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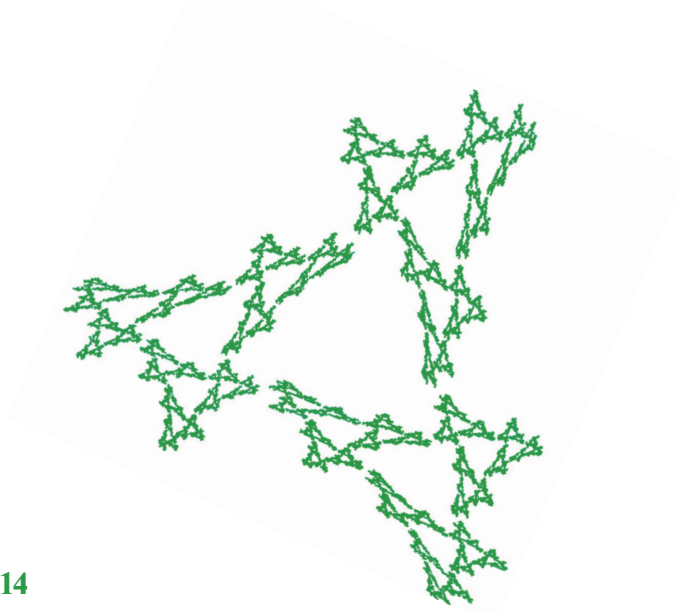
SLOVAK UNIVERSITY OF
TECHNOLOGY IN BRATISLAVA



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**GALILEO GALILEI'S LOCATION,
SHAPE AND SIZE OF DANTE'S INFERNO:
AN ARTISTIC AND EDUCATIONAL PROJECT**

ANGELINI Alessandra (I), MAGNAGHI-DELFINO Paola (I), NORANDO Tullia (I)

Key words. Art, Geometry, Geometrical Shapes, Dante Alighieri, Galileo Galilei.

Mathematics Subject Classification: AMS_01A99.

Mathematics and Art have a long historical relationship, which goes as far back as the ancient Greeks. It suffices to think, for instance, to their use of the golden ratio, regarded as an aesthetically pleasing canon and incorporated into the design of many monuments and temples. With the Renaissance we can see a rebirth of Classical (Greek and Roman) culture and ideas, and among them the study of Mathematics as a relevant subject needed to understand the nature and the arts.

Two major reasons drove Renaissance artists towards the pursuit of Mathematics. Firstly, painters needed to figure out how to depict three-dimensional scenes on a two-dimensional canvas. Secondly, philosophers and artists alike were convinced that Mathematics was the true essence of the physical world so that the entire universe, including the arts, could be explained in geometrical terms. For instance, Galileo Galilei in his *Il Saggiatore* wrote that “[The universe] is written in the language of Mathematics, and its characters are triangles, circles, and other geometric figures.”

Thus, there is a close relation between Mathematics and Fine Arts during the Renaissance: mathematical knowledge is applied in drawings and paintings with the use of symmetry, producing ratios and proportions.

Within the study of such a context arises the artistic and educational project “Galileo: location, shape and size of Dante’s Inferno” as a collaboration between the FDS Laboratory for Mathematical Education and Science Communication at the Department of Mathematics of the Politecnico di Milano and Accademia di Belle Arti di Brera.

The project is inspired by the first of two lectures held by Galileo Galilei at the Accademia Fiorentina in 1588. These lectures were commissioned by the Accademia to solve a literary controversy concerning the interpretation of Dante's Inferno. In these lessons Galileo took the opportunity to show his mathematical abilities combined with his strong background in Humanities. His ultimate aim was to show that Mathematics is not merely useful from a technical point of view, but can also give a contribution to nobler cultural debates, thus acquiring an intellectual status comparable to that of the Humanities.

When giving his lectures Galileo probably used drawings to explain how to map Dante's Inferno, because of "la difficoltà del soggetto che non patisce esser con la penna facilmente esplicato" (the difficulty of the subject which does not admit of easy explication in writing). Galileo's manuscript survives and is catalogued in the Filza Rinucciniana 21 of the Biblioteca Nazionale di Firenze, but the drawings are lost.

