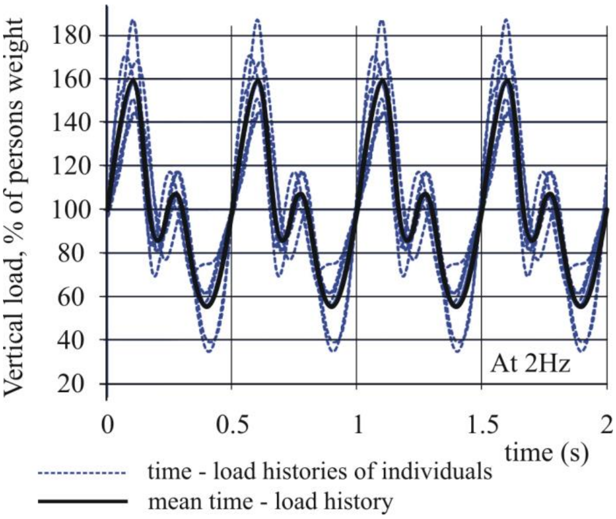
2.3.The results of footfall induced forces on stairs

The developed new methods that are described in previous sections 2.1 and 2.2 for experimental identification of footfall forces and data processing were used to obtain the equivalent continuous walking force histories (step 1 to 6 from section 2.2) for the stair ascending and descending cases (Figures 2.19 to 2.24). Summary of input parameters are presented in the Table 2.2.

Table 2.2.

Summary of input parameters of stair walking force experiment

Description	Parameter	Notes
Place of the experiment	RTU staircases	Figures 2.5 to 2.8
Stair inclination	25^o	
Number of stair flights for tests	2	When recorded measurements
Test subjects	18	11 men and 7 women
Average weight of test subjects	71	kN
Number of recorded continuous walking force histories totally	216	For data processing taken only second stair flight each time
Pacing frequency	Freely chosen by test subject	
Number of walking force histories taken for data processing to obtain mean value of DLF	16	At pacing frequency near 2,0 Hz (ascending) and near 2.15 Hz (descending)
Number of walking force histories taken for data processing to obtain force amplitude dependence from pacing frequency	108	Pacing frequencies from 0,9 Hz to 2,33 Hz (normal climbing)



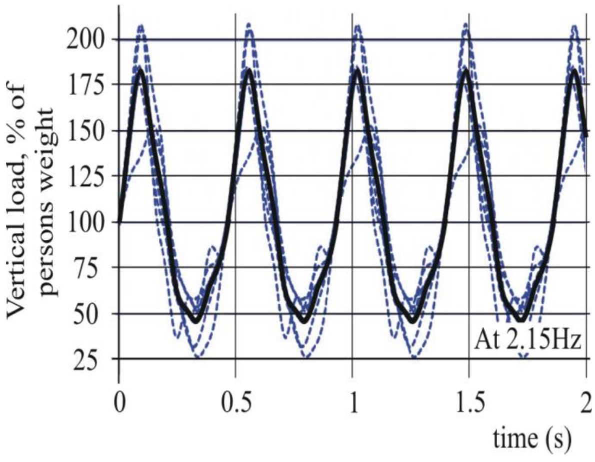
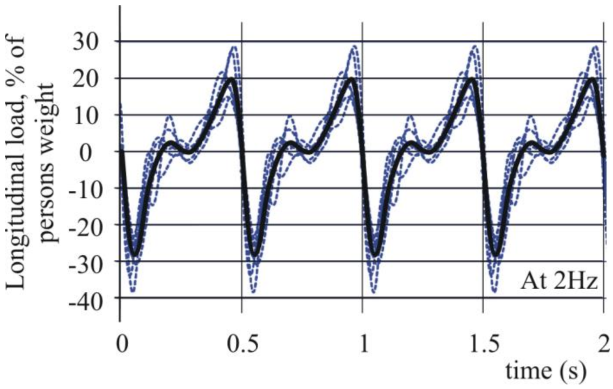


Figure 2.19. Analytical function of continues human walking force time history in a vertical direction during the stair ascent

Figure 2.20. Analytical function of continues human walking force time history in a vertical direction during the stair descent



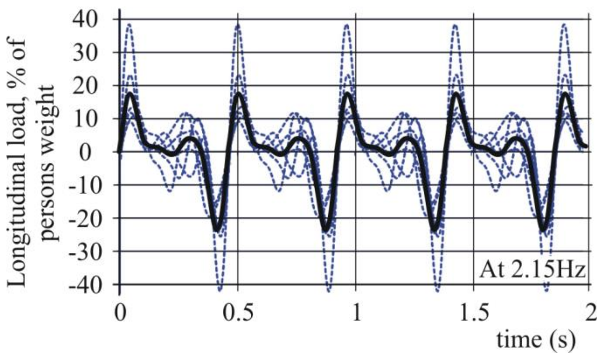
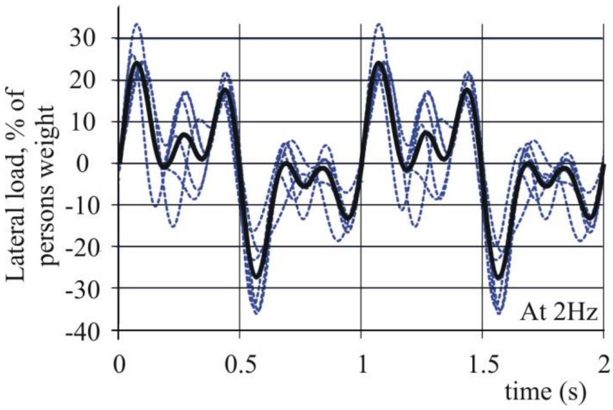


Figure 2.21. Analytical function of continues human walking force time history in a longitudinal direction during the stair ascent

Figure 2.22. Analytical function of continues human walking force time history in a longitudinal direction during the stair descent



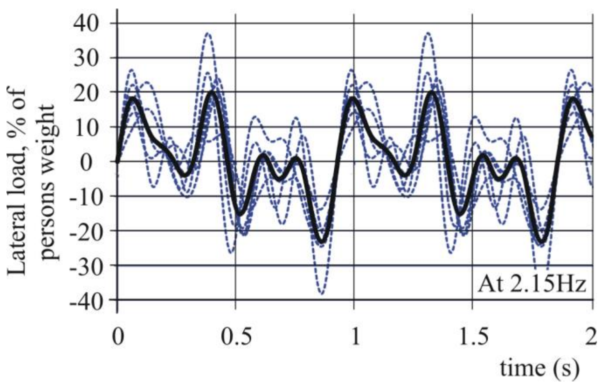


Figure 2.23. Analytical function of continues human walking force time history in a lateral direction during the stair ascent

Figure 2.24. Analytical function of continues human walking force time history in a lateral direction during the stair descent

Table 2.3 presents maximum peak amplitudes $F_{peak}^{\%}$ and root-mean-square amplitudes $F_{rms}^{\%}$ of equivalent (mean) walking force histories to highlight the differences in generated loading form stair ascending and descending. Root-mean-square amplitude of force gives indication to the amount of time the system is subjected to this level of force and defined as follows:

$$F_{rms}^{\%} = \sqrt{\frac{1}{T}\int_0^T F^{\%}(t)^2\,dt} \tag{28}$$

Forces $F_{peak}^{\%}$ and $F_{rms}^{\%}$ are presented as percentage of person weight.

Table 2.3.

Maximum force amplitudes of loading due to stair ascending and descending

Description	$F_{peak}^{\%}$	Δ, %	$F_{rms}^{\%}$	Δ, %
Generated vertical force during the stair ascent	158.8	-14.2	104.5	-4.8
Generated vertical force during the stair descent	181.3		109.7	
Generated longitudinal force during the stair ascent	29.1	19.9	12.5	20
Generated longitudinal force during the stair descent	23.3		10.0	
Generated lateral force during the stair ascent	29.0	19.7	8.4	9.5
Generated lateral force during the stair descent	23.3		7.6	

Where $\Delta = \dfrac{F_{peak\_ascend}^{\%} - F_{peak\_descend}^{\%}}{F_{peak\_ascend}^{\%}} \cdot 100\%$

Table 2.3 reveals that during stair descending higher peak amplitude of vertical force is generated but lower peak amplitude of longitudinal and lateral force component is generated if compared to stair ascending. If analyzed $F_{rms}^{\%}$ values, the difference between vertical force in ascending and descending process is only about 5%. That means that the amount of time system is subjected to higher level of force amplitude is shorter.

In the case of lattice observation tower there is particular interest in the footfall lateral and longitudinal component as they are a cause of structure vibration. Therefore figures 2.25 and 2.26 contain valuable information where a path of the mean walking force vector end point in horizontal plane also obtained from equivalent walking force history is plotted. The weight of the person was taken 740N. A closer look at the middle part (small loops) of the figures reveals that during the ascending process a person tries to balance oneself in the lateral direction but during descending more in the longitudinal direction due to the inertia. The lack of the symmetry confirms that the presented method has taken into account the "leading leg" effect that especially becomes apparent for the stair ascending process.

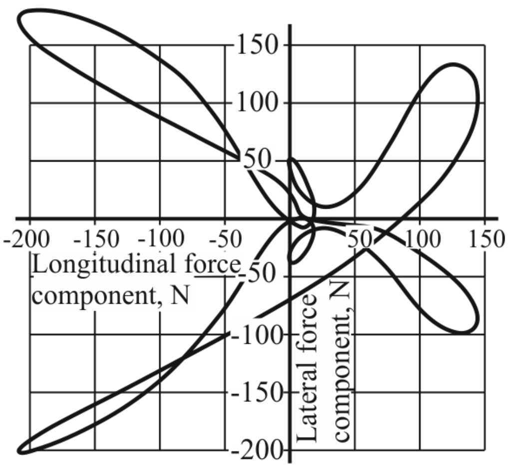


Figure 2.25. Path of the mean walking force vector end point (ascending case at rate 2Hz)

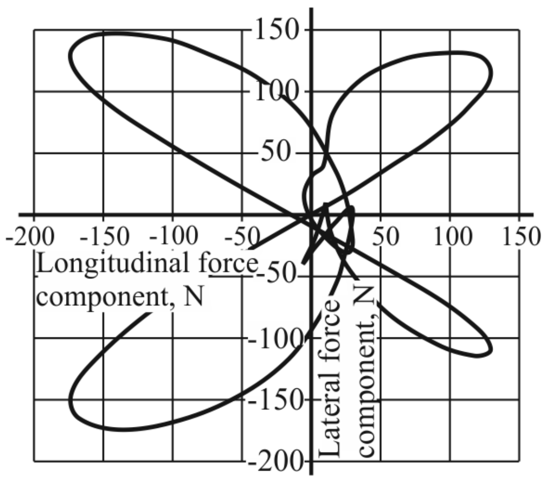


Figure 2.26. Path of the mean walking force vector end point (descending case at rate 2,15Hz).

According to the step 7 (section 2.3) from the time domain to the frequency domain via FFT relevant DLF and phase shifts of first five harmonics for vertical and lateral force component and six for lateral force component of the equivalent (mean) walking force history were obtained and presented in Table 2.4. Graphical illustrations of force frequency spectrum are presented in Figures 2.27 and 2.28 if the weight of a person is 740N.

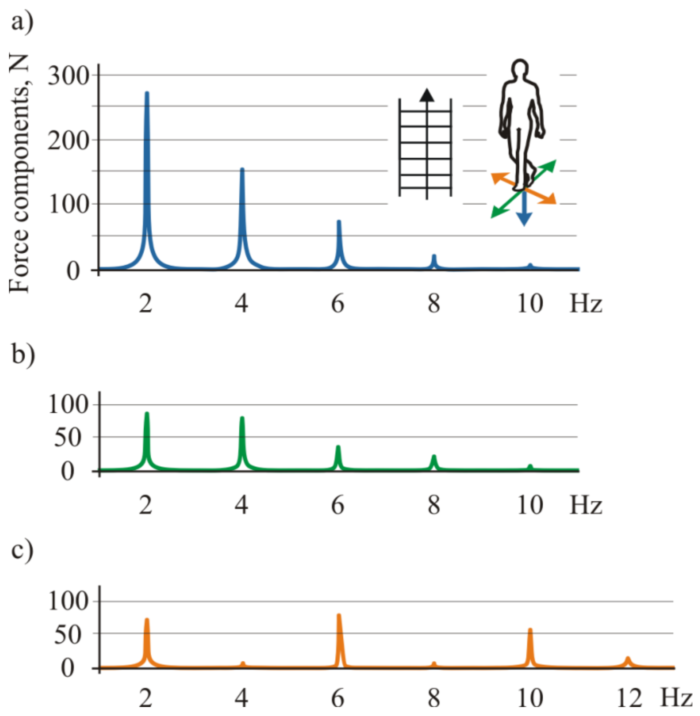


Figure 2.27. Frequency spectrum of force component ascending stair at 2 Hz
a) vertical component; b) longitudinal component; c) lateral component

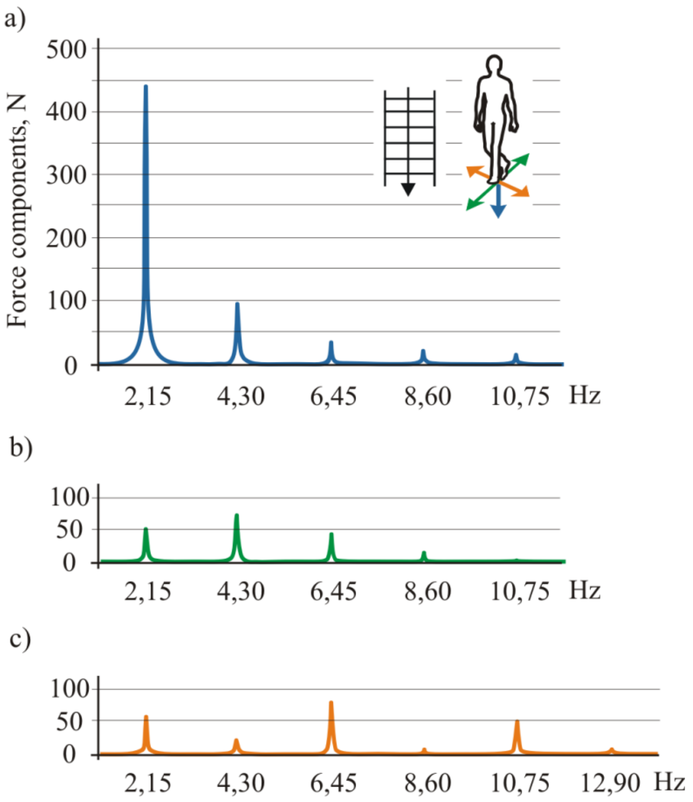


Figure 2.28. Frequency spectrum of force component descending stair at 2.15 Hz
a) vertical component; b) longitudinal component; c) lateral component

Table 2.4.

Parameters for analytical function of equivalent (mean) walking force history

Description	Harmonic №	Ascending (2Hz)		Descending (2,15Hz)	
		DLF, λ_{eq}^{i}	Phase, φ_{eq}^{i}	DLF, λ_{eq}^{i}	Phase, φ_{eq}^{i}
Vertical component	1	0.37	9.66	0.6	20
	2	0.21	2.15	0.13	-60.3
	3	0.1	-142	0.05	-84.5
	4	0.03	84.5	0.03	-125
	5	0.01	18.5	0.02	93.4
Longitudinal component	1	0.12	-166	0.07	-11.7
	2	0.11	-169	0.1	5.5
	3	0.05	173	0.06	31.4
	4	0.03	169	0.02	31
	5	0.01	146	0.002	76
Lateral component	1	0.1	28	0.08	-6.02
	2	0.01	-133	0.03	10
	3	0.11	18.9	0.11	3.02
	4	0.01	-123	0.01	-90.3
	5	0.08	4.95	0.07	34.7
	6	0.02	-129	0.01	149

By analyzing the parameters given in Table 2.4 for obtaining the walking force histories, the harmonics with the highest DLF values and approximately the same phase shifts are the 1st and 2nd harmonic for the longitudinal force component with DLF values of 0.12 and 0.11 accordingly (ascending) and the 1st, 3rd and 5th harmonic, with DLF values varying from 0.08 to 0.11 for the lateral force component. The same critical harmonics were pointed out by Bachmann and Ammann [8].

To verify the new methods of obtained DLF_i values of mean walking force history results for ascending (2Hz) and descending (2.15Hz) cases were compared with the S.C. Kerr's obtained results [72] of the vertical force component DLF_i values. S.C. Kerr experimental data is based on individual footstep force plate measurements opposite to the new method that is based on the indirect measurement technique – inverse dynamic. The obtained results found to be in a very good agreement for the first harmonics. The results of the second harmonics slightly differ that correlate with the proposition of Davis [29] to take a higher value for the second harmonic. The mean value obtained by the presented methodology is plotted on the S.C. Kerr's results in Figures 2.29 to 2.32. The shaded area in the figures represents the typical range of frequencies of relevant harmonics for stair ascending or descending. This is also the range investigated in this thesis.

There are still no results of DLFs to compare the longitudinal and lateral force component directions in the case of stair ascent/descent.

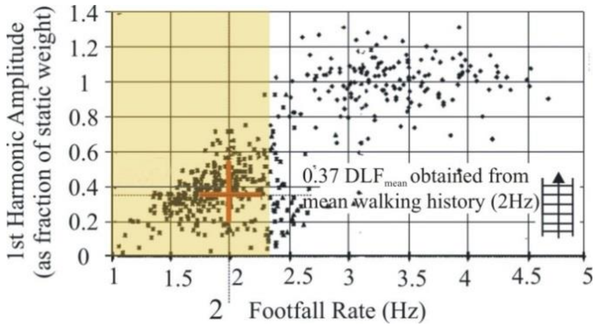


Figure 2.29. First harmonic value comparison for ascending stairs

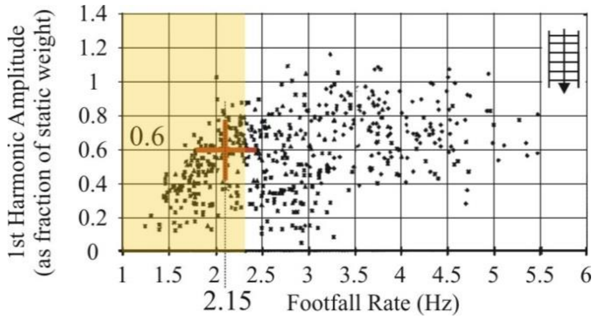


Figure 2.30. First harmonic value comparison for descending stairs

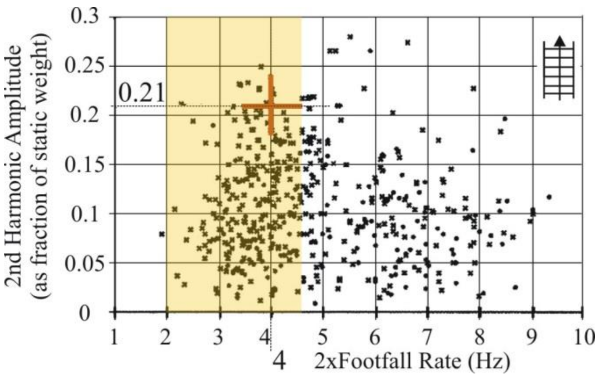


Figure 2.31. Second harmonic value comparison for ascending stairs

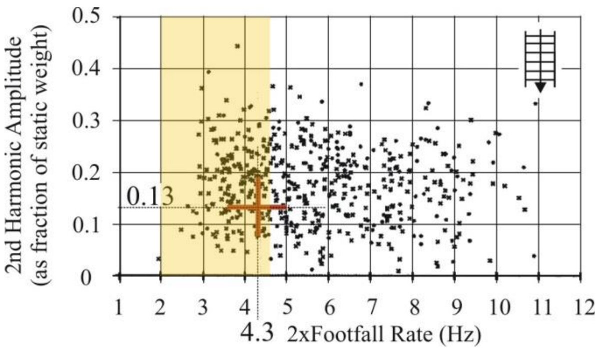


Figure 2.32. Second harmonic value comparison for descending stairs

The Figures 2.29 to 2.32 revile that the amplitudes of DLFs are dependent from the pacing frequency, especially for the first harmonic.

To obtain the relationship between the walking force amplitudes A and pacing frequencies it is again suggested to not look at the separate DLFi values corresponding to the relevant frequency but to take the mean value of the individual's experimental walking force history of n periods expressed as a range from maximum to minimum amplitude (29):

$$A = \frac{1}{n}\sum_{i=1}^{n}(A_i^{poz} - A_i^{neg}), \tag{29}$$

where amplitudes A_i^{poz} and A_i^{neg} are presented in Figure 2.33.

This is a true reflection of force peak amplitudes opposite to the individual DLF values where connections between the harmonics are not taken into account.

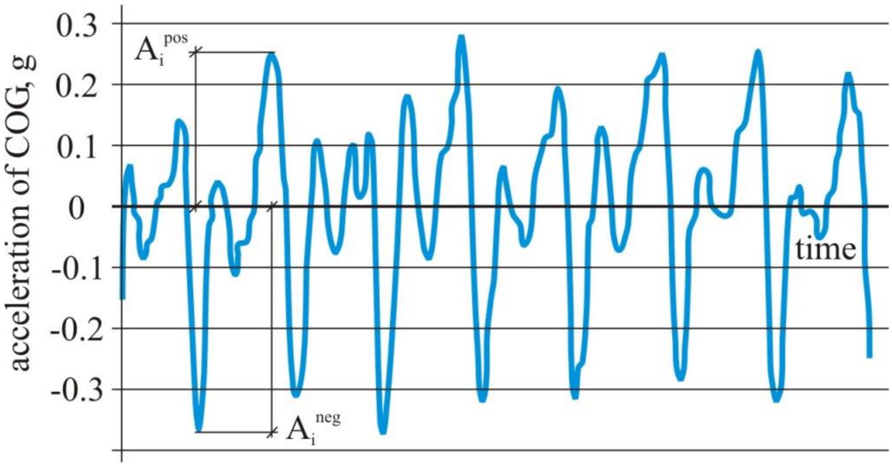


Figure 2.33. Experimental walking force history of individual (longitudinal component)

The relationship between the walking force amplitudes *A* and pacing frequencies is obtained from the experimental data of the recorded walking histories of individuals and presented in Figures 2.34 to 2.39. The experimental input parameters were presented in the Table 2.2. Each of the experimental points on the chart is the mean value of 12 periods (number of steps in the stair flight).

The vertical dynamic walking forces are sensitive to the pacing frequency until the point of noticeable scatter in the values of each test subject and therefore the mean value might be taken as constant. The shape of the mean value function is similar to other researchers' works. The longitudinal and lateral force component is sensitive to changes of the pacing frequency with a tendency to increase by adding the walking speed in the case of the stair ascending. For the descending case these components seem to be very slightly dependent on the persons' pacing frequency and therefore could be regarded as a constant.

Most of the data correlate very well except the second harmonic for the descending case. By analyzing the mean values obtained by Kerr for a vertical direction (in Figure 18 [72], it follows that the second harmonic does not depend on the pacing frequency for the stair descending case and therefore the shape of the walking force history should differ dramatically even if there is a slight change in the walking speed. Apparently, it is due to the rapid change of the first harmonic values. However, this does not appear in the experimental data for fundamental pacing frequency range of $1Hz \leq f \leq 2.3Hz$. On the other hand, Kerr states in his paper that “…the weight is transferred quickly from one leg to the other, which creates a deeper hollow between the humps. The greater the distance between the hollow and the humps, the greater the second harmonic”. Therefore, the second harmonic amplitude should change if the pacing frequency changes for the stair descending or ascending case.

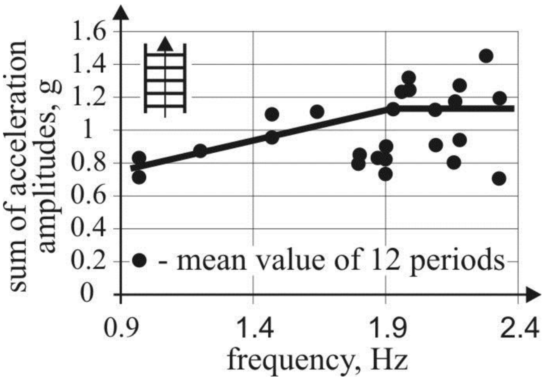


Figure 2.34. Relationship of amplitude and pacing frequency (vertical direction, ascent)

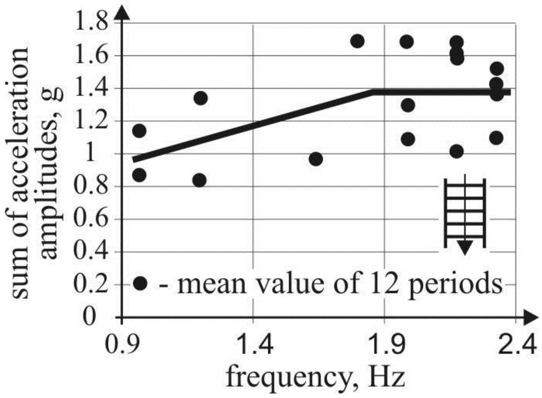


Figure 2.35. Relationship of amplitude and pacing frequency (vertical direction, descent)

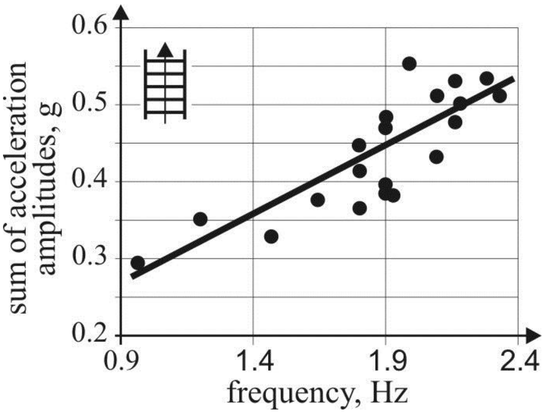


Figure 2.36. Relationship of amplitude and pacing frequency (longitudinal, ascent)

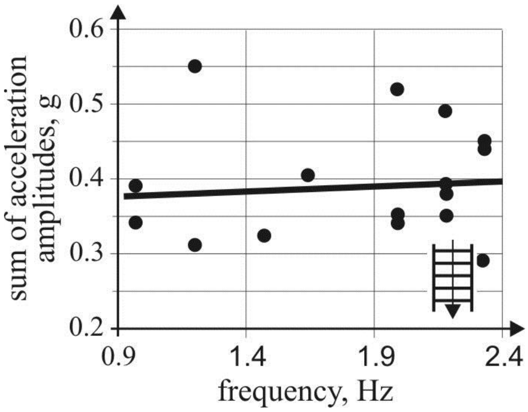


Figure 2.37. Relationship of amplitude and pacing frequency (longitudinal, descent)

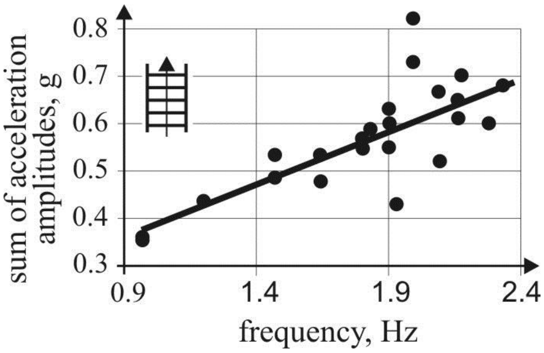


Figure 2.38. Relationship of amplitude and pacing frequency (lateral, ascent)

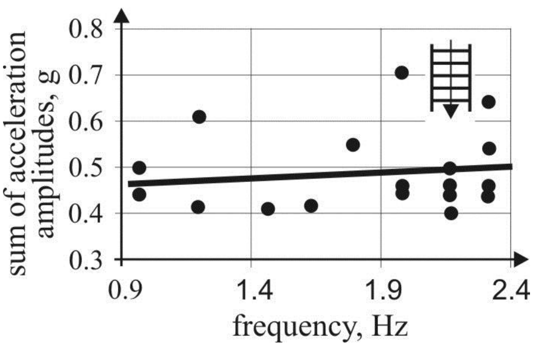


Figure 2.39. Relationship of amplitude and pacing frequency (lateral, descent)

Experimental walking histories at a pacing frequency range of $1Hz \le f \le 2.3Hz$ for few test persons were decomposed into Fourier series and it was found that the following harmonics are sensitive to the pacing frequency changes:

- 1st, 2nd and 3rd harmonic of walking force history vertical component (ascending and descending);
- 1st, 2nd harmonic of walking force history longitudinal component (ascending);
- 1st, 3rd and 5th harmonic of walking force history lateral component (ascending);

Amplitudes of the rest of the harmonics can be treated as constant and taken from Table 2.4. If assumed that changes in the force amplitude due to different pacing frequencies divide proportionally between the harmonics and therefore DLF values that are sensitive to the pacing frequency change, it is possible to find relationship that describes each of the relevant DLF value dependence from the person's pacing frequency. These relationships are presented in the Table 2.5. Additionally, as an example, DLF values for the pacing frequency 1.6Hz were calculated and compared to the Kerr data.

Table 2.5.

DLF values of stair ascending and descending dominant harmonics

Action	Proposed, $DLF_n(f)$	Calculated DLF_i at 1.6Hz	Average DLF by Kerr at 1.6Hz [72]
Ascending, vertical	$DLF_n(2Hz)\cdot(0.94f-0.88)$; $1 \le f < 1.95$ for $n=1...3$	$0.23(DLF_1)$	$0.27(DLF_1)$
	$DLF_n(2Hz)$; $\begin{cases}1 \le f \le 1.95 & for\ n=4,5\\ 1.95 \le f \le 2.3 & for\ n=1...5\end{cases}$	$0.13(DLF_2)$	$0.12(DLF_2)$
Descending, vertical	$DLF_n(2.15Hz)\cdot(0.99f-1.13)$; $1 \le f < 1.85$ for $n=1...3$	$0.27(DLF_1)$	$0.24(DLF_1)$
	$DLF_n(2.15Hz)$; $\begin{cases}1 \le f \le 1.85 & for\ n=4,5\\ 1.85 \le f \le 2.3 & for\ n=1...5\end{cases}$	$0.06(DLF_2)$	$0.22(DLF_2)$

Ascending, longitudinal	$DLF_n(2Hz)\cdot(1.49f-1.98)$; $1\le f<2.3$ for $n=1...2$	$0.049(DLF_1)$	-
	$DLF_n(2Hz)$ for $n=3...5$; $1\le f\le 2.3$	$0.044(DLF_2)$	-
Descending, longitudinal	$DLF_n(2.15Hz)$ for $n=1...5$; $1\le f\le 2.3$	$0.07(DLF_1)$	-
		$0.1(DLF_2)$	-
Ascending, lateral	$DLF_n(2Hz)\cdot(2.2f-3.4)$; $1\le f<2.3$ for $n=1, 3, 5$	$0.012(DLF_1)$	-
	$DLF_n(2Hz)$ for $n=2, 4, 6$; $1\le f\le 2.3$	$0.013(DLF_3)$	-
Descending, lateral	$DLF_n(2.15Hz)$ for $n=1...5$; $1\le f\le 2.3$	$0.08(DLF_1)$	-
		$0.11(DLF_3)$	-
		$0.07(DLF_5)$	-

where f – pacing frequency, Hz;

n – number of the harmonic;

$DLF_n(f)$ - dynamic load factor at pacing frequency f for the harmonic n;

$DLF_n(2Hz)$ - dynamic load factor at pacing frequency 2 Hz (ascending) and corresponding phase shifts for the harmonic n found from Table 2.4.

$DLF_n(2.15Hz)$ - dynamic load factor at pacing frequency 2.15 Hz (descending) and corresponding phase shifts for the harmonic n found from Table 2.4.

2.4.Summary of the chapter

A new method of obtaining analytical functions of continuous walking histories that is based on experimentally obtained continuous walking histories of individuals is presented. The experimentally obtained continuous walking histories are found by the developed method in the thesis that is from the branch of inverse dynamics and uses kinematics of the human center of gravity. Instead of the traditional approach when the relationship between the walking pace and force amplitudes is based on the average harmonic (DLF) amplitude, this method proposes averaging between continuous walking histories priory proceeded in order to take into account the imperfections of repeated footfall. This way information about the phase shifts – necessary parameter to obtain analytical function based on the Fourier series is preserved. The main advantages of using the presented method are as follow:

- Possibility to estimate continuous human–induced forces of different actions applied to the structure under a wide range of conditions due to the non-laboratory restrictions;
- The measurement devices do not have a strong influence on human ability to move naturally;
- Requirement of a low cost instruments: few accelerometers capable of storing and downloading data with relatively small time intervals;
- Allows to obtain not only dynamic load factors but also the phase shift values associated with the mean walking history;
- The obtained analytical mean function contains information about imperfections of the person's footfalls and differences between the continuous walking histories but still it is a deterministic force model. Unlike the probabilistic force models it is more convenient to handle when performing analytical or numerical calculations of the structure under consideration.

To test the method equivalent DLFs and their dependence on the walking pace for all three force directions were obtained (the stair ascending and descending case): vertical, longitudinal and lateral. Vertical results are compared with the measurements of the vertical component done by Kerr using the force plate technology. The overall results correlate very well except the second harmonic were Kerr's data has a very significant scatter and the mean value does not depend on the walking pace (stair descending case) that seems to be quite unrealistic.

Descending the stair produces higher vertical force amplitudes than ascending that is logical and in agreement with other researchers' works. The lateral and longitudinal direction force amplitudes strongly depend on the walking pace only in the case of the stair ascent. In the case of the stair descent these might be considered as constant but with smaller amplitudes. The authors are not aware of any information that could be compared with the obtained results for these two directions. Recent concerns about some of the light-weight public observation towers excessive vibrations and dissatisfaction of the visitors' comfort criteria call for greater attention to longitudinal and lateral force components during a long stair ascending or descending process.

3. HUMAN-INDUCED LATTICE LIGHT–WEIGHT TOWER VIBRATIONS

It is important to understand the typical dynamic behavior of the existing observation towers to be able to develop the methodology of tower response calculations to human induced loads. Therefore experimental investigations to find the typical dynamic parameters of existing lattice observation towers have been carried out, as well as theoretical and experimental investigations of application and structure response to the human induced loads obtained in previous section of thesis. Based on those investigations the algorithm of the methodology for calculation of maximum response of structure to typical human induced loads has been developed. Verification of the developed methodology is performed by comparing the theoretically and experimentally obtained results of the structures maximum response.

3.1. Experimental investigation of lattice observation tower parameters and responses to human induced loading

During the experiments the vibration accelerations of 19 observation towers were measured to assess the existing observation towers dynamic performance (Figure 1). Vibrations of slender lattice towers are low frequency vibrations therefore it was convenient to use the same accelerometers described in the section 2.1. In total five 3-axis light-weight (55g) USB accelerometers were used (Model X6-1A) to record the accelerations for each experiment. Devices were fully glued by means of adhesive tape on the load bearing structure on upper platform of towers (Figure 3.1).

Figure 3.1. Arrangement of accelerometers on the tower in Dzintari

The static calibration method described in the section 2.1 was used. The measurement sample rate was selected 160 Hz. Each accelerometer simultaneously recorded vibration accelerations in three directions. The typical arrangement of accelerometers is presented in Figure 3.2. The majority of experimental measurements of towers were taken during the summer of 2012.

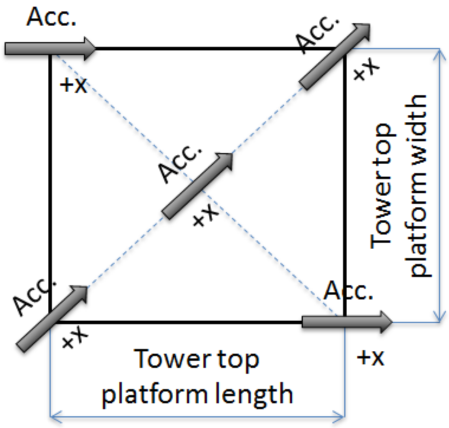


Figure 3.2. Accelerometers arrangement scheme

Experimental program for each tower consisted of the following:

- Visual assessment of tower technical condition;
- Measurement of the tower geometry: height, dimensions in plan, dimensions of main load bearing elements and parameters of the tower stairs;
- Recorded weather conditions during the experiment: wind speed and temperature
- Measured acceleration amplitudes of tower top platform under the following conditions:
    a. mild wind and no visitors on the tower;
    b. two visitors moving downstairs along the full tower height;
    c. two visitors moving upstairs along the full tower height;
    d. natural behavior of visitors on the upper sightseeing platform;
    e. organized movement on the upper sightseeing platform in a circular direction (first clock-wise, then anti clock-wise for 30 seconds);
    f. organized movement on the upper sightseeing platform in two transversal directions for 30 seconds;
    g. free decay after 10 seconds of intentional swaying of the tower.
    h. natural behavior of group of tower visitors (ascending, descending and moving on the upper sightseeing platform) when it was possible.

There were some examples of mixed structures e.g. timber structure (columns, beams, and cladding) with a steel rod lateral resisting system but majority of the observation towers can be divided in timber (70% of the inspected towers) and steel structures. The slope of the observation towers' stairs was in the range of 30^0 to 70^0 but most of the observation towers' slope of the stairs was around 45^0. Although most of the observation towers are less than ten years old, their technical condition widely varies. Only the timber towers that were less than five years old with a deeply treated timber are in good technical condition. Most of the damages are located in the main column areas, whereas the steel towers columns' splice connection seems to be affected by the tower's vibrations (Figure 3.3).It correlates with information given in [135] that without deep treatment of structural elements timber tower service life is around five to ten years.

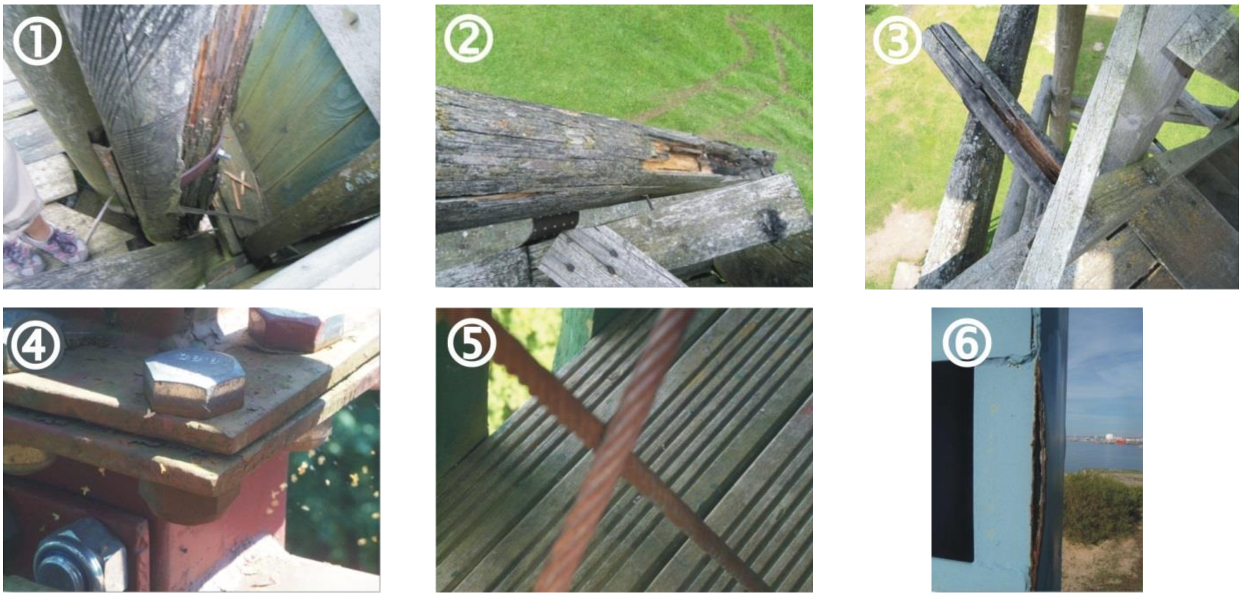


Figure 3.3. Typical damage of timber (1-3) and steel (4-5) observation towers

Further for the dynamic analysis 12 observation towers that are in satisfactory technical condition were taken (no major defects found by visual inspection). A list of the towers is presented in the Table 3.1. The tower height is taken from the upper platform to the ground level.

The example of recorded time histories of the observation tower in Ligatne is presented in Figure 3.4 and Krustkalnu observation tower is presented in Figure 3.5 (raw measurements).

Table 3.1.

List of the observation towers taken for the dynamic analysis

№	Name of the tower	Height, m	Structural material	Plan dimension on the ground	Plan dimension at the upper platform level
1.	Krustkalnu t.	25	Steel	6.0 m x 6.0 m	3.0 m x 3.0 m
2.	Kalsnavas t.	25	Steel	7.6 m x 7.6 m	7.6 m x 7.6 m
3.	Dzintaru t.	34	Steel	4.24 m x 4.24 m	4.24 m x 4.24 m
4.	Egļu kalna t.	26.5	Timber	6.5 m x 6.5 m	2.85 m x 2.85 m
5.	Priedaines t.	32	Timber	6.8 m x 6.8 m	1.82 m x 1.75 m
6.	Kamparkalna t.	26.5	Timber	6.8 m x 6.8 m	2.90 m x 2.80 m
7.	Ūdru kalna t.	26.5	Timber	6.9 m x 6.9 m	2.57 m x 2.50 m
8.	Ventspils t.	12	Steel	9.0 m x 9.0 m	9.0 m x 9.0 m
9.	Kuldiga t.	16.3	Mixed	7.5 m x 7.5 m	3.90 m x 3.90 m
10.	Lielais liepu t.	34	Timber	9.4 m x 9.3 m	2.75 m x 2.65 m
11.	Ligatne t.	22	Timber	5.0 m x 5.0 m	3.80 m x 3.45 m
12.	Lozmeteju t.	28.5	Timber	6.8 m x 6.8 m	2.70 m x 2.70 m

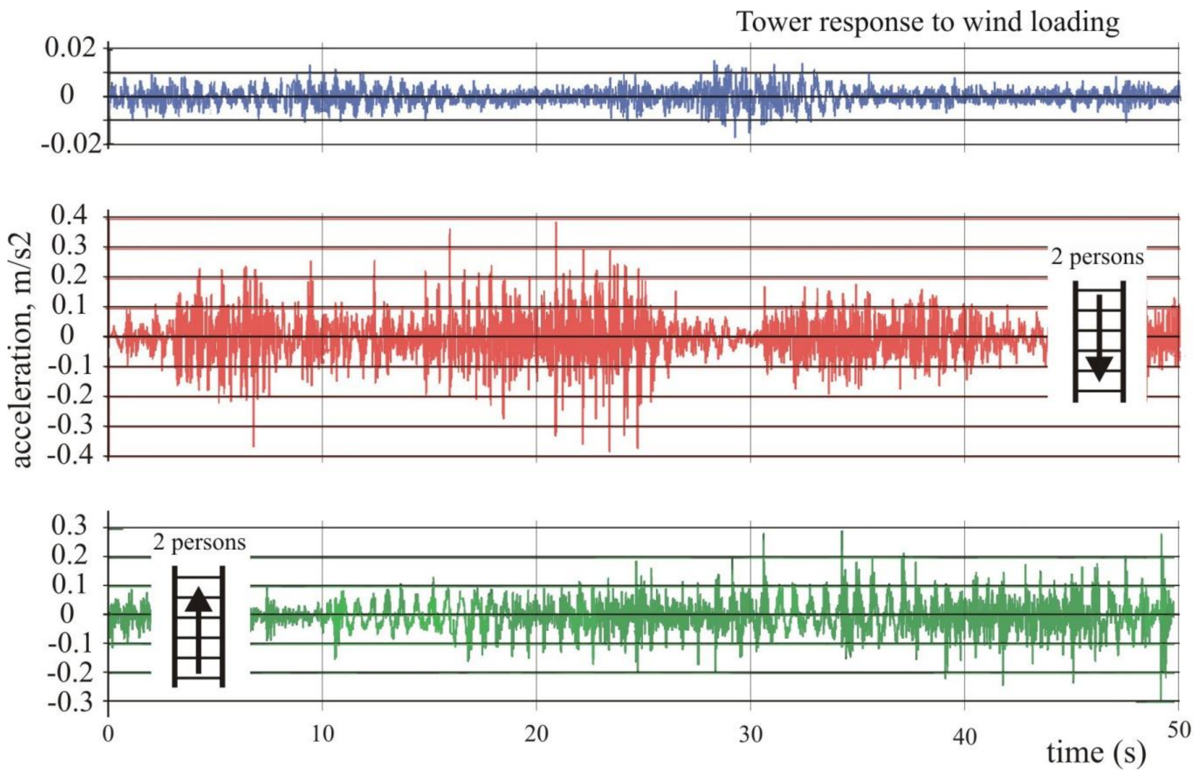


Figure 3.4. Acceleration time history of observation tower in Ligatne

Almost all recorded time histories of the structure vibration revealed significant response to human movement along the towers height. For example, the recorded peak accelerations (one direction) under mild wind (4 m/s) conditions on Ligatne tower was about 20 times less than only from two person movement upstairs and downstairs but in this case most of the tower's height was sheltered by surrounding trees. Where this kind of shelter was not provided wind induced vibrations (max 10 m/s) still produced 3 to 4 times lower vibration amplitudes than from 2 person movement along the tower height.