The project here presented included an accurate analysis of Galileo's work and was meant as an opportunity for the students of Graphic to investigate the relationship between geometric representation and artistic interpretation. They made scale drawings of the Inferno, by using different paper media and drawing techniques of their choice. Later they produced original art works resulting from a personal artistic interpretation of the subject, free of pure scientific representation. The results reflect various artistic and creative sensibilities: drawings, paintings, engravings. The students' works were gathered, accompanied by short sentences associated with the selected quotes of Inferno and displayed on the exhibition that was held at Politecnico di Milano (May 2012). After the works were exhibited at the Museo Dantesco of Ravenna (September 2013) and at the Bergamo Science Festival (XI Edition, October 2013).

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More information and photos should reader find on

<http://fds.mate.polimi.it/>

<https://www.facebook.com/pages/Mostra-Galileo-Polimi/539643792737239?ref=ts&fref=ts>

<https://www.facebook.com/pages/MatematicArte/581215691915905?ref=ts&fref=ts>

DETERMINATION OF ACTUAL MOBILITY OF INCORRECT MECHANISMS USING MATRIX LIE GROUPS

BALKO Ľudovít (SK), PALČÁK František (SK)

Abstract. The correct mechanism has geometric constraints, which are enforced according their type and computed mobility is actual. When diameters and configurations of links and geometrical constraints cause their partial or total passivity, then mechanism is incorrect and computed mobility is not actual. In this paper some properties of matrix groups is used for determination of actual mobility of incorrect mechanisms.

Keywords. Mobility of mechanism, passivity of geometrical constraints, matrix Lie groups.

Mathematics Subject Classification: Primary 22E99; Secondary 20G99, 17B99.

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PROBLÉMY STUDIA V PRVNÍM ROČNÍKU VŠB-TU OSTRAVA

BOHÁČ Zdeněk (CZ), DOLEŽALOVÁ Jarmila (CZ), KREML Pavel (CZ)

Key words. . Studium, I. ročník, matematika, přednášky, cvičení, zkoušky.

Mathematics Subject Classification: Primary 97U99.

Na klesající úroveň matematických znalostí studentů technických vysokých škol si stěžují všichni, koho se tato skutečnost týká. S nedostatečnými vědomostmi z matematiky se potýkají rovněž vyučující odborných předmětů. Příčiny tohoto stavu jsme se pokusili zjistit formou anonymního dotazníku, který vyplnili studenti tří fakult VŠB-TU: Fakulty strojní (FS), stavební (FAST) a bezpečnostního inženýrství (FBI).

Dotazníkem, který studenti vyplnili ve vyšším ročníku, jsme zjišťovali rok nástupu na vysokou školu (pouze 82% respondentů postoupilo do vyššího ročníku bez opakování nebo přerušování studia), názor studentů na úroveň jejich vstupních znalostí z matematiky a geometrie, přístup studentů k přednáškám, cvičením a zkouškám.

Složení studentů: Téměř 60% studentů absolvovalo střední průmyslové školy, na gymnáziích maturovala necelá čtvrtina studentů. Na FS a FAST tvoří absolventi SPŠ tři čtvrtiny všech studentů.

Zájem o studium: Je potěšující, že téměř 90% respondentů se na příslušnou fakultu přihlásilo z vlastního zájmu. Škoda jen, že se zájem o studium neprojevuje ve studijních výsledcích.

Maturitní zkoušku z matematiky složilo téměř 80% studentů, při tom s výborným výsledkem 17%, s chvalitebným výsledkem 31% (obr. 1). Tato informace je zářezující, protože výsledky, kterých studenti dosahují v prvním ročníku (viz dále), jí neodpovídají.