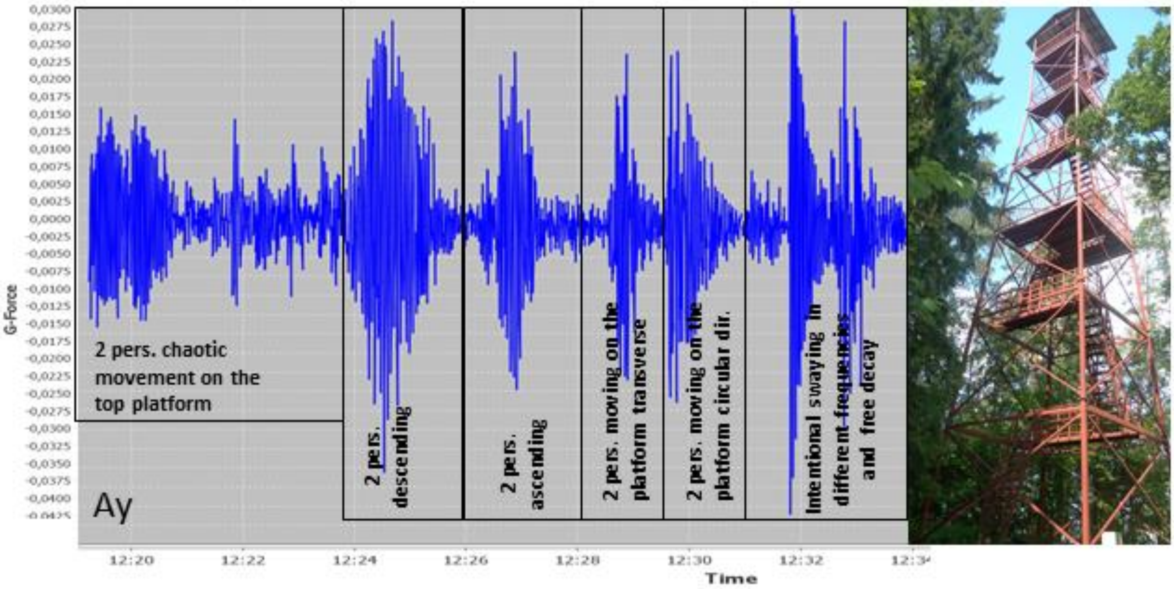


Figure 3.5. Acceleration time history of Krustkalnu observation tower

To see the vibration direction changes it is useful to plot both horizontal accelerations on the chart. For example, in the Figure 3.6 the measurement of the steel lattice tower tip (Dzintari tower) acceleration path in the 20sec time is presented and this reveals a chaotic nature of the observation tower vibration. Therefore in a case of a cantilever vibration with no constant direction in time, the human and structure synchronization phenomenon described in the section 1.1.2 possibly will not develop at all.

By analyzing the recorded histories of the vibration accelerations it was noticed that the response of the system does not reach the steady state vibrations when the response wave form has settled down (for example see Figure 3.4). It is due to the fact that system is not subjected to the constant cyclic force and in most of the cases the transient response amplitudes are greater.

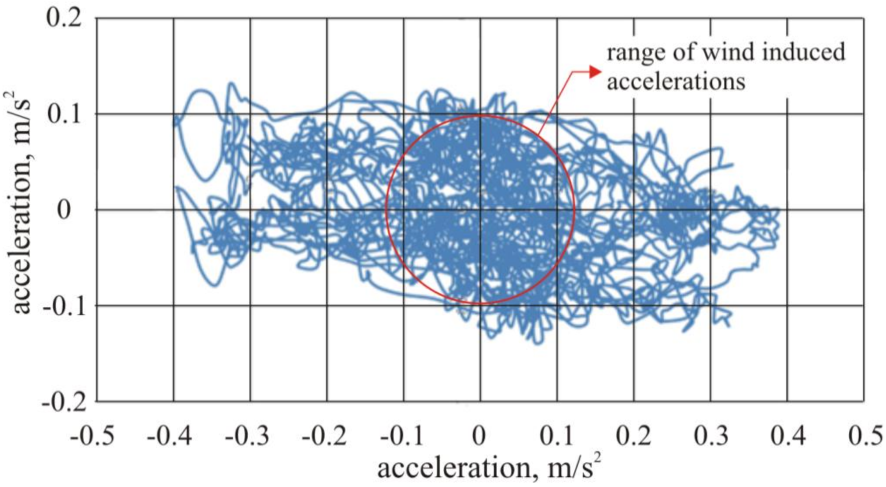


Figure 3.6. Measurement of tower (Dzintari) tip acceleration path in 20 sec

Further, recorded experimental vibration acceleration histories of 12 towers were processed with the methods described in section 3.1.1 to extract the dynamic parameters and excited frequencies of the tower due to human movement along its height.

3.1.1. Processing technique of experimental data

The structural dynamic behavior denotes the modal parameters of the structure (natural frequencies, damping ratios and mode shapes). The field of research referred to as "modal analysis" is dealing with identification of those parameters. The branch of modal analysis is operational modal analysis that aims to determine the dynamic characteristics of structure under operational conditions.

Excitation force of a person's movement along tower's height is weak compared to the self-weight and stiffness of observation towers therefore peaks in the output spectrum will be responses in the structural modes. For example self-weight of Kalsnava tower is ≈260kN but excitation force from two visitors according to Table 3.2 is 0.22 kN (for two people with mean weight of 740N and DLF of 0.12 for longitudinal walking force component).

The spectral analysis was performed using software package ME'scopeVES to determine the excited frequency content of simultaneously recorded time traces of observation tower's top platform accelerations.

The autocorrelation functions (30) of the time traces that show how the mean power in a signal is distributed over frequency were obtained. It is also a very handy tool to detect the harmonic signals buried in the noise [59].

$$G_{AA}(f) = A(f) \cdot A^*(f), \tag{30}$$

where *A(f)* is the Fourier transform of the time trace *a(t)* defined as:

$$A(f) = \int_{-\infty}^{+\infty} a(t) e^{-ift} dt, \quad (31)$$

The "*" indicates the complex conjugate and:

$$e^{ift} = \cos(ft) + i\sin(ft), \quad (32)$$

where *i* is a unity imaginary number.

To reduce the leakage effects due to non-periodicity of the time signal records the "Hanning window" was applied to each sampling window before the FFT (Fast Fourier Transform) was applied. In the ME'scopeVES the modal parameters are extracted from the cross channel measurement functions using FRF-based curve fitting methods. The DeConvolution window was applied to remove the "second half" of the time domain correlation function associated with the measurement.

To check the reliability of obtained natural frequencies the stabilization diagram that subsequently assumes an increasing number of poles was used. The physical poles (excited frequencies) always appear as "stable poles" consequently the unrealistic poles are filtered out. Examples of obtained auto spectrum and stabilization diagrams are presented in Figures3.8 and 3.9.

Damping ratios of the towers were obtained from free decay time histories using formula (33) [21]:

$$\xi = \frac{1}{2\pi n} \ln \frac{a_0}{a_n} \quad (33)$$

where,

n – number of relevant periods in time history;

a_0- max amplitude;

a_n - min amplitude.

Example of recorded free decay time history presented in Figure 3.7.

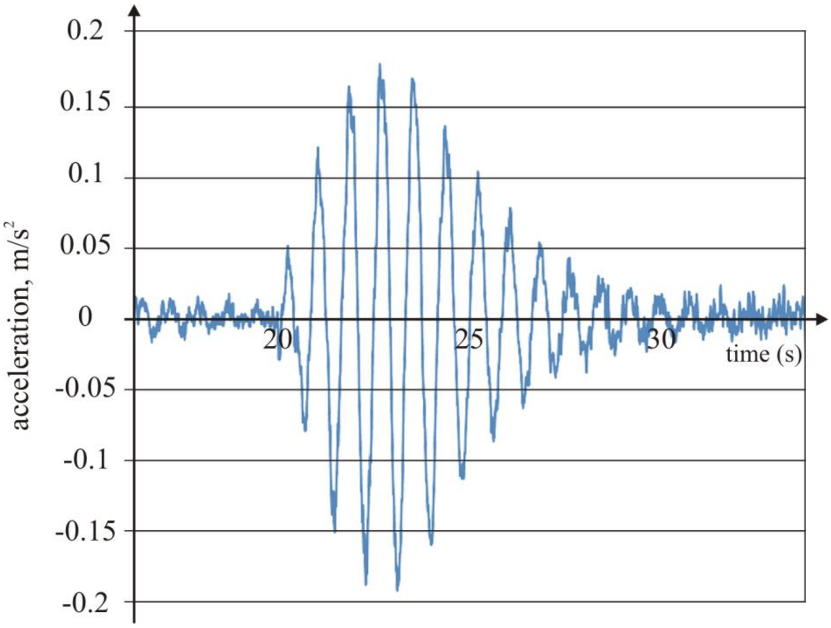


Figure 3.7. Recorded forced vibration and free decay time history of Egļu kalns tower

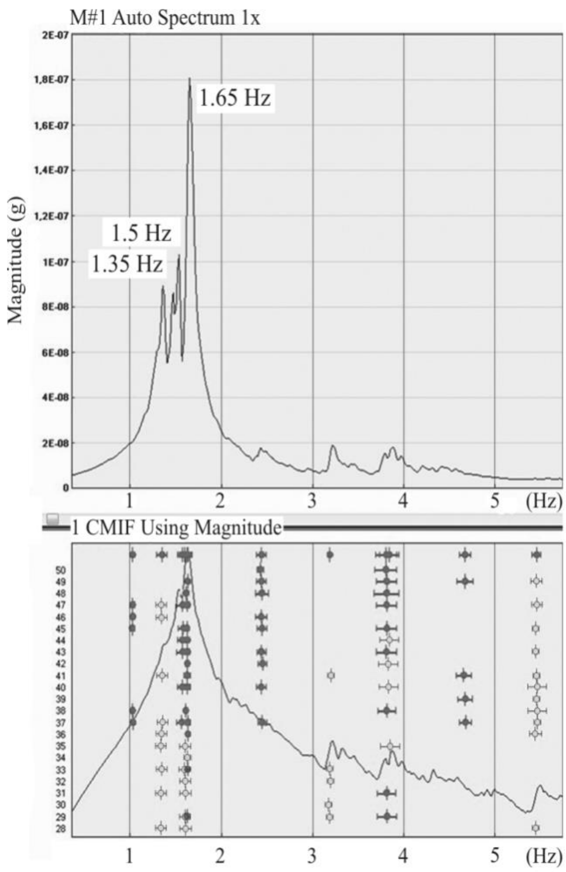


Figure 3.8. Response spectrum and stabilization diagram of Ligatne tower due to 2 persons descending

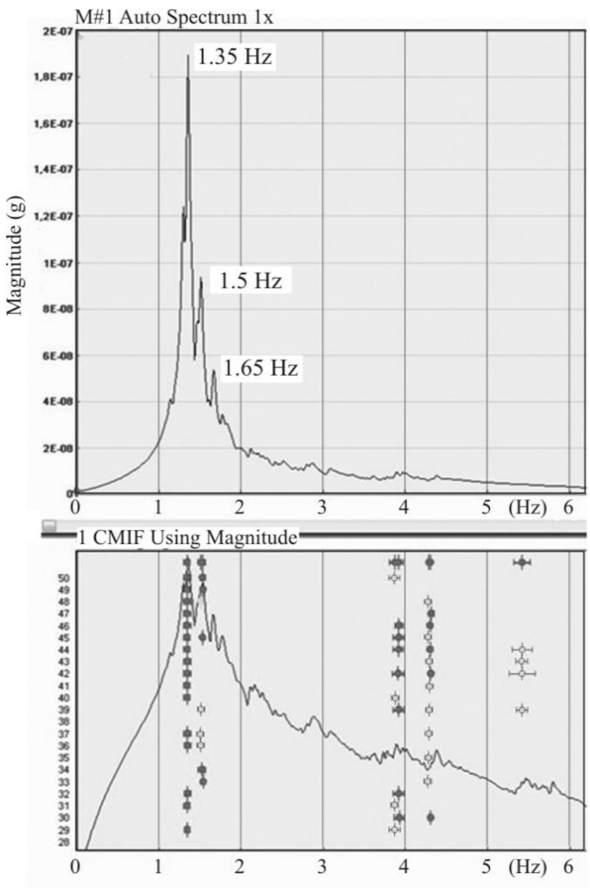


Figure 3.9. Response spectrum and stabilization diagram of Ligatne tower due to 2 persons ascending

3.1.2. Obtained dynamic parameters and response to human induced loading

It is important to find out exactly which of the human activities on the structure produce the highest accelerations and displacement to be able to develop a reliable methodology of assessing tower dynamic response to human induced loads.

Generally speaking, ascent and descent cases initiated the maximum acceleration amplitudes, with a slightly higher response level in descent. Higher values were recorded only when nonstop movement in a circular or transversal direction was organized on the upper platform. However, when the sightseers' on the upper platform behaved naturally, the maximum response was never reached. Experimental results revealed no linear relationship between the number of tower visitors and the response level of the structure. This illustrates Figure 3.10 where peak acceleration from 7 people descending or ascending the tower in Ūdru kalns are around 0.25 m/s^2 but from two people it is around 0.15 m/s^2. As it is expected the highest acceleration and displacement values were reached when intentional swaying on the upper platform of the towers was performed. In the case of the Ūdru kalns intentional swaying performed on the upper platform for 10 seconds produced maximum acceleration of just 0.29 m/s^2.

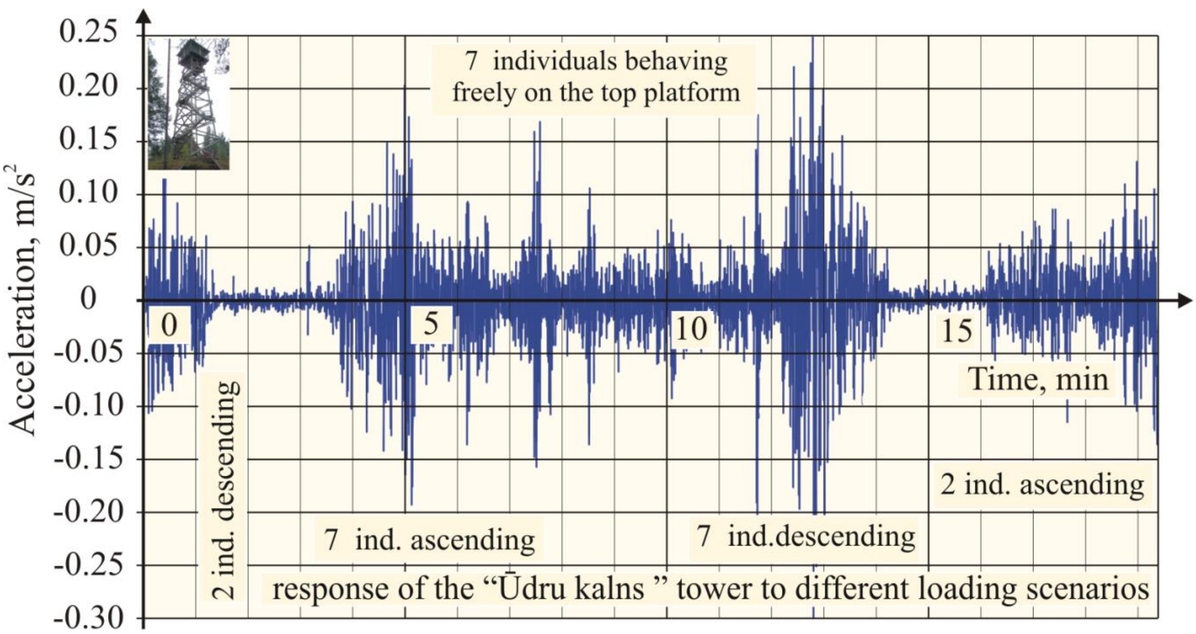


Figure 3.10. Tower response to different loading scenarios

From the recorded measurements of towers it follows that human induced vibration is clearly the matter of serviceability limit sate. When acceleration as high as 0.6m/s^2 was reached by intentional swaying for the Dzintari tower, the maximum corresponding displacement was only 50mm. Deflection is only 1/720 of the towers height and therefore corresponding stresses in the members will be far from critical.

In reality, intentional swaying of the structure is classified as a case of the vandal loading. This is an accidental design situation. There is no interest in providing the acceptable comfort level of the structure users in such case therefore vandal loading is outside the scope of this thesis.

Summing up, the highest acceleration values as determined from the experimental results were reached when the group of visitors descended or ascended the stairs if no organized activity of tower visitors was performed.

Table 3.2 presents three main excited natural frequencies of observation towers and maximum accelerations observed due to two person movement up and down the tower staircase as well as presented damping ratios if such were obtained. The damping ratios of timber observation towers are roughly twice (ξ ≈4%) of steel ones.

Table 3.2.

Dynamic parameters and response to human loading of observation towers

Tower name and height of the top platform above the ground level	Excited frequencies of two persons movement, Hz						A_{max}, m/s^2	ξ, %
	Ascending			Descending				
Krustkalnu tower (25m)	2.6	2.8	-	2.6	2.8	-	0.47	0.8
Kalsnava tower (25m)	1.7	-	-	1.6	1.7	2.2	0.22	1.7
Dzintari tower (34m)	0.75	0.8	1.15	0.8	3.3	-	0.39	2.3
Eglu kalns (26.5m)	1.3	4.2	-	1.3	2.5	4.2	0.3	4
Priedaine (32m)	1.2	2	3.1	1.1	2	2.2	0.15	-
Kamparkalns (26.5m)	1.35	1.45	-	1.45	2.85	-	0.3	3.1
Udru kalns (26.5m)	1.35	2.6	-	1.35	1.55	2.6	0.15	3.85
Ventspils tower (12m)	Excitement is negligible; fundamental frequency is 3.4Hz							-
Kuldiga tower (16.3m)	0.8	1.1, 1.2	2.6	0.8	1.2	2.6	0.26	-
Lielais liepu tower (34m)	1.1	1.3	2.1	1.1	1.5	4.7	0.13	-
Ligatne tower (22m)	1.35	1.5	1.65	1.35	1.5	1.65	0.25	5.4
Lozmeteju tower (28.5)	1	1.1	2.2	1	1.1	-	0.3	-

The lowest excited frequencies of human movement are generally the fundamental frequencies of the observation towers. To recognize it, obtained from ambient response data (where as an input force was considered the wind loading) a frequency spectrum of each tower was analyzed. Therefore it is confirmed that lattice light-weight observation tower responds in the structural modes to human movement along its height.

According to [72], footfall rate may be roughly broken into three regions: walking (below 2.3 Hz), mixture region where the subject could comfortably walk or run up or down the stairs (2.3 Hz - 3.5 Hz) and running. These results are applicable if the stair is relatively short. As noted before, the stairs of observation towers are "long stairs" with a large number of flights. Therefore it is obvious that the walking rate changes along the height of the observation tower and depends on the physical and emotional condition, as well as the motivation of the visitor.

Summarizing the findings about typical walking harmonics in Section 1.2, it is possible now to distinguish the range of walking frequencies for the harmonics that mostly influence the observation tower performance evaluating the serviceability limit state (SLS). The ranges are presented in Table 3.3, where f_1 (Hz) is a person's frequency of stair ascent or descent. All fundamental frequencies of the inspected towers are in the typical human walking range as given in the Table 3.3.

Table 3.3.

Typical frequencies range of harmonics due to the observation tower stair ascending or descending

Frequency range title	Frequency f_1, Hz
1st harmonic of force longitudinal component (a)	$0.5 \leq f_1 \leq 3$
2nd harmonic of force longitudinal component (b)	$1 \leq 2f_1 \leq 6$
1st harmonic of force lateral component (c)	$0.25 \leq f_1/2 \leq 1.5$
3rd harmonic of force lateral component (d)	$0.75 \leq 3f_1/2 \leq 4.5$
5th harmonic of force lateral component (e)	$1.25 \leq 5f_1/2 \leq 7.5$

It was noticed that generally during the stair ascent case the lowest natural frequency excited with the highest acceleration amplitude but in descending case there could be a different dominant natural frequency. It corresponds well with the observations during the experiment that visitors move downstairs faster than upstairs. The differences in the excited natural frequencies of the towers with very similar structure and the same fundamental frequency due to human movement (Kamparkals tower and Udru kalns tower) indicate the stochastic nature of human dynamic loading.

Although the inspected towers vary in the structural arrangements and materials, the maximum response level (acceleration amplitude) of two people's movement is close to $0.3 m/s^2$. The exception is two steel towers (Krustkalns and Dzintaru) that are considerably lighter and have higher acceleration amplitudes and a tower in Ventspils that is shorter with no visible effect from human movement.

The observation tower in Dzintari has the most interesting structural arrangement in the sense that it is not built as traditionally as most observation towers in Latvia. Also it is one of the most sensitive to human induced vibrations. Therefore it was chosen for the additional experiment that aimed to identify the importance of human walking frequency synchronization with the natural frequency of the tower by comparing the human walking ground reaction force harmonic frequencies with the frequencies of the tower at its maximum response.

On May 2010 36.48 m high sightseeing tower in Dzintari, Latvia was opened for public use (Figure 1.1). Since the tower was opened, there have been complaints from visitors about the tower's excessive vibrations even in non-windy days. All of its elements – the inner and outer core, platforms and stairs are made of steel, except the wooden cladding on the facades of the steel cores. The structural configuration of the tower can be seen in Figure 3.12.

The structure consists of the braced inner core with the dimensions of 1500x1500 mm, made from tubes with the cross section of 200x200x8, and the outer core with the dimensions of 4240x4240 mm, made from tubes with the cross section of 140x140x5. The outer core has no vertical bracing, as it was required by the architectural concept. The inner and outer cores are connected together only by steel stairs. At the level of 33,5m there is a platform for sightseers. The platform is placed offset from the central core and therefore makes the tower an eccentric structure.

During the experiment the vibration accelerations of the tower eccentric platform were measured in three directions. The accelerometers were placed on the upper sightseer's platform based on the results of numerical analysis in order to find the highest acceleration values and identify whether the critical mode shape is a torsional mode shape (see Figures3.1 and Figures 3.12).

To evaluate the fundamental frequencies and the critical mode shapes of the existing tower theoretically, a three dimensional finite element model created by the structural analysis software STRAP 12.5. To extract the eigenvalues, the structural analysis software uses the subspace iteration technique.

Numerically calculated and experimentally measured natural frequencies of the tower are presented in Table 3.4, and the first five mode shapes are presented in Figure 3.11.

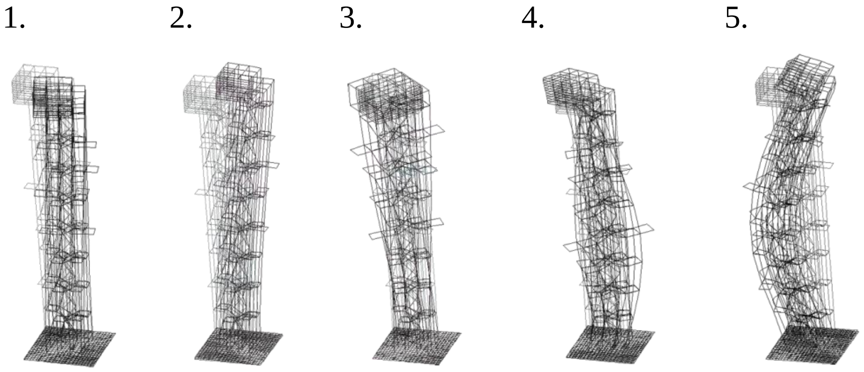


Figure 3.11.Mode shapes of eccentric tower in Dzintari

Due to the eccentrically placed visitor's platform mode shape with the lowest frequency is torsional with the center of rotation outside of the tower's geometry. The calculated frequencies are higher than the measured ones because the structure is sensitive to the accuracy of the simulation. In numerical calculations, an effect such as partial stiffness of the connections has not been taken into account.

Table 3.4.

Experimentally obtained and numerically calculated eccentric tower natural frequencies

Mode shape Nr.	Measured natural frequency, Hz	Calculated natural frequency without foundation stiffness , Hz	Calculated natural frequency with foundation stiffness, Hz	Mode shape
1.	0,76	0,98	0,86	Torsional + transverse
2.	0,79	1,04	0,90	Transverse
3.	1,15	1,45	1,41	Torsional
4.	2,96	4,20	4,00	Torsional
5.	3,23	4,71	4,30	Transverse

Simultaneously with the tower's stair ascent and descent the accelerations of the person's center of gravity (COG) in vertical, lateral and longitudinal directions were measured and recorded. The method of measuring, measuring devices and experimental data analysis methods of the person's ground reaction forces are described in the section 2 of thesis.

The surrounding air temperature during the experiment was 7° C and almost constant wind speed of 1.2 m/s was observed.

The experiment program was to measure the tower's response to different amount of people (7, 8 and 11) moving upstairs and downstairs with their natural choice of walking pace frequency. Afterwards in order to identify and analyze the initiated tower frequencies and compare them with separate harmonics of human walking force. The total time necessary to ascend the tower with pacing frequency 1,21 Hz was 3,32 minutes.

Table 3.5 presents the tower's maximum acceleration measurements at the visitors' upper platform while different amount of people in the close group were ascending or descending the tower's stairs. It is also summarized which tower frequencies are excited and increases during the stair ascending or descending process of the group. And there are extracted and presented the corresponding frequencies of human walking harmonics.

Table 3.5.

Eccentric tower response to the human induced dynamic loads

Number of tower visitors	Action	Tower platform max acceleration during event, [m/s²]	Corresponding human walking harmonic frequencies, f_p [Hz]		Obtained tower frequencies corresponding to dynamic load application, f_t [Hz]	$\frac{f_p - f_t}{f_p} \cdot 100\%$
			Longitudinal force direction	Lateral force direction		
7	Ascend	0,39	-	**0,875(1st)**	**0,76***; 0,79	13%
			1,75(1st)	-	-	-
			3,5 (2nd)	-	3,3	6%
8	Descend	0,4	-	**0,97(1st)**	0,76; **0,79***	19%
			1,94(1st)	-	1,16	40%
			3,88 (2nd)	2,91(3rd)	3,3	12%
8	Descend	0,44	-	-	0,76; 0,79	-
			2,29(1st)	**1,15(1st)**	**1,16***	1%
			6,87 (3rd)	-	7,6	10%
11	Ascend	**0,48**	-	0,605 (1st)	0,79; 0,76	21%
			1,21 (1st)	-	**1,16***	4%
			-	3,025 (5th)	2,96	2%
			3,63 (3rd)	-	3,23	11%

The frequencies with higher amplitudes are marked with "*"

This experimental investigation of 36 m high steel observation tower revealed that the key parameter – the ratio between relevant walking harmonic and the excited natural frequency of the structure was mainly found to be in the range of $0.8 \leq \Omega/\varpi_n \leq 1.2$ and it correlates with the coefficient of variation $\mu = 0.277/1.416 \approx 20\%$ from the "long stair" experiment in Hannover [75].

Experimentally torsional mode shapes of vibration were identified by plotting the vibration amplitudes on the plan of the upper platform. Figure 3.12 also presents measurement of the tower's tip vibration acceleration changes (m/s^2) during the tower ascending conducted by 11 people.

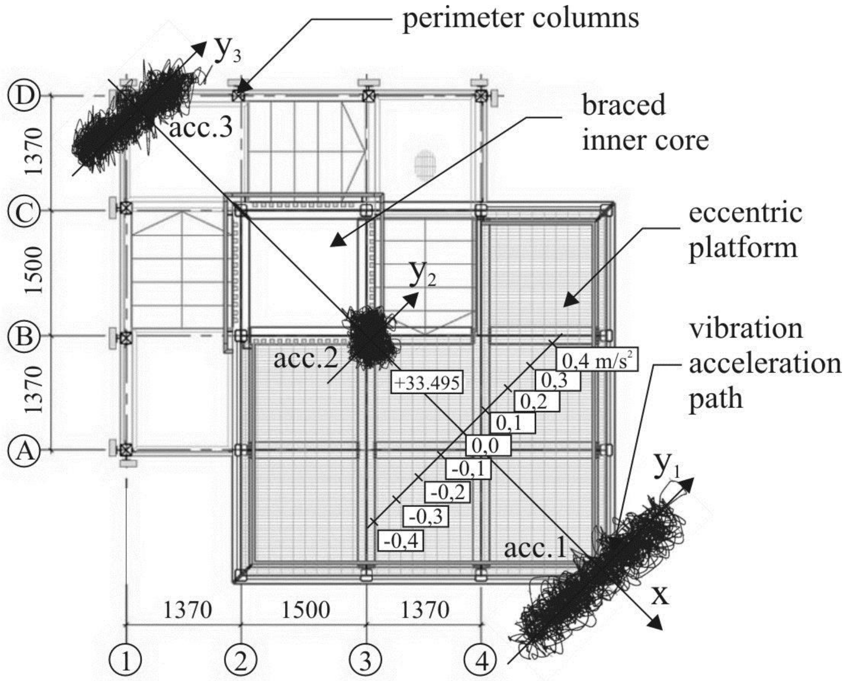


Figure 3.12.Measured tower tip vibration acceleration (m/s^2) and tower structural scheme

During all the tests it was observed that people's movement on stairs activated the tower's resonant frequencies and it was not entirely connected with the number of the test individuals. Therefore the tower's maximum response to an induced dynamic load does not depend only on the number of individuals but more on the degree of human and structure synchronization.

During the experiment the scatter of individual pacing frequencies was remarkable but always the first harmonic of human walking lateral or longitudinal force component approximately corresponds to the tower's resonant frequency with the highest vibration amplitude. It has been noticed that the second or third harmonic of human walking activated the tower's higher resonant frequencies as well but with smaller amplitudes. In some cases the tower torsional mode shape frequencies were activated and in some cases were activated the tower's transverse mode shape frequencies and in some cases human walking lateral force component induced these mode shapes but in some cases longitudinal force component.

As different tests present a humans tendency to synchronize pacing frequency to a different resonance frequency of the tower therefore seems like human and structure synchronization have more an accidental nature and human induced loading itself has a stochastic nature.

3.1.3. Criteria of limiting the vibration amplitude

The final goal of the vibration analysis of observation towers is to reduce or remove discomfort for the structure users. Therefore an expected vibration of the observation tower requires limitations to meet human comfort criteria. As discussed in the section 1.1.3, in the international codes the limit values of acceleration are directly related to the pedestrian comfort.

To assess the preliminary recommended maximum vibration acceleration limit that would ensure the comfort of the visitors on the observation towers, the visitors' subjective assessment of the vibration felt on the upper platform of the tower were investigated. The individuals were asked to describe the level of vibration and their comfort as follows:

- no vibration;
- sensible vibrations;
- discomfort;
- disturbing discomfort.

The test subjects were random tourists of 12 towers that visited the particular tower during the day of the experiment. It was noticed that often the visitors discussed the sense of vibration even before they found out about the experiment.

Although subjective assessment of vibrations depends on many factors, in the case of observation towers the tolerance depends mostly on visual clues and the height of the structure. It was noticed in the responses that women seem to be more sensitive to vibration than men. In general, visitors started to feel uncomfortable when the vibration acceleration exceeded 0.2 m/s^2. Therefore, to ensure comfort for the majority of the visitors, it is proposed to set the limit on the peak acceleration value during the design process for structures with a fundamental frequency close to typical human walking frequency.

Figure 3.13 presents the peak acceleration limit (green line) proposed by authors. It depends on the excited natural frequency of the tower and is based on the experimentally obtained peak acceleration values of different towers when the visitors experienced some degree of discomfort. On the vertical axes of the chart is plotted peak acceleration but on the horizontal axes is the fundamental frequency of the buildings or observation towers. The shaded area corresponds to typical range of fundamental frequencies of lattice observation towers that were obtained from the experimental results earlier. Additionally, the chart shows the typical frequency ranges of the walking harmonics according to Table 3.3 to illustrate that at least one of the walking harmonics will be usually close to the fundamental frequency of the tower.

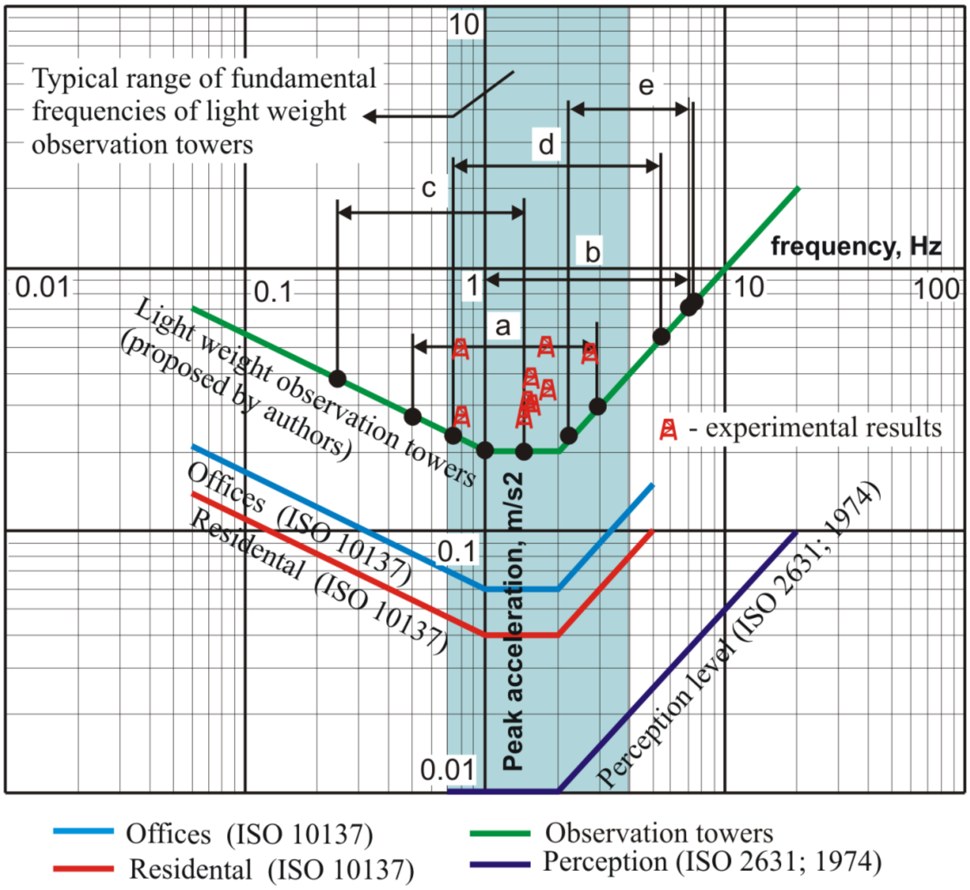


Figure 3.13.Comfort criteria (for “a” to “e” see Table 3.3)

The presented curve can be further modified by factors that would take into account the type and location of the particular structure, the required comfort level or other factors. For practical calculations more convenient is a use of “frequency weighting” – peak acceleration is attenuated to take into account the variation of sensitivity of vibration due to varied frequencies. Then the vibration response can be considered as satisfactory when the peak acceleration multiplied by the weighting factor W_d does not exceed a limiting value of $0.2 m/s^2$. Weighting factor W_d is defined as follows:

$$W_d = \begin{cases} 1.0 & for \quad 1Hz < f < 2Hz \\ \dfrac{2}{f} & for \quad f \geq 2Hz, \end{cases} \tag{34}$$

where f – considered natural frequency of the structure.

3.2.Theoretical investigations on predicting observation tower dynamic response to the human induced loading

From the theoretical point of view, according to the generally accepted design processes for low frequency structures it is convenient to consider the maximum level of the resonant response that can be induced by a person under repeated footfall and to limit it to the acceptable level. There are several issues to be considered:

- how to anticipate the resonance response of the structure with several modes;
- the effect of separate walking harmonics on the total vibration;
- the typical number of the "successful footfalls" (a footfall coinciding with the vibration frequency of the structure) to progress resonant build – up;
- the presence of the human-structure synchronization;
- the phenomenon and response of the structure induced by a group of people;
- the parameters that mostly influence the structure response to human induced loading.

Therefore to investigate the above mentioned issues theoretical investigations were carried out.

In the previous study of the author, the tower response (acceleration) due to human-induced loads was calculated as a "steady state response". Considerable discrepancies between the measured and calculated values suggest that a tower does not reach a steady state vibration due to the inconsistent periodicity of the applied loading and its direction, and this should be taken into account. This also corresponds with the observations done in the experimental part of the research.

Slender sightseeing towers are line-like structures and for the purpose of response analysis it is modeled as a cantilever with mass uniformly distributed along the height. Foundation stiffness has not been taken into account. The loading scheme and mode shapes of the observation tower for analytical investigation are presented in Figure 3.14.

The natural frequencies and the corresponding transverse mode shapes of the tower can be found from the equations of an ideal Euler – Bernoulli prismatic cantilever [81].

The response of the system with viscous damping to induced harmonic excitation can be written in the form of a well-known non-homogenous differential equation [133]:

$$\frac{\delta^2}{\delta y^2}\left[EI(y)\frac{\delta^2 x}{\delta y^2}\right]+m(y)\frac{\delta^2 x}{\delta t^2}+c(y)\frac{\delta x}{\delta t}=F(y,t)\,, \tag{35}$$

where ,

EI – stiffness of the tower, Nm^2;

$m(y)$ – mass of the structure per meter length, kg/m;

$c(y)$ – viscous damping per unit length;

x – displacement, m;

$F(y,t)$ – human-induced force, N.

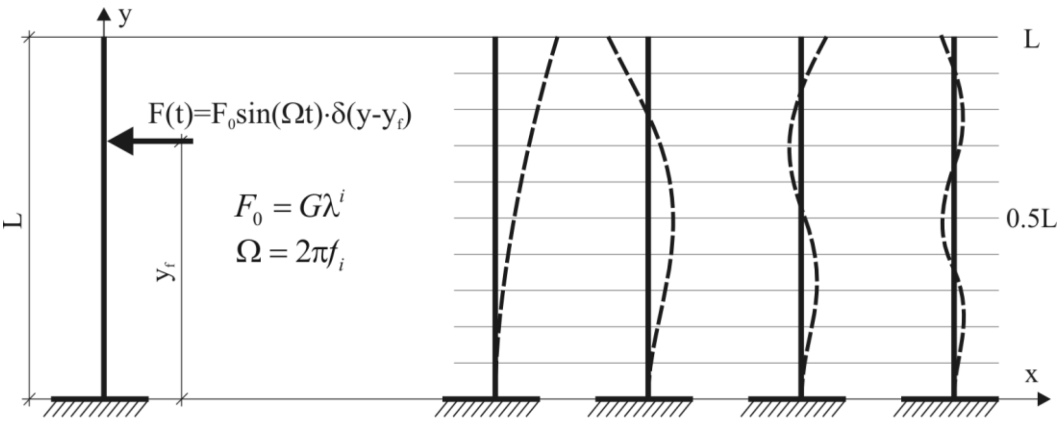


Figure3.14.Calculation scheme and mode shapes

(G - static weight of the subject body, N ; λ^i- the Fourier coefficient of the i^{th} harmonic, often referred to as the dynamic loading factor (DLF); f_i - human walking frequency of the i^{th}harmonic, Hz)

Parameters EI , m and c are assumed constant for the further theoretical analysis.

Function $F(y,t)$ can be expressed as series. In this way it is possible to approximate almost any load [85]:

$$F(y,t)=\sum_{n=1}^{\infty}\Phi_n(y)\cdot Q_n(t)\,, \tag{36}$$

where Φ_n – mode shape of structure.

To obtain the function of time $Q_i(t)$, both sides of the equation are multiplied by $\Phi_i(t)$ and integrated along the beam length. By employing orthogonality properties of the mode shape $\Phi_n(t)$ [98], only one sum element with index *i* will remain on the right side of the equation, and if the system is loaded with concentrated load, then:

$$Q_i(t) = \frac{F(t)\cdot\Phi_i(y_f)}{\int_0^L \Phi_i^2(y)dy}. \tag{37}$$

The solution of the differential equation of beam motion is searched in the form of a series:

$$x(y,t) = \sum_{n=1}^{\infty}\Phi_n(y)\eta_n(t). \tag{38}$$

Taking into account that every element of a series generates motion that is described by a particular element of the series, an equation can be written for every $\eta_i(t)$:

$$\ddot{\eta}_i(t) + 2\xi_i\omega_i\dot{\eta}_i(t) + \omega_i^2\eta_i(t) = \frac{Q_i(t)}{m}, \tag{39}$$

where $i = 1,2,...,\infty$;

ξ - damping ratio;

ω_i - i^{th} natural angular frequency of the tower, rad/s

The general solution of the equation (39) is:

$$\eta_i(t) = e^{-\xi_i\omega_i t}(A_i \sin\bar{\omega}_i t + B_i \cos\bar{\omega}_i t) + \frac{1}{m\bar{\omega}_i}\int_0^t Q_i(\tau)e^{-\xi_i(t-\tau)}\sin\bar{\omega}_i(t-\tau)d\tau, \tag{40}$$

where:

$$\bar{\omega}_i = \omega_i\sqrt{1-\xi_i^2}. \tag{41}$$

Further, by bringing the constant part Q_0 of the Q_i before the integral and taking into account that it varies in accordance with principle *sin* Ω*t*, the solution of the equation (39) can be rewritten in the following form:

$$\eta_i(t) = e^{-\xi_i\omega_i t}(A_i \sin\bar{\omega}_i t + B_i \cos\bar{\omega}_i t) + \frac{Q_0^i}{m\bar{\omega}_i}\int_0^t e^{-\xi_i(t-\tau)}\sin\Omega\tau\sin\bar{\omega}_i(t-\tau)d\tau, \tag{42}$$

where Ω - human walking angular frequency, rad/s;

Coefficients *A* and *B* can be determined from the initial conditions of the beam.