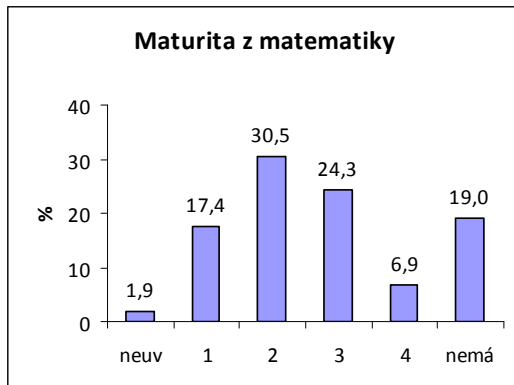
Připravenost z matematiky a geometrie na SŠ: Jen necelá polovina ze všech respondentů hodnotí svou přípravu z matematiky jako kvalitní (19% výborně, 29% chvalitebně). Připravenost z geometrie hodnotí studenti ještě hůře – 17% známkou

nedostatečná (na FS dokonce 36%). Přitom studenti gymnázií se cítí připraveni z matematiky i geometrie lépe než studenti SPŠ (obr 2).

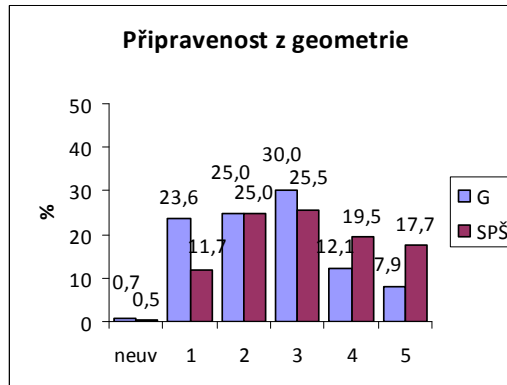
Matematika 1: Účast na přednáškách je nízká, jen 41% studentů uvedlo účast mezi 80%-100%. Rovněž přípravu na cvičení přiznávají studenti velmi slabou: 12% se nepřipravuje vůbec, 45% jen nepravidelně. Stejně je to s přípravou na zkoušku. Ačkoliv matematiku považuje mnoho studentů za obtížný předmět a nejsou na její studium podle vlastního názoru dostatečně připraveni, 55 % studentů se na zkoušku připravovalo méně než 20 hodin. Přitom jen 8% studentů považuje za snadnou praktickou část, 12% teoretickou část zkoušky. Nevalné přípravě na zkoušku pak odpovídají výsledky.

Matematika 2: Účast na přednáškách se ještě snížila s výjimkou FS, kde polovina studentů uvedla docházku mezi 80%-100%. Příprava na cvičení (obr. 3) stejně jako počet hodin přípravy na zkoušku (obr. 4) zůstaly v podstatě beze změny, přestože se podle studentů zvýšila obtížnost praktické části zkoušky. Ve výsledcích zkoušek se zvýšilo procento studentů, kteří se na zkoušku nedostavili (obr. 5).

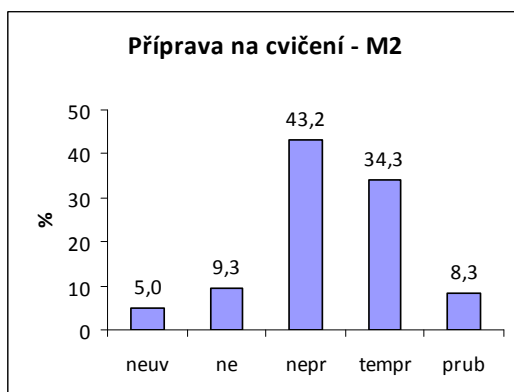
Závěr: Uvedené skutečnosti podle nás nejsou příliš povzbudivé. Výsledky studentů v matematice jsou slabé, studenti si sami uvědomují nedostatečné základy ze střední (i základní) školy, jsou si vědomi i z toho plynoucí náročnosti zkoušky z matematiky. Nechává je to však většinou v klidu, nic je nenutí k vyššímu úsilí. Studenti nejsou zvyklí samostatné práci, neumí studovat z literatury předpokládají, že se všechno naučí ve škole. Teorie je nebaví a pro praktické úlohy nemají zase potřebný teoretický základ. S daným stavem věcí jsou spokojeni. Uvádějí, že studium na dané fakultě odpovídá jejich představám: 46% je zcela spokojeno, 44% je spokojeno s výhradami (obr. 6).



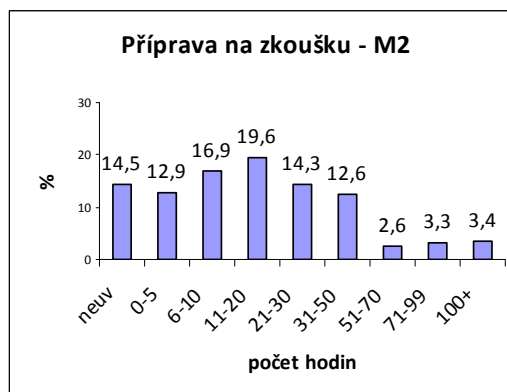
obr. 1



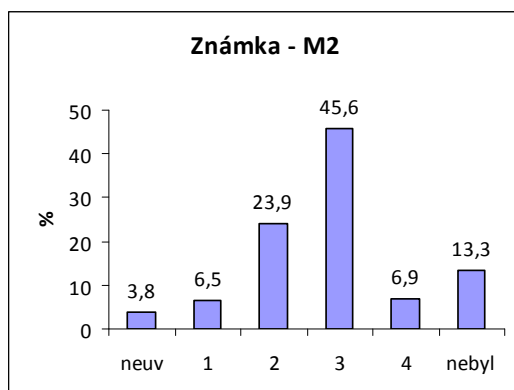
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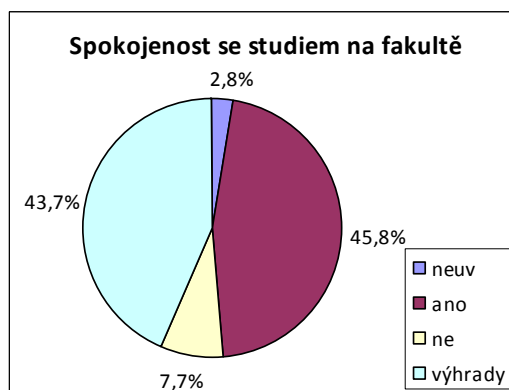
obr. 3



obr. 4



obr. 5



obr. 6

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**EXPERIENCE OF THREE-DIMENSIONAL VISION IN THE ERA OF
DIGITAL INTERACTION.
NEW DEVICES: OPPORTUNITY AND CHALLENGES
FOR APPLIED MATHEMATICS AND VISUAL**

Federico Alberto Brunetti (I)

Key words. Digital stereoscopic viewers, augmented reality, virtual vision, science communication, visual design, 3D animation, LHC - CERN, particle Physics, particle collision detector, Higgs boson.

Mathematics Subject Classification: Primary AMS01A99; Secondary: 97M80.

1 New generation of digital interactive stereoscopic viewers

It will soon be presented to the public a new version of stereoscopic viewers that will update a hardware product already widely distributed, but relatively known only in expert groups. These stereoscopic viewers were designed for viewing files and video projected images through a system of transparent optical prisms - sent by a compact wired remote control comes with internal memory, touchpad and android wi-fi - which allow the simultaneous perception of the surrounding environment.

2 Conceptual references to the interaction with the visual techniques

In this context I would underline some preliminary methodological reflections to a possible technical and practical experimentation which can be coordinated with the appropriate research facilities. In particular I would like to point out some factors and conceptual references to the interaction with the visual techniques, that it will allow new visual experience in such apparatus.

3 Opportunities and challenges for applied mathematics and visual design

Several stereoscopic devices have preceded the upcoming technological innovations in three-dimensional viewers. Progress must retain the memory of some aspects of what has preceded it, at least under a typological profile to facilitate its comprehension, in order not to be perceived as a foreign object and simply incomprehensible. The real challenge for applied mathematics and visual designers will be to prefigure how to use possible applications of these new devices that can actually enable a more deep visual experience for the observer - and not just an augmented reality of a possible isolated and only passive user- to become a protagonists of new knowledge and real communication (word and idea which etymologically means “shared action”!).

4 Case study: Collisions 3D visualization from LHC-CERN of Geneve

A specific case study that we would like to present concerns the visualizations of particle collisions at the LHC at CERN, selected to verify the traces theorized Higgs boson.