In case the initial displacement and speed is zero, by solving the integral, the solution would be:

$$\eta_i(t) = \frac{Q_0^i}{m\overline{\omega}_i}\left[\frac{\left(p_1 \sin\Omega t - 2\xi_i \Omega \cos\Omega t\right)\overline{\omega}_i}{D} + \frac{\Omega\left(p_2 \sin\overline{\omega}_i t + 2\xi_i \overline{\omega}_i \cos\overline{\omega}_i t\right)e^{-\xi_i t}}{D}\right], \quad (43)$$

where:

$$\begin{cases} p_1 = \overline{\omega}_i^2 + \xi_i^2 - \Omega^2 \\ p_2 = \Omega^2 + \xi_i^2 - \overline{\omega}_i^2 \\ D = \left(\xi_i^2 + \Omega^2 + \overline{\omega}_i^2\right)^2 - 4\Omega^2 \overline{\omega}_i^2 \end{cases} . \quad (44)$$

Then the solution of the equation of motion (35) e.g. displacement if the initial conditions are zero and viscous damping has been taken into account is:

$$x(y,t) = \sum_{i=1}^{\infty} \frac{\Phi_i(y) Q_0^i}{m\overline{\omega}_i}\left[\frac{\overline{\omega}_i(p_1^i \sin\Omega t - 2\xi_i \Omega \cos\Omega t) + \Omega(p_2^i \sin\overline{\omega}_i t + 2\xi_i \overline{\omega}_i \cos\overline{\omega}_i t)e^{-\xi_i t}}{D}\right]. \quad (45)$$

Then vibration acceleration of the tower is:

$$\ddot{x}(y,t) = \frac{\delta^2 x(y,t)}{\delta t^2}, \quad (46)$$

The correctness of the equation (45) is checked against the finite element calculations of cantilever presented in the Figure 3.14.

For non-homogenous differential equations with constant coefficients like (35) the particular solution may be found separately for each function on the right side of the equation. Therefore, the maximum response from each walking harmonics may be found from equations (45) and (46), and then summed to obtain the total response of the structure after a specific number of "successful footfalls" (expressed as a moment of time) (47):

$$\frac{\delta^2}{\delta y^2}\left[EI(y)\frac{\delta^2 x}{\delta y^2}\right] + m(y)\frac{\delta^2 x}{\delta t^2} + c(y)\frac{\delta x}{\delta t} = \sum_{i=1}^{n} G\lambda_i \sin(\Omega_i t + \varphi_i), \quad (47)$$

where G - static weight of the subject body, N;

λ_i - dynamic loading factor (DLF) of the i^{th} harmonic;

Ω_i - human walking angular frequency of the i^{th} harmonic, rad/s;

φ_i - phase shift of human walking i^{th} harmonic, rad.

3.2.1. Observation tower response to human-induced walking loads

The methodology described in the previous section is used to carry out the following theoretical investigations:

- typical mode shapes that respond to the human induced loading;
- location of load application that causes highest amplitudes of vibration;
- various walking harmonic influence on the total vibration of the tower;
- human-structure synchronization effect on the tower response;
- analysis to establish dynamic parameters that mostly influence the dynamic response amplitudes due to human movement.

Towers with mass distributed approximately uniformly over the height and the fundamental frequency close to one of the human walking harmonic frequencies mostly have a resonant response in mode shape (frequency ϖ_n) with only one "node" (at foundation level), see Figure 3.14. It is due to the fact that the next transverse normal mode shapes, according to [81], would be $6.27\varpi_n$, (two "nodes") $17{,}6\varpi_n$, (three "nodes"), which results in a frequency that responds to resonant excitation with small displacements and high acceleration values (felt like a trembling). According to Figure 3.13 the acceleration limit for those frequencies is considerably higher. Another constraint in reaching significant vibration amplitudes with higher frequencies is the small probability of continuation of the "successful footfalls", which can be initiated only by a group of people not walking in phase with each other. The same assumptions can be applied to the torsional normal mode shapes. Therefore theoretically the typical mode shapes that respond considerably to the human induced loading are first transverse normal mode shape and first torsional normal mode shape or coupled first transverse and torsional mode shape. Experimental investigations in section 3.1.2 confirm it.

Obviously, the last flight of the tower stairs is a critical place where to apply the walking load for the critical mode shapes mentioned above.

The next step is investigation of various walking harmonic influences on the total vibration of the tower to obtain the critical ones. According to Table 3.3, ideally there are five possible cases of walking dynamic loading on stairs of observation towers that should be considered:

- Case № 1: 1st harmonic of longitudinal force component coincides with the fundamental frequency f_n of structure;
- Case № 2: 2ndharmonic of longitudinal force component coincides with the fundamental frequency f_n of structure;

- Case № 3: 1st harmonic of lateral force component coincides with the fundamental frequency f_n of structure;
- Case № 4: 3rd harmonic of lateral force component coincides with the fundamental frequency f_n of structure;
- Case № 5: 5th harmonic of lateral force component coincides with the fundamental frequency f_n of structure.

Figure 3.15 presents an example of the resonant response level (displacements and accelerations) of lattice tower if one of the above mentioned cases emerges. For each case five subsequent "successive steps", when the structure's fundamental frequency coincides with one of the walking harmonics, were considered. The variation of vibration perception sensitivity corresponding to the relevant frequency was taken into account by multiplying the acceleration with a "frequency weighting" according to (34).

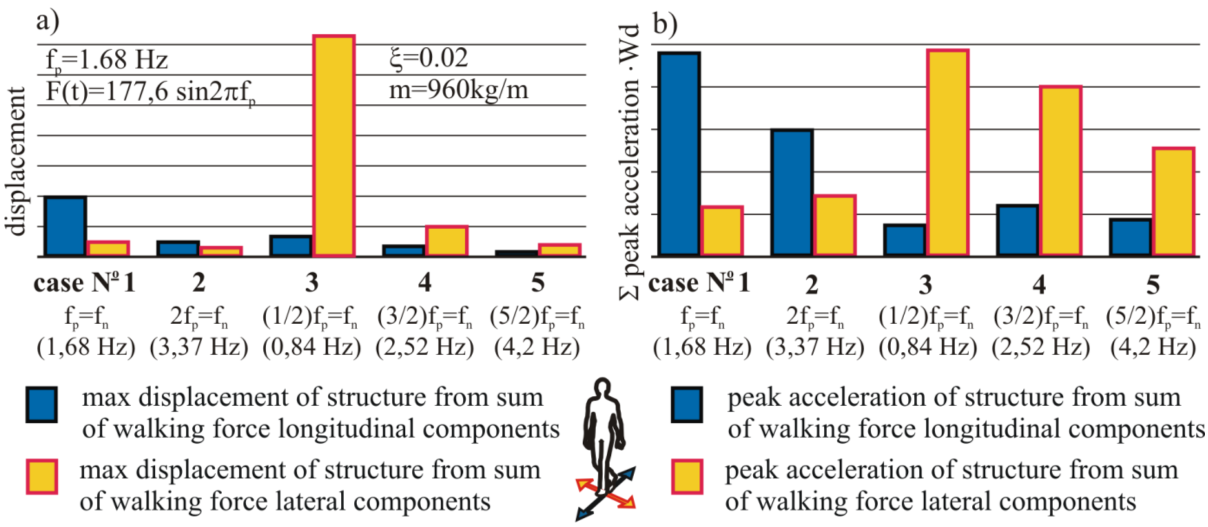


Figure 3.15.Comparison of the effect of longitudinal and lateral walking harmonic components on the total vibration of the structure for different cases

(f_p - human walking frequency, Hz; f_n -natural frequency of the tower, Hz; *Wd* - *w*eighting factor)

Depending on the fundamental frequency of structure, when comparing the input of lateral and longitudinal walking forces to the total vibration of the structure, it turns out that all cases described above might be the critical ones. By comparing Table 3.3 (typical fundamental frequencies) with the mean walking frequency of 1.416 Hz ("long stair" case), it follows that case № 1 would be the most common situation, and therefore it is further analyzed in Figure 3.16.

Various walking harmonic inputs to the total vibration greatly depend on the fundamental frequency of the structure itself. Figure 7 a) and b) present a percentage of the input of each walking force longitudinal harmonic to the total tower vibration amplitude when case № 1 emerges (1st harmonic of longitudinal force component coincides with the fundamental frequency f_n of structure). Figure 7 c) and d) show the percentage relationship between the input of the relevant walking force lateral harmonic and the first longitudinal harmonic to the total tower vibration amplitude.

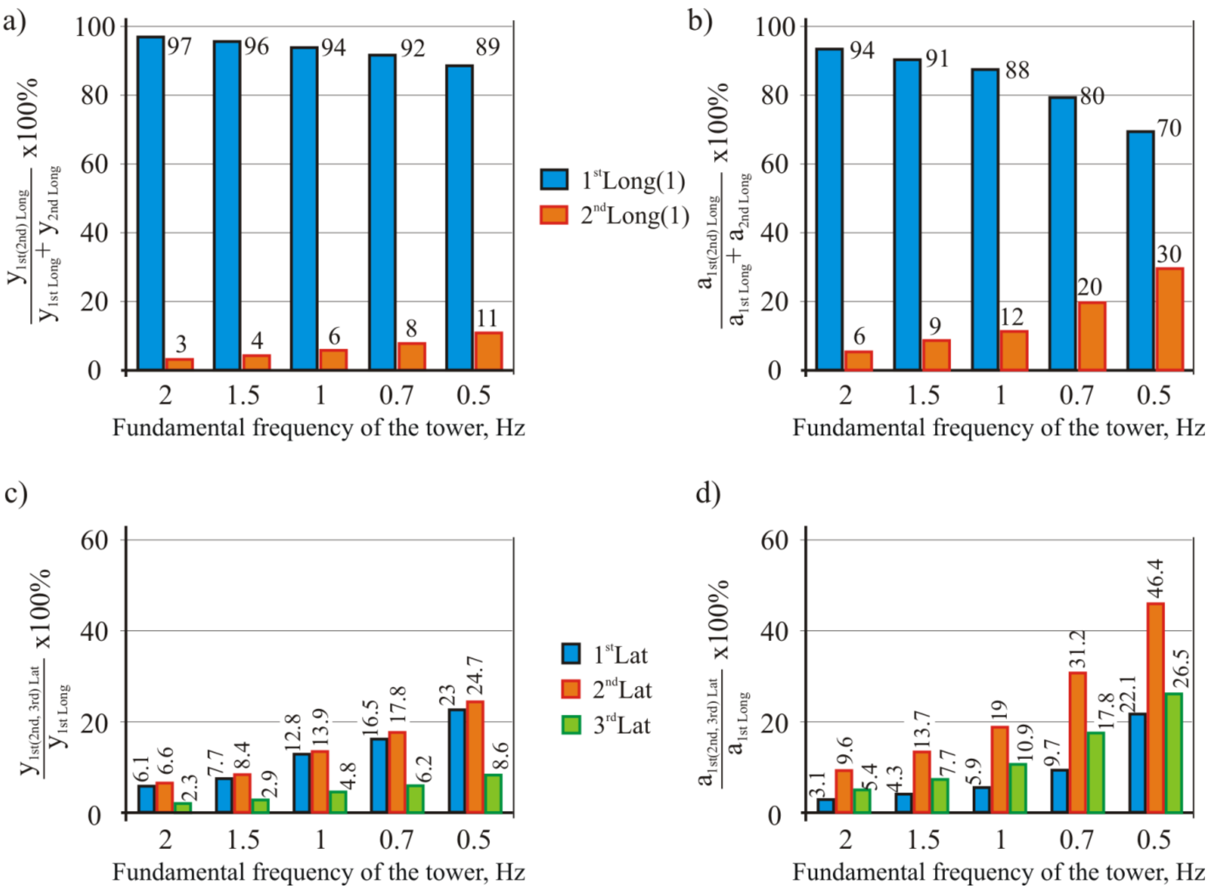


Figure 3.16.The relationship between the fundamental frequency of a structure and the input of separate walking harmonics to the total vibration for case № 1

(y – displacement, mm and a – acceleration, m/s^2)

Figure 3.16 reveals that for stiffer structures (above 2 Hz), the effect of harmonics other than the first walking force longitudinal harmonic on the total vibration is small. The input of other harmonics should be taken into consideration during the design process for very slender observation towers (fundamental frequency less than 1 Hz).

In reality, the case when one of the walking harmonics coincides with one of the natural frequencies of the tower has a stochastic nature. Almost complete conjunction for a longer period of time would have a minor probability due two main reasons:

- individual walking upstairs or downstairs vary the speed especially walking upstairs due to the physical reasons;
- the stair has turns (landing platforms) that influence the pacing rhythm.

Because of the stochastic nature of excitation, the loading duration (number of subsequent “successive” steps that is close to the one of natural frequencies of the tower) has a major effect on the tower response as well as the key parameter - the ratio between the relevant walking harmonic and the excited natural frequency. Therefore the variation of the tower peak response with the ratio between relevant walking harmonic and the excited natural frequency Ω / ϖ_n was investigated, also taking into account a possible variation in damping ratios ξ and the number of subsequent “successive” steps n. During the experimental investigations it was noticed that two persons walking close to each other tend to fully synchronize their step. Therefore, as a starting point for the tower response calculations it is reasonable to take the dynamic loading of two persons walking at the same speed and phase and one of the walking harmonic close to the natural frequency of the tower.

Results of the Kalsnava tower is presented in the Figure 3.17. Here at the location of the last stair flight force longitudinal walking harmonic of two visitors whose walking speed and phase shift match exactly was applied.

To take into account a possible variation in the ratio between the relevant walking harmonic and the excited natural frequency of the structure, it is suggested to calculate the mean value of the tower response for the selected number of steps in the ratio range of $1 \pm 20\%$ (based on the experimental results described in 3.1.2).

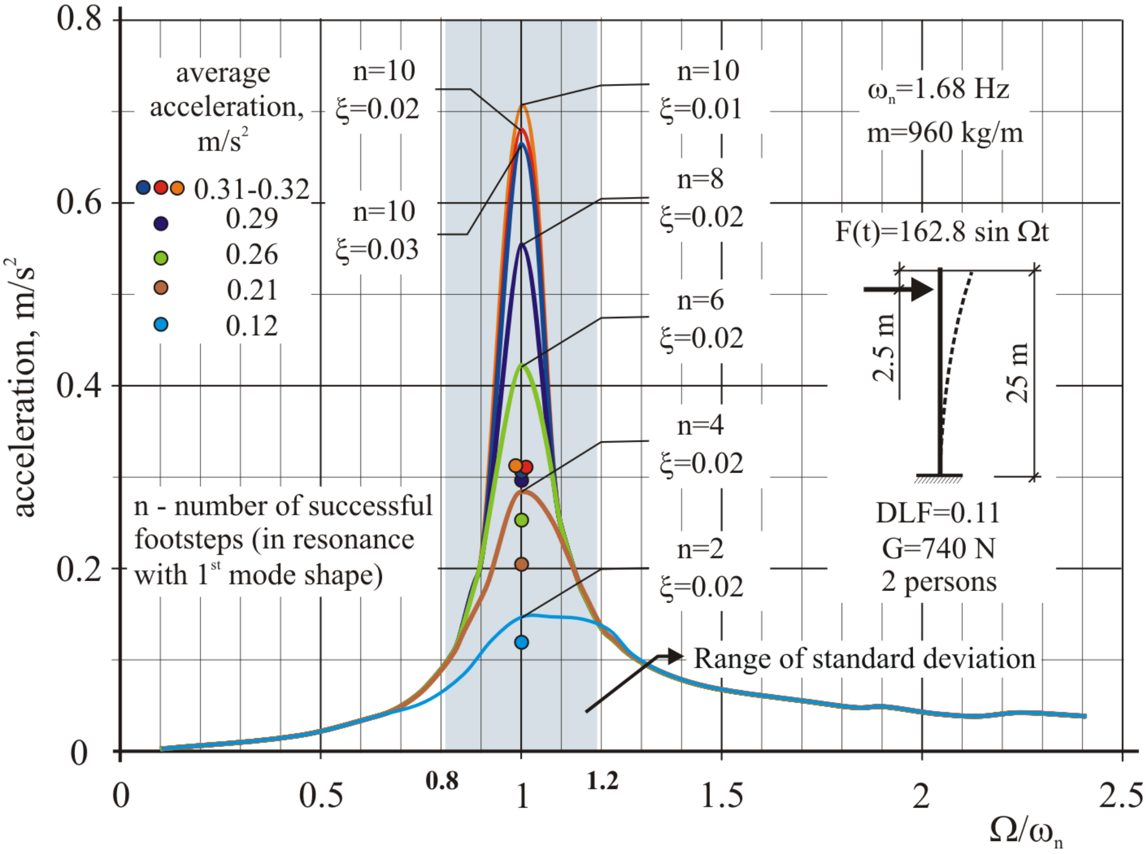


Figure 3.17.Tower response at different frequency ratios

In the Figure 3.17 it appears as a shaded area and could be defined as follows:

$$a_{mean} = \frac{1}{1.2-0.8}\int_{0,8}^{1,2} a_{\max}(z)d(z) = 2{,}5\int_{0,8}^{1,2} a_{\max}(z)d(z), \qquad (48)$$

where

a_{mean} - mean peak acceleration, m/s^2;

$a_{\max}(z)$ - peak acceleration at time *t*, m/s^2

$z = \dfrac{\Omega}{\varpi_i}$ - the ratio between relevant walking harmonic and the excited natural frequency;

$t = \dfrac{2\pi n}{\varpi_i}$ - time after *n* subsequent “successive” steps, s;

ϖ_i - considered natural frequency of the tower, rad/sec;

As expected, the slight variation in the damping ratios of the structure does not make any considerable difference in the tower response amplitude due to the fact that after n subsequent "successive" steps the structure does not reach a steady state of the vibration.

A rather good agreement is obtained between the calculated peak acceleration of $a_{mean} = 0.21 m/s^2$ (Figure 3.17 for n=4) and experimental results presented in Figure 3.18 ($a_{\max} = 0.22 m/s^2$). During the experiment wind induced background vibration amplitude was measured: $a_{wind} = 0.03 m/s^2$.

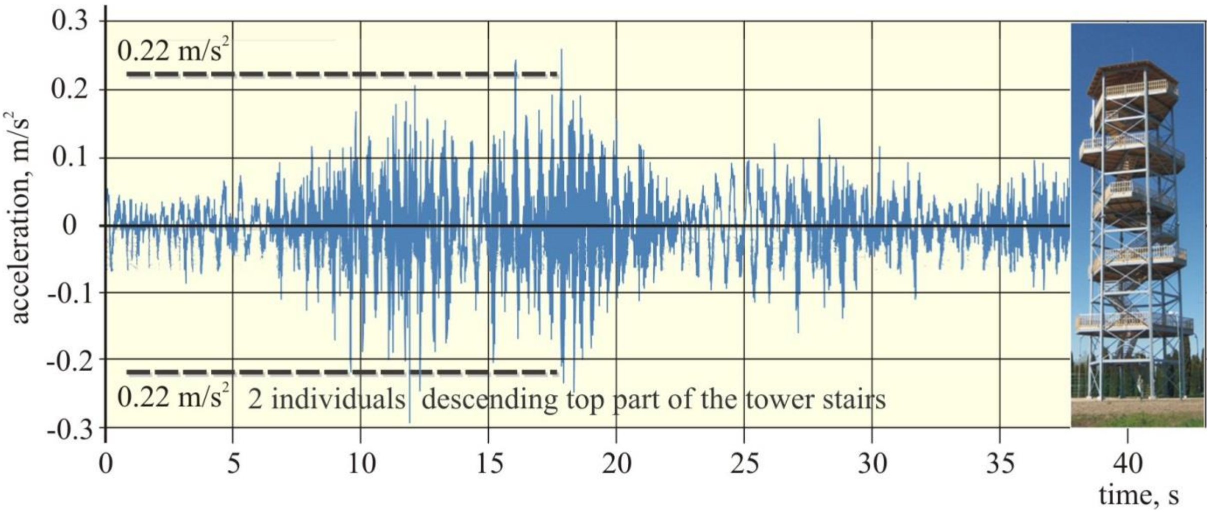


Figure 3.18.Tower response at different frequency ratios

Based on the comparison of theoretically and experimentally obtained peak acceleration amplitudes due to the human movement along the tower height, the most reasonable number of subsequent "successful footfalls" to be considered is four for the design calculations. See also the full example of other tower response calculations in the Section 3.2.4 of thesis.

3.2.2. Observation tower response to the group loading

The worst case of loading as determined from the experimental results earlier was the loading from group of visitors descending the stairs. In case of an observation tower, where the stairs usually aren't usually so wide, and even if only one person from the group of visitors accidentally or naturally synchronizes one of the force harmonics with the natural frequency of the structure, he/she causes the entire group to follow the same speed. The more compact a group is the bigger is the possibility that the individuals have the same pacing frequency. Because the stride length is determined by the tread of the stairs and therefore high degree of synchronization of pacing rate is inevitable. However, there will be discrepancy in the phase shift among individuals of the group.

If all individuals are walking with a perfectly synchronized pacing rate and half of the group has a phase shift difference of exactly 180^0, they would cancel each other out and, because of the force equilibrium there would be no action on the structure in the considered direction. Such an idealized situation has very low probability but it still highlights the fact that, due to the probabilistic nature of the phase shifts among individuals, some of the loading would be cancelled out. This correlates with results from the experimental investigations. Response of the structure would not have a linear relationship with an increasing number of individuals in the group.

Currently a widely recognized tool to take into account uncertainties for various aspects of analysis and design of structure is the Monte Carlo simulation (MCS) technique [107, 132]. Therefore MCS was utilized to simulate the stochastic property of the phase shift redistribution with a different number of individuals in the group in order to find the equivalent number of persons H_{eq} in the considered direction.

To find the equivalent number of persons H_{eq} in the group, whose one of the walking frequencies coincides with the natural frequency of the structure, random walking phase shift redistribution among individuals in the group was generated to simulate the stochastic property of the walking force phase shift. (Figure 3.19). Details and parameters of the performed simulations are as follows:

- the performed number of simulations: $n = 10^5$ for each of visitors group;
- group size varies from 2 to 20 persons;
- the phase shift of each individual in the group was a randomly generated number within the range of $0 < \varphi \leq 360$;
- software where written code for simulations: Mathcad 14.0.

The scheme of calculation for an 8 person group is presented in Figure 9, a. The obtained relationship between the actual number of persons *m* in the group and the equivalent number of persons H_{eq} may be approximated by a trend line (Table 3.6). It is valid for a group of 2 to 20 people (range of the simulations).