It is methodologically an exceptional interesting representation path. From a physical phenomenon at the microscopic extreme limits of events in space-time –through the major detectors ATLAS and CMS- the data of the tracks collisions have been recorded, collected and selected: these data obtained from the detectors have been recomposed in sequences of 3D modeling with chromatic highlights of different types of tracks detected. The modeling of these sequences were visually reproduced in the progressive dynamics of the collisions returned finally in 3D animation. This complex elaboration is an innovative visual product, between science and visual arts, that carries us from the invisible to the infinitesimal space-time the immersive vision of the physical events.

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**MEMORIES OF LOST CUBES AND DOMES.
SELF-SIMILARITY IN MODULAR CONTEMPORARY BUILDINGS,
THE PERSISTENCE OF FORM
BETWEEN MATH AND ARCHITECTURE**

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Abstract. After the revolutions of contemporary architecture, and the inventive and daring design experimentations obtained by using, for example, genetic algorithms and computer evolutionary simulations, at the basis of even apparently complex compositions there are simple mathematical rules or forms. In this paper, to be considered as a preliminary study with a preeminent architectural/artistic bias, we will take into consideration a few examples of buildings, in which self-similar, reiterated and even fractal cubes have been used to create very outstanding compositions. A deeper analysis reveals similarities to and influences by very ancient paradigms or iconic symbols and the persistence of form through centuries and different cultures.

Key words. architecture, art, geometrical inspiration.

Mathematics Subject Classification: Primary 00A67.

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**PHASE MERGER PRINCIPLE FOR NONSTATIONARY
IMPULSE MARKOV DYNAMICAL SYSTEMS**

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Abstract. The paper deals with impulse type non-stationary dynamical systems fast switched by step Markov process having m invariant measures. We have derived more simple Markov dynamical system in a form of ordinary differential equation with right part switched by merger homogeneous Markov process with m states and have proved that for sufficiently fast switching this dynamical system approximates initial system.

Key words. Averaging principle; impulse dynamical systems; merger procedure.

Mathematics Subject Classification: Primary 60H10, 60H30; Secondary 37H10.

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**LOCAL / GLOBAL IN PICASSO'S PAINTINGS,
A RIEMANNIAN VIEW**

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Abstract. Simultaneity of points of view, or polycentric views, characterizes much of the twentieth century visual culture, both in geometry and in the arts, allowing the description of complex spatial relationships. We will review the analysis of two drawing of Picasso, their local charts, and re-assemble the charts in atlases, to reconstruct the position in three dimensions of the represented object, much following the methodology established in Riemannian geometry to glue local information, when possible, into a global one. The second painting had never been analyzed before, to our knowledge. This talk is dedicated to Mauro Francaviglia, who encouraged us with stimulating questions about curvature

Key words. Riemannian Geometry, Polycentric view, Visual analysis, Local Charts, Global Atlases.

Mathematics Subject Classification: 00A66, 58-02, 53A99.

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IMPLICIT SOLUTIONS OF DIFFERENTIAL EQUATIONS

DOBRAKOVOVÁ Jana (SK), ZÁHONOVÁ Viera (SK)

Abstract. This paper is devoted to some aspects of teaching ordinary differential equations of the first order, especially properties of their solutions in implicit form and some special tools for visualization of the obtained results, with the aid of the programming system MATHEMATICA.

Key words. differential equations, integral curves, direction field, isoclines, solutions in implicit form.

Mathematics Subject Classification: Primary 97U50, 97U70.

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**DERAS VERSUS MS EXCEL SOLVER
IN LEVELLING THE LIFE EXPECTANCY AT BIRTH**

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Abstract. The article deals with using of MS Excel Solver for demographic calculations. We will focus on the levelling of specific mortality rates for higher ages. The reason is that there is a gradual improvement in mortality. In our study, we will focus on levelling specific mortality rates by Gompertz–Makeham (and modified Gompertz–Makeham) function. An important part of the article will provide the process for estimating the parameters of mentioned functions using the procedure of MS Excel Solver. Our goal is to present the methodology that we use for estimating parameters of these functions. The procedure will be applied on data about mortality of Czech population. The results will be compared with the estimates, obtained from the software DeRaS. The aim is to compare the results obtained from both procedures. There will be presented the possible consequences of current development of mortality in the conclusion.