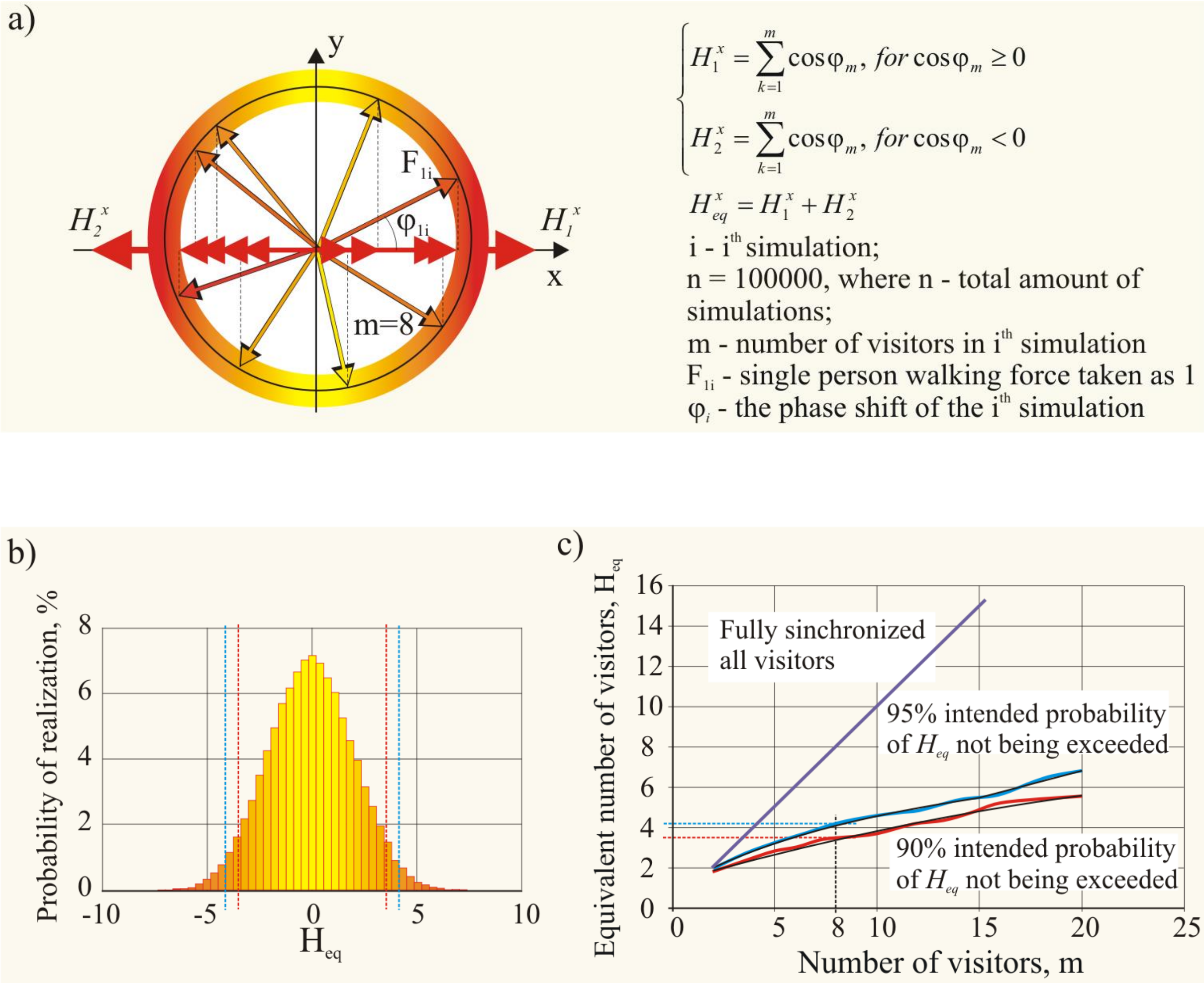


Figure 3.19.Simulation of the phase shift redistribution between individuals, a) scheme of the calculation for an 8 person group, b) the obtained density function of the equivalent number of persons in this group, c) relationship between the number of persons in the group and the equivalent number of visitors

Table 3.6.

Equivalent number of persons H_{eq} in a group of m persons

Intended probability of H_{eq} not being exceeded	Trend line	Coefficient of determination (R – squared value)
95%	$H_{eq} = 0.001m^3 - 0.0353m^2 + 0.6249m + 0.845$ (49)	0.9983
90%	$H_{eq} = -0.0035m^2 + 0.2831m + 1.3419$ (50)	0.9929

* - valid for a group of 2 to 20 people

There is a rather good agreement between the experimental data and the obtained function of the equivalent number of people H_{eq} in a group of m persons. For instance in

Figure 3.10 and Table 3.2 where the peak acceleration ratio between a group of 7 visitors and a group of two visitors descend the timber tower is r_a=0.25/0.15=1.7. But the theoretically obtained ratio of equivalent number of persons between a group of 7 visitors and a group of two (Figure 9, c) with the intended probability of 95% or 90% is $r_{95\%} = H^7_{eq}/H^2_{eq} = 3.83/1.96 = 1.95$ and $r_{90\%} = H^7_{eq}/H^2_{eq} = 3.15/1.89 = 1.67$ respectively. Comparison between theoretical and experimental results as well for other towers is presented in Table 3.7.

Table 3.7.

Comparison of theoretical and experimental results

Tower name	Sizes of groups considered	Theoretical results of ratio with intended probability		Experimental results of ratio, r_a	Difference, %	
		$r_{90\%}$	$r_{95\%}$		$\frac{r_{90\%} - r_a}{r_{90\%}} \cdot 100\%$	$\frac{r_{95\%} - r_a}{r_{95\%}} \cdot 100\%$
Ūdru k.	7 & 2	1.67	1.95	$0.25/0.15 = 1.7$	-1.8	12.8
Priedaine	3 & 2	1.24	1.14	$0.21/0.15 = 1.4$	-12.9	-22.8
Dzintari	11 & 7	1.28	1.25	$0.48/0.39 = 1.23$	3.9	1.6
Kalsnava	9&2	1.9	2.21	$0.19/0.1 = 1.9$	0	14.0

The structures of observation towers are essentially different (Ūdru and Priedaine towers have a timber structure, Dzintari and Kalsnava have a steel structure) but the theoretically obtained equivalent number of persons H_{eq} in the group agrees well with the experimental data. As expected, the more people in the group are considered, the better the agreement between the simulated and experimental results due to the narrower probability density function.

3.2.3. Methodology of tower response calculations

The developed algorithm of the methodology for calculation of maximum response of structure to typical human induced loads is based on the experimental and theoretical investigations presented in the previous sections of the thesis. The algorithm will be a useful tool for the structural engineers designing the lattice observation towers. For the first time there is a possibility to assess the actual maximum vibration acceleration levels produced by the movement of tower visitor group and compare them to the limiting acceleration value, therefore ensuring fulfillment of serviceability limit state requirements. During the design stage it allows to set a limit on the number of tower visitors that is justified by the calculations or alter the structural arrangement. The possibility to predict the vibration amplitudes of the lattice observation towers gives the designers confidence about the dynamic behavior of structure in service and therefore allows to design more interesting structures from an architectural point of view.

The verification of the developed methodology is performed by comparing the theoretically and experimentally obtained results of the structure maximum response and presented in the next section (3.2.4).

The following procedure can be used to determine whether the designed lattice observation tower fulfills the serviceability requirement: acceptable comfort level of structure users.

- **Step 1: To determine the input parameters**

Input parameters needed for calculation are as follows:

- Geometry of the structure and structural elements;
- Dynamic parameters of the observation tower: fundamental and natural frequencies, mode shapes, tower self-weight, stiffness and damping ratio;
- Mean weight of the visitors (recommended $G=746N$).

If the structure is generally symmetric and has close to uniform stiffness and mass distribution along its height, it is possible to use the analytical method presented in section 1.1.1.1 to determine the natural frequencies and mode shapes. Use of FEM model is always appropriate. Recommended damping ratios are $\xi = 3.5\%$ for timber towers and $\xi = 1.5\%$ for steel towers;

- Assumed number of the tower visitors in the group;
- Number of subsequent "successive steps" n. Generally, for the typical lattice observation towers, the recommended value of $n=4$. But for towers with frequency bellow 0.8Hz or specific character of the structure number of subsequent "successive steps" n could be increased.

- **Step 2: To determine the most possible design situations**
  - Response to the human movement induced loads should be assessed when the fundamental frequency of a lattice observation tower is less than 3.3 Hz and when self-supporting tower parameters are in the following range: stiffness $EI \leq 2 \cdot 10^6 L^{2.968}$ and mass per meter $m \leq 206241 \cdot L^{-1.032}$;

Increasing the mass over the denoted range human induced transversal force magnitude of 20 visitors (group size) applied at the tip of the tower will not be sufficient to set the structure in motion with critical accelerations (if provided that the structure are flexible enough to be moved at least for 2mm). Lattice structures with higher fundamental frequencies than 3.3 Hz are usually too stiff to be considerably excited by typical human induced dynamic walking loads.

  - Theoretical pacing frequency f_p when one of the five possible cases emerges should be found (here f_n is the fundamental frequency of the structure):

    • Case № 1: 1st harmonic of longitudinal force component coincides with the fundamental frequency f_n of structure (with recommended DLF value of 0.12), then:

$$f_p = f_n . \tag{51}$$

    • Case № 2: 2nd harmonic of longitudinal force component coincides with the fundamental frequency f_n of structure (with recommended DLF value of 0.11), then:

$$f_p = 0.5 f_n . \tag{52}$$

    • Case № 3: 1st harmonic of lateral force component coincides with the fundamental frequency $f_n$of structure (with recommended DLF value of 0.1), then;

$$f_p = 2 f_n . \tag{53}$$

    • Case № 4: 3rd harmonic of lateral force component coincides with the fundamental frequency f_n of structure (with recommended DLF value of 0.11), then:

$$f_p = \frac{2}{3} f_n . \tag{54}$$

    • Case № 5: 5th harmonic of lateral force component coincides with the fundamental frequency f_n of structure (with recommended DLF value of 0.08), then:

$$f_p = \frac{2}{5} f_n \,. \quad (55)$$

- Further analysis should be performed for the cases when pacing frequency is in the following range:

$$1.0 \le f_p \le 2.3 \quad (56)$$

- **Step 3: Determine the equivalent number of persons H_{eq}**

The highest response of the tower can be reached when a compact group of visitors moves along the height of the structure. To find the equivalent number of persons H_{eq} in the group, whose relevant walking harmonic frequency is close to the natural frequency of the structure equations (49) and (50) might be used (Table 3.6).

- **Step 4: Maximum dynamic force and its application to structure**

Maximum force F_i from the selected number of tower visitors for harmonic i and relevant design situation should be applied horizontally at the last stair flight level and is following:

$$F_i = H_{eq} \cdot G \cdot DLF^i_{lat\ or\ long} \sin(2\pi f^i_{caseNr} t) \,. \quad (57)$$

where

f^i_{caseNr} -frequency of the walking force harmonic for design situation under consideration, Hz;

$DLF^i_{lat\ or\ long}$- dynamic loading factor of i^{th} harmonic for lateral or longitudinal walking force component;

G - mean weight of the visitor, N.

Load application duration t depends on the selected number of subsequent "successive steps" n and is defined as follows:

$$t = \frac{n}{f_{caseNr}} = \frac{2\pi n}{\Omega_{caseNr}} \quad (58)$$

where

Ω_{caseNr} - angular frequency of the walking force harmonic for design situation under consideration, rad/s.

For example, if in the *Step 2* it is determined that case № 3 is critical, load at the last stair flight level should be applied with frequency of $f_{caseNr}^{i} = f_{caseNR3}^{1stlat} = 0.5f_p = f_n$ for the 1st harmonic of lateral walking force (then 1st harmonic of lateral force component coincides with the fundamental frequency f_n of structure). Simultaneously other four harmonics with relevant frequency according to Table 3.3 should be applied when fundamental frequency of tower is below 2 Hz.

- **Step 5: Determine peak acceleration from the applied dynamic loading**
  - The peak accelerations for the ratio between relevant walking harmonic angular frequency and the fundamental angular frequency Ω/ω_n should be determined in the ratio range of $0.8 \leq \Omega/\omega_n \leq 1.2$ to take into account a stochastic nature of the loading. If the structure is generally symmetric and has a uniform stiffness and mass distribution along its height, it is possible to use the analytical method presented in section 3.2 to determine peak responses. The use of FEM calculations is always appropriate.
  - The mean value of peak acceleration a_{mean} of ratio $z=\Omega/\omega_n$ range $0.8 \leq \Omega/\varpi_n \leq 1.2$ can be calculated according to the equation (48).

- **Step 6: Limiting the peak acceleration**

The vibration response can be considered satisfactory when the obtained mean peak acceleration multiplied by the weighting factor W_d does not exceed a limiting value of $0.2 m/s^2$. Weighting factor W_d is defined in (34).

A limiting value of $0.2 m/s^2$ is only the recommended value (majority of people will perceive motion). It can be further modified by factors that take into account the type and location of the particular structure, the required comfort level or other factors.

3.2.4. Calculation examples of observation tower peak response to human–induced loads

As a verification of developed methodology and also practical guide for the peak acceleration calculation of lattice observation tower response to human induced loads in this section the analytical calculation example (Example 1) and numerical calculation example (Example 2) are presented. The further calculated peak accelerations compared to experimentally obtained accelerations in the case of 7 visitors descending or ascending the tower stairs.

Example 1.

Ūdru kalna observation tower is a 4-legged self-supporting timber tower in good technical condition (no visible damages) and randomly selected for the analytical calculation. Stairs are located in the center of the tower therefore mostly the bending mode shapes will be excited. The geometrical parameters are presented in the Table 3.1 but experimentally obtained acceleration history is presented in Figure 3.10. Basic data for the calculation is presented in Table3.8.

Tower structure is centric therefore mode shapes are well separated. According to the investigations of thesis the first bending mode shape is the critical one and according to Table 3.3, ideally there are five possible cases of walking dynamic loading on stairs of observation towers that should be considered (see page 97). But realization of some of them particularly for this tower is highly unlikely. Therefore, the possible design situations are analyzed in Table 3.9 by finding the pacing frequency when one of the walking harmonics looked at, coincides with the fundamental frequency of the tower.

Table 3.8.

Basic data of Ūdru kalns tower

Parameter description	Value		Notes
Number of tower visitors	7	pcs	
Action	descending	-	
Fundamental frequency	1.35	Hz	From experiment
Peak acceleration	0.25	m/s^2	From experiment
Height of the tower (to the upper platform), L	26.5	m	
Plan dimension at the upper platform level	2.5	m	
Plan dimension on the ground	6.9	m	
Diameter of the column (at ground level)	0.35	m	
Diameter of the column (at top)	0.22	m	
Damping ratio	3.85	%	From experiment
Mean weight of visitors, G	746	N	Assumed
Weight of the tower	1050	kg/m	Calculated

By comparing fundamental frequency of the tower (1.35Hz) with the mean walking frequency of 1.416 Hz ("long stair" case), follows that most likely the case № 1 when 1st harmonic of longitudinal force component coincides with the fundamental frequency of the structure will realize.

Relevant walking force dynamic load factors (DLF) for stair descent are according to Table 2.4.

Table 3.9.

Loading situation analysis

Case	Walking harmonic	Pacing frequency, Hz*	Relevant DLF	Notes
№ 1	**1st longitudinal**	$f_p = 1.35$	**0.07**	**Should be checked**
№ 2	2nd longitudinal	$f_p = 1.35 \cdot 0.5 = 0.675$	0.1	Small probability (pacing too slow)
№ 3	1st lateral	$f_p = 1.35 \cdot 2 = 2.7$	0.08	Small probability (pacing too fast)
№ 4	3rd lateral	$f_p = 1.35 \cdot \frac{2}{3} = 0.9$	0.11	Small probability (pacing too slow)
№ 5	5th lateral	$f_p = 1.35 \cdot \frac{2}{5} = 0.54$	0.07	Small probability (pacing too slow)

* - Pacing frequency when relevant walking harmonic coincides with fundamental frequency of the tower;

Next step is to find the equivalent number of persons H_{eq} in the group, whose 1st longitudinal walking harmonic frequency is close to the natural frequency of structure (equations (49) and (50)).

$$H_{eq}^{95\%} = 0.001m^3 - 0.0353m^2 + 0.6249m + 0.845 = \\ = 0.001 \cdot 7^3 - 0.0353 \cdot 7^2 + 0.6249 \cdot 7 + 0.845 = 3.83;$$

$$H_{eq}^{90\%} = -0.0035m^2 + 0.2831m + 1.3419 = -0.0035 \cdot 7^2 + 0.2831 \cdot 7 + 1.3419 = 3.15.$$

Thus maximum force from 7 people group with intended probability of 90% not being exceeded is:

$$F_{90\%} = H_{eq}^{90\%} \cdot G \cdot DLF_{long}^{1} = 3.15 \cdot 746 \cdot 0.07 = 164.5N.$$

And maximum force from 7 people group with intended probability of 95% not being exceeded is:

$$F_{95\%} = H_{eq}^{95\%} \cdot G \cdot DLF_{long}^{1} = 3.83 \cdot 746 \cdot 0.07 = 200N \,.$$

To take into account possible variation in the ratio between the relevant walking harmonic and the excited natural frequency of the structure, as it is suggested in the section 3.2.1 the mean value of the tower response for the selected number of steps in the ratio range of $1 \pm 20\%$ should be calculated. Each acceleration peak response for relevant ratio between walking harmonic and the excited angular natural frequency Ω / ϖ_n calculated according to methodology given in the section 3.2. Example of calculation for ratio $\Omega / \varpi_n = 1$ is as follows:

1. Angular fundamental frequency:

$$\varpi_n = 2\pi f_n = 2 \cdot \pi \cdot 1.35 = 8.48\, rad / s \,;$$

2. Pacing frequency:

$$\Omega = 1 \cdot \varpi_n = 8.48\, rad / s \,;$$

3. Load applied at tower height of 26.5m, thus function of time $Q(t)$ maximum value is calculated by the equation (37):

$$Q_{\max} = \frac{F_{\max}^{95\%} \cdot \Phi_1(L)}{\int_0^L \Phi_1^2(y)dy} = \frac{200 \cdot 2}{26.5} = 15.095N \,,$$

where $F_{\max}$ - maximum force applied, N and first mode shape defined as (2) :

$$\Phi_1(y) = \cosh(a_1 y) - \cos(a_1 y) - \delta_1(\sinh(a_1 y) - \sin(a_1 y)) \,,$$

where according to Table 1.1

$$a_1 = \sqrt{\frac{c_1}{L^2}} = \sqrt{\frac{3.516}{26.5^2}} = 0.073 \text{ and } \delta_1 = 0.734096 \,.$$

4. To calculate modal participation factor $\eta(t)$ the following values should be found according to equations (41) and (44):

$$\overline{\omega}_n = \omega_n \sqrt{1 - \xi_1^{\,2}} = 8.48\sqrt{1 - 0.0385^2} = 8.476 \,;$$

$$\begin{cases} p_1 = \overline{\omega}_i^{\,2} + \xi_i^2 - \Omega^2 \\ p_2 = \Omega^2 + \xi_i^2 - \overline{\omega}_i^{\,2} \\ D = \left(\xi_i^2 + \Omega^2 + \overline{\omega}_i^{\,2}\right)^2 - 4\Omega^2 \overline{\omega}_i^{\,2} \end{cases}$$

$$\begin{cases} p_1 = 8.476^2 + 0.0385^2 - 8.48^2 \\ p_2 = 8.48^2 + 0.0385^2 - 8.476^2 \\ D = (0.0385^2 + 8.48^2 + 8.476)^2 - 4 \cdot 8.48^2 \cdot 8.476^2 \end{cases} = \begin{cases} -0.105 \\ 0.108 \\ 0.438 \end{cases}.$$

5. Next modal participation factor $\eta(t)$ calculated according to the equation (43). Then the acceleration history in time *t* can be obtained and the peak value found:

$$a_{\max} = \ddot{x}(y,t) = \frac{\delta^2 x(y,t)}{\delta t^2}$$

where $x(y,t)$ - maximum displacement according to equation (45);

$t = \dfrac{2\pi n}{\varpi_n} = \dfrac{2\pi \cdot 4}{8.48} = 2.96\,s$ - time after *n* subsequent "successive" steps;

$y = L$ - height above the ground level considered.

The calculated values for rest of the ratios Ω / ϖ_n are presented in Tables 3.10 and 3.11.

Table 3.10.

Peak acceleration with intended probability of 95% not being exceeded from 1st longitudinal walking force harmonic

№	Loading frequency (f_p), Hz	Ratio of angular frequencies (Ω / ϖ_n)	Peak acceleration at time interval t ($a_{\max}(t)$), m/s^2	Time after *n* subsequent "successive" steps ($t = \frac{2\pi n}{\Omega}$), s
1.	1.08	0.8	0.11	3.70
2.	1.15	0.85	0.156	3.48
3.	1.215	0.9	0.237	3.29
4.	1.28	0.95	0.311	3.12
5.	1.31	0.97	0.329	3.055
6.	1.34	0.99	0.340	2.99
7.	**1.35**	**1**	**0.341**	**2.96**
8.	1.38	1.02	0.32	2.905
9.	1.42	1.05	0.295	2.82
10.	1.485	1.1	0.261	2.69
11.	1.55	1.15	0.21	2.58
12.	1.62	1.2	0.165	2.47

The relevant peak acceleration mean value in the ratio range of $0.8 \leq \Omega / \varpi_n \leq 1.2$ can be calculated according the equation (48):

$$a_{mean}^{long.1st} = 2{,}5\int_{0{,}8}^{1{,}2} a_{\max}(z) d(z) = 0.2402\, m/s^2,$$

where

$z = \dfrac{\Omega}{\varpi_n}$ - the ratio between relevant walking harmonic and the excited frequency;

$a_{\max}(z)$ - peak acceleration at time t, m/s^2.