Key words. DeRaS, MS Excel, Life Expectancy at Birth, Gompertz–Makeham function.

Mathematics Subject Classification: 90C30.

Used methodology

For the analysis of mortality is most commonly used the life expectancy at exact age x , which determines how long will person lives in average. Calculating of life tables proceed in several steps. The first step is calculation of specific mortality rates (see e.g. Fiala [2])

$$m_x = \frac{M_x}{S_x}, \quad (1)$$

where M_x is the number of deaths at age x , \bar{S}_x is the number of persons at age x . Among the most frequently used models for levelling mortality rates is included Gompertz–Makeham

function (G–M function) and modified (mG–M function). The G–M function (Boleslawski, Tabeau [1]) is

$$\mu_x = a + bc^x, \quad (2)$$

where μ_x is the intensity of mortality, x is age and a , b , and c are parameters. For the intensity of mortality is true the equation:

$$\mu\left(x + \frac{1}{2}\right) \doteq m_x, \text{ where } x = 1, 2, \dots, 59. \quad (3)$$

For higher ages is used for the levelling and extrapolating mortality curve the G–M function. In the first way will be used the Solver in MS Office Excel (for more see e.g. Šimpach [3]). We have to find the initial estimates of parameters in the first step and we will improve them using by the ordinary least squares. The last step is the calculation of the weighted squared deviations (WSD), through which we will optimize the parameter of G–M (respectively mG–M) model

$$WSD = \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2 \text{ for } x \in \langle 60; y \rangle, \quad (4)$$

where $S_{t,x}$ is the number of living at age x in year t , $S_{t+1,x}$ is the number of living at age x in year $t+1$, m_x are specific mortality rates, $\tilde{m}_x^{(GM)}$ are levelled values of specific mortality rates using by G–M or mG–M model, y is the highest age for which we have a non-zero value of m_x . When we optimize the parameters using by the OLS, it is necessary to create two instrumental sums,

$$S_1 = \sum_{x=60}^{82} \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2 \text{ and} \quad (5)$$

$$S_2 = \sum_{x=83}^y \frac{S_{t,x} + S_{t+1,x}}{2 \cdot m_x \cdot (1 - m_x)} \cdot (m_x - \tilde{m}_x^{(GM)})^2$$

which will help us to optimize the initial values of parameters. For continuation of analysis, let us explain mG–M function (see e.g. Thatcher et al. [4]) as

$$\mu_x = a + bc^{\frac{x_0 + \frac{1}{\gamma} \ln[\gamma(x-x_0)+1]}{\gamma}}, \quad (6)$$

where a , b and c are the parameters of the original G–M function, x_0 is the age from which will be performed equalization and γ is a parameter of mG–M function.

Expected results

In our study we focused on the analysis of the mortality trend of the Czech population from 1970 to 2009. The results obtained from the analysis will be published separately for males and females. From the Fig. 1 and Fig. 2 is clear, that the life expectancy at birth increased during the reporting period. For males, its value has changed more than for females.

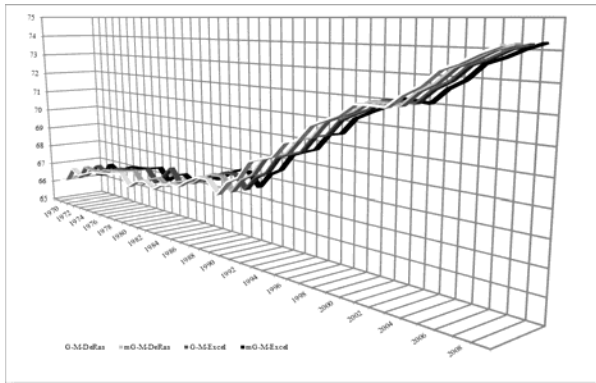


Fig. 1. Levelled life expectancy at birth, males. Source: authors' calculations and illustration.