At the same time the effect of walking force second harmonic should be calculated as it acts simultaneously and has similar phase. According to the section 3.2.1 the rest of the walking harmonics with similar phases can be ignored only if the fundamental of the structure is more than 2 Hz. Then the maximum force from 7 people group with intended probability of 90% not being exceeded is:

$$F_{90\%} = H_{eq}^{90\%} \cdot G \cdot DLF_{long}^{2} = 3.15 \cdot 746 \cdot 0.1 = 235 N.$$

And maximum force from 7 people group with intended probability of 95% not being exceeded is:

$$F_{95\%} = H_{eq}^{95\%} \cdot G \cdot DLF_{long}^{2} = 3.83 \cdot 746 \cdot 0.1 = 285.7 N.$$

Table 3.11.

Peak acceleration with intended probability of 95% not being exceeded from 2nd longitudinal walking force harmonic

№	Loading frequency ($2 f_p$), Hz	Ratio of angular frequencies ($2\Omega / \varpi_n$)	Peak acceleration at time interval t ($a_{\max}(t)$), m/s^2	Time after n subsequent “successive” steps ($t = \frac{2\pi n}{\Omega}$), s
1.	2.16	1.6	0.112	3.70
2.	2.295	1.7	0.102	3.48
3.	2.43	1.8	0.093	3.29
4.	2.565	1.9	0.088	3.12
5.	2.62	1.94	0.085	3.055
6.	2.65	1.98	0.082	2.99
7.	**2.7**	**2**	**0.077**	**2.96**

8.	2.75	2.04	0.079	2.905
9.	2.84	2.1	0.079	2.82
10.	2.97	2.2	0.077	2.69
11.	3.10	2.3	0.073	2.58
12.	3.24	2.4	0.072	2.47

The relevant peak acceleration mean value in the ratio range of $0.8 \leq \Omega / \varpi_n \leq 1.2$ can be calculated according the equation (48):

$$a_{mean}^{long.2nd} = 2{,}5 \int_{0{,}8}^{1{,}2} a_{\max}(z) d(z) = 0.0875\, m/s^2,$$

where

$z = \dfrac{\Omega}{\varpi_n}$ - the ratio between relevant walking harmonic and the excited frequency;

$a_{\max}(z)$ - peak acceleration at time *t*, m/s^2.

Therefore calculated peak acceleration with intended probability of 95% not being exceeded is:

$$a_{mean}^{95\%} = a_{mean}^{long.1st} + a_{mean}^{long.2nd} = 0.2402 + 0.0875 = 0.33\, m/s^2.$$

Ratio between the equivalent number of people with 95% and 90% with intended probability of 95% not being exceeded is:

$$r = \frac{H_{eq}^{95\%}}{H_{eq}^{90\%}} = \frac{3.83}{3.15} = 1.22\,.$$

Thus the calculated peak acceleration with intended probability of 90% not being exceeded is:

$$a_{mean}^{90\%} = \frac{a_{mean}^{95\%}}{r} = \frac{0.33}{1.22} = 0.27\, m/s^2.$$

Comparison of the obtained results is summarized in the Table 3.12.

Table 3.12.

Comparison of theoretically and experimentally obtained results for Ūdru kalns tower

№	Calculated peak acceleration	Experimental peak acceleration	$\frac{a_{\exp} - a}{a_{\exp}} \cdot 100\%$
1.	$a_{mean}^{95\%} = 0.33 m/s^2$	$a_{\exp} = 0.25 m/s^2$	24.2%
2.	$a_{mean}^{90\%} = 0.27 m/s^2$		7.4%

Despite simplifications and assumptions made to perform analytical calculations there is a good agreement with the experimental results. Because, by lowering the limit of the intended probability not being exceeded, an experimental value can be reached easily.

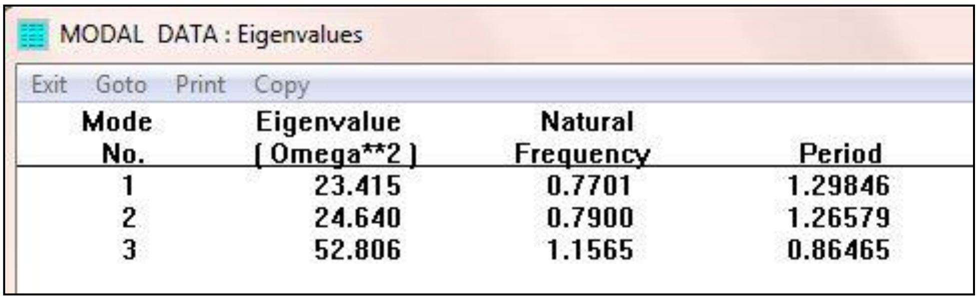
The vibration response can be considered satisfactory when the peak acceleration multiplied by the weighting factor W_d does not exceed a limiting value of 0.2m/s^2. In this case weighting factor W_d=1 according to the (34), therefore peak value of acceleration is more than 0.2m/s^2 and majority of people will perceive motion. Some of the visitors will feel uncomfortable and long-term exposure may produce motion sickness.

Example 2.

For the numerical calculation, selected eccentric steel tower located in Dzintari (see Figure 1.1). Experimental results of this tower can be found in section 3.1.2.

Stairs are located around the central stiffness element of the tower and therefore it can be excited in bending and also torsional mode shapes. The geometrical parameters are presented in Table 3.1. Basic data for the calculation is presented in Table 3.13.

First the finite element model of the observation tower should be constructed taking into account the recommendations given in the section 1.1.1.1. Special attention should be paid to the mass and stiffness properties of the tower. In this case the natural frequencies of the structure obtained experimentally and finite element model is adjusted (by reducing the stiffness of structure due to the foundation and connection flexibility) to make experimentally and theoretically obtained results comparable. The calculated natural frequencies of the first three mode shapes (first coupled torsional and flexural, first flexural and first torsional) presented in the Figure 3.20 accordingly. Mode shapes and finite element model are presented in Figure 3.11.


MODAL DATA : Eigenvalues

Exit Goto Print Copy

Mode No.	Eigenvalue [Omega**2]	Natural Frequency	Period
1	23.415	0.7701	1.29846
2	24.640	0.7900	1.26579
3	52.806	1.1565	0.86465

Figure 3.20.Numerically calculated natural frequencies of the observation tower in Dzintari

Table 3.13.

Basic data of eccentric observation tower in Dzintari

Parameter description	Value		Notes
Number of tower visitors	7	pcs	
Action	ascending	-	
Fundamental frequency	0.76	Hz	From experiment (torsion + bending)
Peak acceleration	0.39	m/s^2	From experiment
Height of the tower (to the upper platform), L	33.5	m	
For plan dimensions see Figure 3.12; for element description see page 86.			
Damping ratio	2.3	%	From experiment
Mean weight of visitors, G	746	N	Assumed
Weight of the tower	720	kg/m	Calculated

Tower structure is not centric therefore mode shapes are not well separated. According to Table 3.3, ideally there are five possible cases of walking dynamic loading on stairs of observation towers that should be considered (see page 97). The possible design situations are analyzed in Table 3.14 by finding the pacing frequency when one of the walking harmonics looked at, coincides with the fundamental frequency of the tower. Relevant walking force dynamic load factors (DLF) for stair ascent are according to Table 2.4.

Analysis reveals that most likely the case № 3 when 1st harmonic of lateral force component coincides with the fundamental frequency of structure will realize. This can also be seen from the experimental results in Table 3.5.

Table 3.14.

Loading situation analysis

Case	Walking harmonic	Pacing frequency , Hz*	Relevant DLF	Notes
№ 1	1st longitudinal	$f_p = 0.76$	0.12	Small probability (pacing too slow)
№ 2	2nd longitudinal	$f_p = 0.76 \cdot 0.5 = 0.38$	0.11	Almost impossible (pacing too slow)
№ 3	**1st lateral**	$f_p = 0.76 \cdot 2 = 1.52$	**0.1**	**Should be checked**
№ 4	3rd lateral	$f_p = 0.76 \cdot \frac{2}{3} = 0.51$	0.11	Small probability (pacing too slow)
№ 5	5th lateral	$f_p = 0.76 \cdot \frac{2}{5} = 0.3$	0.08	Almost impossible (pacing too slow)

* - Pacing frequency when relevant walking harmonic coincides with fundamental frequency of the tower;

Due to the specific character of the structure and fundamental frequency well below 2 Hz the rest of the harmonics also should be applied to the finite element model simultaneously. For this tower it is suggested to take 4 subsequent “successive” steps (one stair flight).

To take into account possible variation in the ratio between the relevant walking harmonic and the excited natural frequency of the structure, as it is suggested in section 3.2.1 the mean value of the tower response for the selected number of steps in the ratio range of $1 \pm 20\%$ should be calculated. Each acceleration peak response for relevant ratio between walking harmonic and the excited angular natural frequency Ω / ϖ_n calculated at the time after n subsequent “successive” steps.

The advantage of finite element calculation is that all relevant walking harmonics can be applied at once. Maximum force of each harmonic from a group of 7 people with intended probability of 95% not being exceeded applied to the last flight of stairs and is as follows:

$$F^1_{95\%} = H^{95\%}_{eq} \cdot G \cdot DLF^1_{long} = 3.83 \cdot 746 \cdot 0.12 = 342.9N\,;$$

$$F^2_{95\%} = H^{95\%}_{eq} \cdot G \cdot DLF^2_{long} = 3.83 \cdot 746 \cdot 0.11 = 314.3N\,;$$

$$F^1_{95\%} = H^{95\%}_{eq} \cdot G \cdot DLF^1_{lat} = 3.83 \cdot 746 \cdot 0.1 = 285.7N\,;$$

$$F^3_{95\%} = H^{95\%}_{eq} \cdot G \cdot DLF^3_{lat} = 3.83 \cdot 746 \cdot 0.11 = 314.3N\ ;$$

$$F^5_{95\%} = H^{95\%}_{eq} \cdot G \cdot DLF^5_{lat} = 3.83 \cdot 746 \cdot 0.08 = 228.6N\ ,$$

where $H^{95\%}_{eq}$ and $H^{90\%}_{eq}$ the same as in Example 1.

Each load $F^1_{95\%}$ is applied horizontally in the appropriate direction at the level of last stair flight. The $F^1_{95\%}$ changes according to the sine wave: $\sin(2\pi f^{\,i}_{case\,Nr\,3} t)$.

The example of obtained acceleration history of the ratio $\Omega^1_{lat} / \varpi_n = 0.8$ is presented in Figure 3.21. The peak value should be found at time interval t:

$$t = \frac{n}{f_n} = \frac{4}{0.616} = 6.492\,s$$ - time after n subsequent "successive" steps.

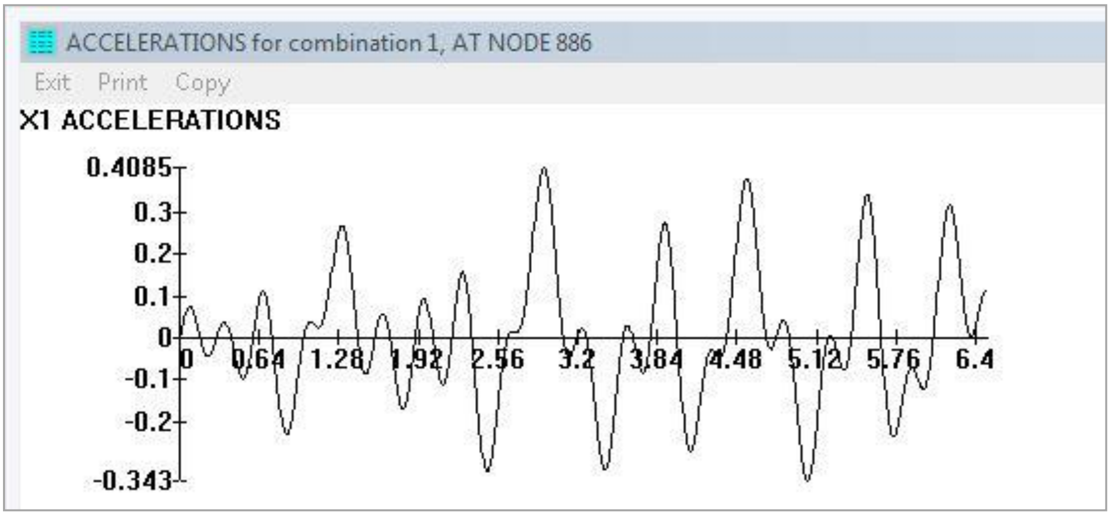


Figure 3.21.Numerically calculated acceleration history of the ratio $\Omega^1_{lat} / \varpi_n = 0.8$

Calculated values for rest of the ratios $\Omega^1_{lat} / \varpi_n$ are presented in Table 3.15

Table 3.15.

Peak acceleration with intended probability of 95% not being exceeded from all five walking force harmonics (Case № 3)

№	Loading frequency ($f_p/2$)*, Hz	Ratio of angular frequencies (Ω^1_{lat}/ϖ_n)	Peak acceleration at time interval t ($a_{mean}(t)$), m/s^2	Time after *n* subsequent "successive" steps ($t=\frac{2\pi n}{\Omega^1_{lat}}$), s
1.	0.616	0.8	0.4085	6.492
2.	0.654	0.85	0.3851	6.110
3.	0.693	0.9	0.384	5.771
4.	0.731	0.95	0.3895	5.467
5.	0.747	0.97	0.3802	5.354
6.	0.762	0.99	0.3919	5.246
7.	**0.770**	**1**	**0.3978**	**5.194**
8.	0.786	1.02	0.414	5.092
9.	0.809	1.05	0.4374	4.947
10.	0.847	1.1	0.3905	4.722
11.	0.886	1.15	0.289	4.516
12.	0.924	1.2	0.2754	4.328

* - only for the 1st lateral walking harmonic; frequencies of other harmonics was according to Table 3.3.

Then the peak acceleration mean value in the ratio range of $0.8 \le \Omega/\varpi_n \le 1.2$ with intended probability of 95% not being exceeded can be calculated according the equation (48):

$$a_{mean} = 2{,}5\int_{0,8}^{1,2} a_{\max}(z)d(z) = 0.39\, m/s^2,$$

where

$z = \frac{\Omega}{\varpi_n}$ - the ratio between relevant walking harmonic and the excited frequency;

$a_{\max}(z)$ - peak acceleration at time *t*, m/s^2.

Ratio between the equivalent number of persons with intended probability 95% and 90% not being exceeded is:

$$r = \frac{H_{eq}^{95\%}}{H_{eq}^{90\%}} = \frac{3.83}{3.15} = 1.216\,.$$

Thus calculated peak acceleration with intended probability of 90% not being exceeded is:

$$a_{mean}^{90\%} = \frac{a_{mean}^{95\%}}{r} = \frac{0.39}{1.216} = 0.32\, m/s^2\,.$$

Comparison of the obtained results is summarized in the Table 3.16.

Table 3.16.

Comparison of theoretically and experimentally obtained results for tower in Dzintari

№	Calculated peak acceleration	Experimental peak acceleration	$\frac{a_{\exp} - a}{a_{\exp}} \cdot 100\%$
1.	$a_{mean}^{95\%} = 0.39\, m/s^2$	$a_{\exp} = 0.39 m/s^2$	0.0%
2.	$a_{mean}^{90\%} = 0.32\, m/s^2$		-21.8%

In this case the agreement with the experimental result is exact if the 95% of intended probability not being exceeded is considered. It is explicable with the fact that during experiment (described in section 3.1.2.) test persons were asked to try to intentionally synchronize their pacing frequency between each other.

The vibration response can be considered as satisfactory when the peak acceleration multiplied by the weighting factor W_d does not exceed a limiting value of 0.2m/s^2. In this case the weighting factor W_d=1 according to the (34), therefore peak value of acceleration is considerably more than 0.2m/s^2 and people will perceive motion strongly, therefore most of them will feel uncomfortable. The complaints from visitors about the tower's excessive vibrations since the tower was opened, confirm it.

CONCLUSIONS

Within the thesis the experimentally obtained human movement induced dynamic loads are studied and based on that the calculation methodology for assessment of light-weight lattice self-supporting tower type structure dynamic response to those typical human induced dynamic loads is developed as well as a limit is set on the observation tower vibration acceleration amplitudes due to the comfort criteria of tower visitors.

As the final summary of the present study the following conclusions are drawn:

- The proposed method of obtaining dynamic forces from human movement, based on kinematics of the motion of human center of gravity (COG), eliminates one of the most important drawbacks of the traditional methods – an influence on human ability to move naturally that has significant effect on character and magnitudes of human induced dynamic forces;
- The proposed experimental data processing method for obtaining the mean continuous walking force histories, based on the averaging between continuous walking histories itself, allows to find not only the mean dynamic load factors (DLF) but also the phase shift values associated with the mean walking force history and therefore allows to obtain analytical expression of human induced dynamic force components;
- For the first time experimentally obtained the mean dynamic load factors and corresponding phase shifts of a person while stair ascent or descent at different pacing frequencies (for Table A.4. of International Standard ISO 10137:2007) reveals that lateral and longitudinal direction force amplitudes strongly depend on the walking pace only in case of the stair ascent. In the case of stair descent, DLF values can be considered as constant when the pacing rate is changed. Descent of stairs creates 14% higher vertical amplitudes than ascent. In order to approximate replication of the real walking force time history for vertical, longitudinal and lateral directions a minimum of three harmonics must be used: 1st, 2nd and 3rd for the vertical and longitudinal direction and 1st, 3rd and 5th for the lateral direction;

- The vertical light-weight cantilever type structures such as public observation towers with fundamental frequency less than 3.3 Hz, stiffness of structure less than $EI \leq 2 \cdot 10^{6} L^{2.968}$ and self-weight less than $m \leq 206241 \cdot L^{-1.032}$ may undergo vibrations induced by human activities that do not satisfy the serviceability limit criteria – required comfort criteria during the structure exploitation. The lattice tower does not reach the steady state vibrations due to inconsistent periodicity of the applied loading and its changing direction. The highest response of the tower can be reached when a compact group of visitors descend the structure. The occurrence, when one of the walking harmonics is close to the natural frequency of the structure, has a stochastic nature. The stochastic nature of the group loading might be taken into account with theoretically obtained equation of a non-linear relationship between the real number of visitors in the group and the idealized equivalent number of persons. The separate walking harmonic input to the total vibration of the structure depends on the fundamental frequency of the structure. For structures with fundamental frequency above 2 Hz only one harmonic input is dominant;
- The proposed preliminary curve - the peak acceleration limit ensuring maximum comfort level of visitors, based on the visitor's subjective assessment of vibration level of all inspected towers and other researchers' findings about human tolerance to vibrations reveal that allowable acceleration limit is around 3.3 times higher than the one of high-rise office buildings where such limitation is set in the building codes;
- The developed algorithm based on the performed experimental and theoretical investigations allows to calculate the maximum dynamic response of structure to typical human induced loads. Depending on the degree of pacing synchronization among visitors agreement between theoretically and experimentally obtained results of the structures maximum response to the stochastic time varying human induced loads are in the range of 0 to 25% which serves as a verification of the developed methodology.

LIST OF REFERENCES

1. Alexander R.M. Principles of animal locomotion.- Princeton University Press: Princeton, New Jersey, USA, 2003.
2. Amiri G., Massah S. R. Seismic response of 4-legged self – supporting telecommunication towers // IJE Transactions B: Aplications, 2002, Vol. 20, No. 2, August 2007.
3. Amin. T., Hatzinakos D. Determinants in human gait recognition // Proceedings from 25th IEEE Canadian Conference on Electrical and Computer Engineering: Vision for a Greener Future, CCECE 2012, Montreal, 2012, pp. 1-4.
4. Appendix C, Vibration Serviceability Requirements for Foot Cycle Track Bridges / British Standards Institution. London: BSI, 1978. BS 5400, Part 2.
5. Augusti G., Bartoli G., Borri C., Gusella V., Spinelli P. Wind load and response of broadcasting antennas: Three years of research work in cooperation with RAI // Journal of Wind Engineering and Industrial Aerodynamics, 1992, Vol. 43, Part 3, pp. 2077-2088.
6. Australian Standard. AS 3995 – 1994: Design of Steel Lattice Towers and Masts, Standards Australia, Sydney, Australia.
7. Ayyappa E. Normal human locomotion, part 1: basic concepts and terminology // Journal of Prosthetics and Orthotics, 1997, Vol. 9, pp. 1-10.
8. Bachman H., Ammann W. Vibration in Structures Induced by Man and Machines // Structural Engineering Document 3e, International Association for Bridge and Structural Engineering (IABSE), Ch. 2: Man Induced Vibrations, Appendix A: Case Reports, 1987.
9. Bachmann H., Vibration Problems in Structures, Birkhauser, Basel: Boston, Berlin, 1995.
10. Barker C. Some observations on the nature of the mechanism that drives the self – excited lateral response of footbridges // Proceedings of the First International Conference Footbridge 2002. Paris, 20-22 November, 2002.
11. Bartlett R. Introduction to Sports Biomechanics. Great Britain: Alden Press, 1977, 289 p.

12. Baumann K., Bachmann H. Dynamic Loads Caused by Humans and their Effect on Beam Structures // Institute of Structural Engineering (IBK), Swiss Federal Institute of Technology (ETH), Zurich, Switzerland, 1988.

13. Beely F., Zur Mechanik des Stehens. Uber die Bedeutung des Fussgewolbes beim Stehen // Langenbecks Archiv fur Klinische Chirurgie, 1882, 27, pp. 457.

14. Bishop N.W.M., Willford M., Pumphrey R. Human induced loading of flexible staircases // Safety Science, 1995, Vol. 18, pp. 261–276.

15. Blaise N., Denoël V. Principal static wind loads // Journal of Wind Engineering and Industrial Aerodynamics, 2013, vol. 113, pp. 29-39.

16. Blanchard J., Davies B.L., Smith J.W. Design criteria and analysis for dynamic loading of footbridges // Proceedings of the DOE and DOTTRRL Symposium on Dynamic Behaviour of Bridges, Crowthorne, UK, May 1977.

17. Bocian M., Macdonald J. H. G., Burn J. F., Redmill D. A novel experimental setup for the identification of human actions on lateral oscillating structures // Proceedings of 11th International Conference on Recent Advances of Structural Dynamics, Pisa, 1-3 July, 2013.

18. Bougard A. Human loading on staircases // Proceedings of the Institution of Civil Engineers, Structures & Buildings, 2002, 152, No 4, pp. 371–380.

19. Brownjohn J., Fok P., Roche M., Omenzetter P. Long span steel pedestrian bridge at Singapore Changi airport – part 2: crowd loading tests and vibration mitigation measures, Crowd dynamic loading on footbridges // The Structural Engineer, 2004, 82 (16), pp. 28-34.

20. Brownjohn J.M.W., Pavic A., Omenzetter P. A spectral density approach for modeling continuous vertical forces on pedestrian structures due to walking // Canadian Journal of Civil Engineering, 2004, Vol. 31, No. 1, pp. 65-77.

21. Buchholdt H. A., Nejad S. E. M. Structural Dynamics for Engineers. London: ICE Publishing, 2012.

22. Butz C., Feldmann M. and et.al. Advanced load models for synchronous pedestrian excitation and optimized design guidelines for steel footbridges / European Commission, Research Fund for Coal and Steel, Tech. Rep, European Communities, 2008.

23. Cappozzo A., Leardini A., et.al. Human movement analysis using stereo-photogrammetry. Part 1: theoretical background // Gait & Posture, 2005, Vol. 21, pp. 186-196.

24. Catbas F. C., Gul M., Sazak H. O. Dynamic Testing and Analysis of a Football Stadium // Dynamics of Civil Structures, Proceedings of the IMAC - XXVIII. Florida USA, Jacksonville, 2010. Vol. 4, pp. 195-203.

25. Charles, P.; Bui, V. Transversal dynamic actions of pedestrians & Synchronisation // Proceedings of Footbridge 2005 – 2nd International Conference, Venice, 2005.

26. Clarke T.E., Frederick E.C., Cooper L.B. Effect of shoe cushioning upon ground reaction forces in running // International Journal of Sports Medicine, 1983 (4), pp. 247–251.

27. Cunha A., Caetano E., et.al. The role of dynamic testing in design, construction and long – term monitoring of lively footbridges // Proceedings of 3rd International Footbridge Conference, Portugal, Porto, 2008.

28. Dallard P., Fitzpatric T., Flint A. The London Millennium footbridge // The Structural Engineer, 2001 (79), pp. 17–33.

29. Davis B., Murray T M. Slender Monumental Stair Vibration Serviceability // Journal of Architectural Engineering, 2009, December, pp. 111–121.

30. Design of Footbridges. Background Document. Human induced Vibrations of Steel Structures. Report RFS2-CT-2007-00033, 2007.

31. DeWitt J.K., Cromwell R.L., Hagan R.D. The effect of manipulating subject mass on ground reaction force during locomotion // American Society of Biomechanics Annual Meeting, Blacksburg, USA, September 2006.

32. Dingwell J.B., Davis B.L. A rehabilitation treadmill with software for providing real-time gait analysis and visual feedback // Journal of Biomechanical Engineering, 1996 (118), pp. 253–255.

33. Donelan J.M., Kram R., Kuo A.D. Mechanical and metabolic determinants of the preferred step width in human walking // Proceedings of the Royal Society B, 2001 (268), pp. 1985–1992.

34. Dumas R, Cheze L., Verriest J. P. Adjusments to McConville et. al. and Young et.al. body segment inertial parameters // Journal of Biomechanics, 2006, ol. 40, pp. 543-553.

35. Dynamic performance requirements for permanent grandstands subject to crowd action: Recommendations for management, design and assessment/ Dougill J. W., Blakeborough A., Cooper P., et.al. London: Institution of Structural Engineers, 2008, pp. 54.

36. Edington K.A., Guler H. C., et.al. Instrumented treadmills: reducing the need for gait labs, USA, 2008.

37. Eriksson P.E. Vibration of Low-frequency Floors - Dynamic Forces and Response Prediction. PhD Thesis. Unit for Dynamics in Design, Chalmers University of Technology, Göteborg, Sweden, 1994.

38. Eurocode 0. Eurocode - Basis of Structural Design. EN 1990.

39. Eurocode 1. Actions on structures - Part 1-4: General actions - Wind actions. LVS EN 1991-1-4:2005/A1:2010.

40. Eurocode 3. Design of steel structures - Part 3-1: Towers, masts and chimneys - Towers and masts. LVS EN 1993-3-1:2007.

41. Feldmann M., Heynemeyer Ch., Butz Chr., et. al. Design of floor structures for human induced vibrations // JRC Scientific and Technical Report. 2009. EUR 24084EN, 64 p.

42. Franck L. Synchronous Lateral Excitation of Footbridges. Swiss Federal Institute of Technology, Swiss, 2009, 5 p.

43. Fritz M., Peikenkamp K. Simulation of the influence of surfaces on measured vertical ground reaction forces during fast movements. Human FactorsintheAgeofVirtualReality, ShakerPublishing: Maastricht,Netherlands, 2003, pp. 233–235.

44. Fujino Y., Pacheco B.M., Nakamura S. & Warnitchai P. Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge // Earthquake Engineering and Structural Dynamics, 1993. Vol. 22, No. 9, pp. 741-758.

45. Galvez C. Static method for aseismic design of self – supporting towers // Project Report G95-08, Dept. of Civil Eng. And Applied Mechanics, McGill University. Montreal, Quebec, Canada, 1995.

46. Gaile L., Radinsh I. Analysis of Dynamic Parameters of Observation Towers in Latvia // Proceedings of the 9th International Scientific and Practical Conference "Environment. Technology. Resources", Latvia, Rezekne, 20-22 Jun, 2013, pp. 57-62.

47. Gaile L., Radinsh I. Eccentric lattice tower response to human induced dynamic loads // 19th International Congress on Sound and Vibration 2012, ICSV 2012, pp. 1139-1146.

48. Gaile L., Radinsh I. Human Induced Dynamic Loading on Stairs // Proceedings of International Conference on Civil and Construction (ICSCE 2012), Stockholm, 2012: Issue 67, pp. 626-632.

49. Gaile L., Radinsh I. Lattice tower dynamic performance under human induced dynamic loading // RASD 2013 11th International Conference on Recent Advances in Structural Dynamics, 2013, Italy, Piza, 1st – 3rd July, 2013, pp.1-15.

50. Gard S.A., Miff S.C., Kuo A.D. Comparison of kinematic and kinetic methods for computing the vertical motion of the body center of mass during walking // Human Movement Science, 2004 (22), pp. 597–610.

51. Giakas G., Bultzopoulos V. A comparison of automatic filtering techniques applied to biomechanical walkingdata // Journal of Biomechanics, 1997 (30), pp. 847–850.

52. Gordon B. An Introduction to Structural Dynamic Comfort Criteria. Information paper IP 21/12,IHS BRE: UK, 2012.

53. Gordon D., Robertson E. 2004. Research methods in biomechanics. Library of Congress Cataloging-in-Publication Data, USA, 2004.

54. Griffin M. J. Handbook of Human Vibration. Elsevier Ltd., 2004

55. Gronley J.K., Perry J. Gait analysis techniques // Physical Therapy, 1984 (63), pp.1831-1838.

56. Hamill J., McNiven S. L. Reliability of selected ground reaction force parameters during walking // Human Movement Science, 1990 Vol. 9, pp. 117–131.

57. Hamill J., Selbie W.S. Three-dimensional kinetics. Robertson D.G.E., Caldwell D.W., Hamill J., Kamen G., Whittlesey S.N. (Eds.), Research Methods in Biomechanics, Human Kinetics, Champaign: IL, USA, 2004.

58. Heinemeyer C., Butz C., Keil A. et. al. Design of Lightweight Footbridges for Human Induced Vibrations // Office for Official Publications of the European Communities, EUR 23984 EN. Luxembourg, 2009, pp. 84.

59. Heylen W., Lammens S. and Sas P. Modal analysis theory and testing. Leuven, Katholieke Universiteit Leuven, 2007.

60. He J., Fu Z. Modal Analysis. Butterworth –Heinemann, 2001, pp. 291.

61. Holmes J. D. Along wind response of lattice towers: Part I –Derivation of expressions for gust response factors // Engineering Structures, 1996, Vol. 18(4), pp. 287-292.

62. Holmes J. D. Along wind response of lattice towers: Part II – Aerodynamic damping and deflections // Engineering Structures, 1996., Vol. 18(7), pp. 483-488.

63. IASS. Recommendations for the Design and Analysis of Lattice Towers. International Association for Shell and Spatial Structures Working Group 4. Madrid, Spain, 1991.

64. Ingólfsson E.T., Georgakis C.T. A stochastic load model for pedestrian-induced lateral forces on footbridges // Engineering Structures, 2011, Vol. 33, No. 12, pp. 3454-3470.

65. Ingolfsson E. T., Georgakis C. T. Experimental identification of pedestrian – induced lateral forces on footbridges // Journal of Sound and Vibration, 2011, Vol. 330, pp.1265-1284.

66. Ingolfsson E. T., Gudmundsson G.V., Živanovic S., Pavic A. Crowd. Induced Vibrations of a Steel Footbridge in Reytkjavik // Proceedings of the 30th IMAC, A Conference on Structural Dynamics, Topics on the Dynamics of Civil Structures, 2012, Volume 1 (26), pp. 61-72.

67. ISO 10137:2007, Bases for design of structures – Serviceability of buildings and walkways against vibration, International Standard Organization, 2007.

68. Kasperski M., Czwikla B. A Refined Model for Human Induced Loads on Stairs // Conference proceedings of the Society for Experimental Mechanics Series, Topics on the Dynamics of Civil Structures, Proulx, Tomson Edition, 2011, pp. 27-39.

69. Kasperski M., Czwikla B. Men – induced loads on stairs // Proc. 8th International Conference on Structural Dynamics, EURODYN 2011, Leuven, Belgium, 2011, pp.949–956.

70. Kavanagh J. J., Menz H., B. Accelerometry: A technique for quantifying movement // Gait & Posture, 2008, Vol. 28, pp. 1-15.

71. Kavanagh J.J., Morrison S., James D.A., Barrett, R. Reliability of segmental accelerations measured using a new wireless gait analysis system // Journal of Biomechanics, 2006, Vol. 39, No. 15, pp. 2863-2872.

72. Kerr S. C., Bishop N. W. M. Human induced loading on flexible staircases // Engineering Structures, 2001, Vol. 23, pp. 37–45.

73. Khedr M. A. Seismic Analysis of Lattice Towers // Ph. D. Thesis, Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, Canada, 1998, pp.242.

74. Khedr M. A. McClure G. Earthquake amplification factors for self-supporting telecommunication towers // Canadian Journal of Civil Engineering.1999, 26(2), pp.208-215.

75. Kretz T., Grunebohm A., Kessel A. et. al., Upstairs Walking Speed Distributions on a Long Stairway // Safety Science, 2008, Vol 46, pp. 72-78.

76. Lafortune M.A., Three-dimensional acceleration of the tibia during walking and running // Journal of Biomechanics, 1991, Vol. 24, No. 10, pp. 877-879 + 881-886.

77. Lattice Towers and Masts. Part 4: Code of Practice for Lattice Masts / British Standards Institution. London: BSI, 1994. BS 8100, Part 4.

78. Leardini A., Chiari.et. al. Human movement analysis using stereophotogrammetry. Part 3. Soft tissue artifact assessment and compensation // Gait & Posture, 2005, Vol.21(2), pp. 212-25.

79. Li H.,. Georgakis C. T, Chen Z. Q. The response spectrum methodology considering inter - and intra - subject variability // Proceedings of The 4th International Conference Footbridge 2011, Wroclaw, Poland, July 6-8, 2011.

80. Macdonald J. Pedestrian – induced vibrations of the Clifton Suspension bridge // Proceedings of the ICE – Bridge Engineering, UK, 1 June 2008, Vol. 161, Issue 2, pp.69-77.

81. Madugula M. K. S. Dynamic Response of Lattice Towers and Guyed Masts. USA, American Society of Structural Engineers, 2002. pp. 257.

82. McClure G., Galvez C. A. Zaugg S. Equivalent static load method for seismic analysis of lattice telecommunication towers // Proceedings of ASCE Structures Congress XV, April 13-16. Portland, Oregon, 1997, pp. 539-543.

83. McMahon T.A., Valiant G., Frederick E.C. Groucho running // Journal of Applied Physiology, 1987 (62), pp. 2326–23337.

84. McRobie A., Morgenthal G., Lasenby J., Ringer M. Section model tests on human–structure lock-in // Bridge Engineering, 2003 (156), pp. 71–79.

85. Melderis I., Teteris G. Būvmehānika, Izdevniecība Zvaigzne, Rīga, 1977.

86. Mendis P., Ngo T., Haritos N., Hira A. Wind Loading on Tall Buildings // EJSE Special Issue: Loading on Structures, 2007, pp. 41-54.

87. Mikus J. Seismic analysis of self – supporting telecommunication towers // Eng. Report G94-10, Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, Canada, 1994.

88. Mouring S.E. Dynamic Response of Floor Systems to Building Occupant Activities. PhD Thesis. The Johns Hopkins University, Baltimore, MD, 1993.

89. Mouring S.E., Ellingwood B.R. Guidelines to Minimize Floor Vibrations from Building Occupants // Journal of structural engineering New York, N.Y., 1994, Vol. 120, No. 2, pp. 507-526.

90. Murray T. M., Allen D. E., Ungar E. E. Floor vibration due to human activity // AISC Design Guide, American Institute of Steel Construction, Chicago, 1997.

91. Nurse M.A., Hulliger M., Wakeling J.M., Nigg B.M., Stefanyshyn D.J. Changing the texture of foot wear can alter gait patterns // Journal of Electromyography and Kinesiology , 2005 (15), pp. 496–506.

92. Pimentel R.L., Vibrational Performance of Pedestrian Bridges Due to Human-Induced Loads. Ph. D. Thesis. University of Sheffield, UK, 1997.

93. Pizzimenti A. D., Ricciardelli F. Experimental evaluation of the dynamic lateral loading of footbridges by walking pedestrians // Sixth European Conference on Structural Dynamics EURODYN, Paris, France, September 2005.

94. Racic V. Experimental Measurements and Mathematical Modeling of Nearperiodic Human-induced Force Signals, Ph. D. Thesis.University of Sheffield, UK, 2009.

95. Racic V., Pavic A, Brownjohn, J. M. W. Experimental identification and analytical modeling of human walking forces: Literature review // Journal of Sound and Vibration, 2009, Vol. 326, pp. 1–49.

96. Racic V., Pavic A, Brownjohn, J. M. W. Modern facilities for experimental measurement of dynamic loads induced by humans: A literature review // Shock and Vibration, 2013, 20(1), pp. 53-67.

97. Rainer J.H., Pernica G., Allen D.E. Dynamic loading and response of footbridges // Canadian Journal of Civil Engineering, 1988, 15, pp. 66–71.

98. Rao S. S. Vibration of Continuous Systems. John Wiley & Sons: Hoboken, New Jersey, 2007.

99. Recommendation for the Design and Analysis of Lattice Towers. Second draft / International Association for Shell and Spatial Structures, Madrid, Spain, 1991.

100. Riener R., Rabuffetti M., Frigo C., Quintern J., Schmidt, G. Instrumented staircase for ground reaction measurement // Medical and Biological Engineering and Computing, 1999, Vol. 37, No. 4, pp. 526-529.

101. Riener R., Rabufetti M., Frigo C. Stair ascent and descent at different inclinations // Gait and Posture. 2002 (15), pp. 32–44.

102. Ronnquist A. Pedestrian Induced Lateral Vibrations of Slender Footbridges. Norwegian University of Science and Technology, Trondheim, Norway, 2005.

103. Sabatini A.M., Martelloni C., Scapellato S., Cavallo F. Assessment of walking features from foot inertial sensing // IEEE Transactions on Biomedical Engineering, 2005, Vol. 52, No. 3, pp. 486-494.

104. Sachse R., Pavic A., Reynolds P. Human-structure dynamic interaction in civil engineering dynamics: A literature review // Shock and Vibration Digest, 2003, 35(1), pp. 3-18.

105. Sackmann V. Prediction of natural frequencies and mode shapes of self – supporting lattice telecommunication towers // Project Report – Diplomarbeit Nr. 76, (Technical University of Munich), Departament of Civil Engineering, McGill University, Montreal, Canada, 1996, pp. 102.

106. Sartor C., Alderink G., Greenwald H., Elders L. Critical kinematic events occurring in the trunk during walking // Human Movement Science, 1999, Vol. 18, No. 5, pp. 669-679.

107. Schueller G. I. Past, Present & Future of Simulation – Based Structural Analysis // Proceedings of the Euro SIBRAM 2002. Colloquium. Prague, Czech Republic, 2002, Vol. 2, pp. 17-27.

108. Setareh M. Evaluation and assessment of vibrations owing to human activity // Proceedings of the Institution of Civil Engineers. Structures and Buildings, 2012, Vol. 165, Issue SB5, pp. 219-231.

109. Smith A.L., Hicks S. J., Devine. Design of Floors for Vibration. A New Approach (Revised Edition, February 2009). SCI Publication P354. Steel Construction Institute, UK, 2009.

110. Specification of USB Accelerometer Model X6-1A. Gulf Coast Data Concepts. November 2010, Rev G, 2010.

111. Stacoff A., Diezi C., Luder G., Stüssi E., Kramers-De Quervain I.A. Ground reaction forces on stairs: Effects of stair inclination and age // Gait and Posture, 2005, Vol. 21, No. 1, pp. 24-38.

112. Talukdar S. Vibration of Continuous Systems / Internet.- http://www.iitg.ernet.in/engfac/rtiwari/resume/lecture%20notes.html.

113. Thorstensson A., Nilsson J., Carlson H., Zomlefer M.R. Trunk movements in human locomotion // Acta Physiologica Scandinavica, 1984, Vol. 121, No. 1, pp. 9-22.

114. Vaughan C. L., Davis B. L. O’Connor J. C. Dynamics of Human Gait, Cape Town, Kiboho Publishers, 1999.

115. Venkateswarlu B., Harikrishna P., Selvi R. S, Satish R. K. Stochastic gust response of microwave lattice towers // Computers and Structures. 1994, 52(5), pp. 1031-1041.

116. Venuti F., Bruno L. Crowd-structure interaction in lively footbridges under synchronous lateral excitation: A literature review // Physics of Life Reviews, 2009. Vol. 6, No. 3, pp. 176-206.

117. Weidman P. Pinelis I. Model equations for the Eiffel Tower profile: Historical perspective and new results // Comptes Rendus – Mecanique. 2004, Vol. 332, No. 7, pp.571-584.

118. Willford M., Young P. A Design Guide For Footfall Induced Vibration of Structures // CCIP – 016, The Concrete Centre, Camberley, 2006.

119. Willford M., Young P., Field C. Predicting footfall-induced vibration: Part I // Proceedings of the Institution of Civil Engineers, Structures and Buildings, April 2007, Issue SB2, pp. 65-72.

120. Whittlesey S.N. Robertson D.G.E. Two-dimensional inverse dynamics. Robertson D.G.E., Caldwell G.E., Hamill J., Kamen G., Whittlesey S.N. (Eds.), Research Methods in Biomechanics, HumanKinetics, Champaign: IL, USA, 2004.

121. Winiarski S, Rutkowska – Kucharska A. Estimated ground reaction force in normal and pathological gait // Acta of Bioengineering and Biomechanics, 2009., Vol. 11, No. 1, pp. 54–60.

122. Wu G., van der Helm F. C. T., et. al. ISB recommendation on definitions of joint coordinate systems of various joints for the reporting of human joint motion – part II: shoulder, elbow, wrist and hand // Journal of Biomechanics, 2005, Vol. 38, pp. 981-982.

123. Wu G., Ladin Z. The study of kinematic transients in locomotion using the integrated kinematic sensor // IEEE Transactions on Rehabilitation Engineering, 1996, Vol. 4, No. 3, pp. 193-200.

124. Xie W., He W., Ai K. Study on characteristics of pedestrian loads of railway station structures Gongcheng Lixue // Engineering Mechanics, 2012, 29(12), pp. 256-264.

125. Yao S., Wright J.R., Pavic A., Reynolds P. Experimental study of human-induced dynamic forces due to jumping on a perceptibly moving structure // Journal of Sound and Vibration, 2006 (296), pp. 150–165.

126. Young P. Improved floor vibration prediction methodologies // Proceedings of Arup Vibration Seminar on Engineering for Structural Vibration - Current Developments in Research and Practice, London, UK, 2001.

127. Zhou B., Ren X., Lu X. Vibration Analysis and Evaluation of the Indoor Spiral Steel Stair // Advanced Materials Research, 2011. Vols. 163-167, pp. 36–43.

128. Zivanovic S., Pavic A. Probabilistic assessment of human response to footbridge vibration // Journal of Low Frequency Noise, Vibration and Active Control, 2009, Vol. 28(4), pp. 255-268.

129. Zivanovic S., Pavic A., Reynolds P. Probability based prediction of multi-mode vibration response to walking excitation // Engineering Structures, 2007, 29(6), pp. 942–954.

130. Živanović S., Pavic A., Reynolds P. Human-structure dynamic interaction in footbridges // Proceedings of the Institution of Civil Engineers: Bridge Engineering, 2005, Vol. 158, No. 4, pp. 165-177.

131. Živanović S., Pavic A., Reynolds P. Vibration serviceability of footbridges under human-induced excitation: A literature review // Journal of Sound and Vibration. 2005.- Vol. 279, No. 1-2, pp. 1-74.

132. Аугусти Г., Баратта А., Кашиати Ф. Вероятностные методы в строительном проектировании, Москва, Стройиздат, 1988, 584 с.

133. Киселев В. А., Строительная механика, Специальный курс, Москва Стройиздат, 1980.

134. Конструкции из дерева и пластмасс Примеры расчета и конструирования: Учеб. пособие для вузов. Под ред. проф. Иванова В. А., 3-е изд., Киев, Вища школа, 1981, 392 с.

135. Карлсен Г.Г., Большаков В.В., Каган М.Е., Свенцицкий Г.В. Деревянные конструкции, Москва, Государственное издательство литературы по строительству и архитектуре, 1952.

136. Карлсен Г.Г., Большаков В.В., Коган М.Е. и др. Конструкции из дерева и пластмасс. Учебник для вузов. Москва, Стройиздат, 1975, 688 с.