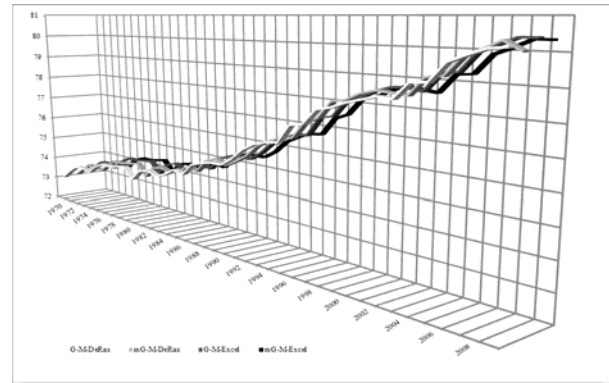


Fig. 2. Levelled life expectancy at birth, females. Source: authors' calculations and illustration

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**“CULTURE OF SOLVING PROBLEMS” –
ONE APPROACH TO ASSESSING PUPILS’ CULTURE
OF MATHEMATICS PROBLEM SOLVING**

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Key words. Problem solving, culture of problem solving, mathematics education, psychological screening.

Mathematics Subject Classification: Primary 97D50; Secondary 97C70.

In the paper, Culture of Solving Problems – a tool for evaluating pupil problem solving in mathematics – is presented and analyzed. As problem solving is in the centre of mathematics education, this issue must be studied in great detail. One of the first steps of achieving any improvement is to understand the factors that influence the culture of solving mathematical problems. The very essence of research focusing on the culture of problem solving asks for collaboration of teachers, researchers and pupils.

In our research, the Culture of Problem Solving is composed from the following four components: Intelligence, Creativity, Reading texts with comprehension, and Ability to use existing knowledge. The following methodological approaches were used: psychological screening, testing the ability to use existing knowledge and evaluation by the mathematics teacher.

Psychological screening consisted of three parts:

- Váňa’s test of intelligence (VIT) is the most general of the used tests. It addresses many components of children’s intelligence but it measures neither creativity, nor reading comprehension or ability to use previous knowledge. It is anchored in what the child learns at school; that is why it correlates with school performance. It tries to address a significant number of indicators of school success.
- Christensen-Guilford test (CGT) measures creativity in the form of divergent thinking. It does not necessarily correlate with VIT.

- Test of the ability reading with comprehension (RC) consisting of condensing text. During the test of the ability to use existing knowledge (AUK) pupils were assigned four pairs of problems (diads). The first problem from the diad tested the presence of certain knowledge, the second its use in a non-algorithmic (non-standard) context. The total evaluation of a pupil expresses his/her success in the use of existing knowledge; it does not evaluate the school success.

The mathematics teachers' evaluation was gained in interviews of the authors with them. Data for this study were collected in mathematics lessons in the 8th grade (pupils aged 14-15) of two lower secondary schools in the Czech Republic: School S1 is situated in Ústí nad Labem, S2 in Prague. S1 although not a grammar school is a selective school for pupils with higher abilities in mathematics (chosen by entrance exams), S2 is an ordinary school on the outskirts of Prague. The participating teachers are both very experienced.

The data collected for this study come from tests assigned and evaluated by a professional psychologist, tests of the ability to use knowledge evaluated by one of the authors of this paper and from recordings of interviews with the two participating teachers.

Each pupil was described by a quartet of values constituted of the gained scores in all four tests (VIT, CGT, RC, AUK). For each class, mutual dependencies between VIT and AUK, CGT and AUK, RC and AUK were statistically tested using chi-squared test of independence and contingency coefficient. This statistical data processing showed the following results:

- a) There is no significant link between VIT and AUK.
- b) There is no significant link between CGT and AUK.
- c) Pupils with low RC show worse results in AUK.

In our analyses, c) is independent on the school level while a) and b) are more significant in case of gifted pupils from S1.

There were some deviations to the typical results gained, e.g. a boy with good results in VIT, CGT and RC but very low performance in AUK. These could only be explained with the help of information from interviews with the mathematics teachers of the classes. In all these cases the teachers' information on the pupil suggested the reasons for these deviation; they were usually attributed to the pupil's personality and/or family background.

The fact that there are differences in the level achieved in VIT and in other components of Culture of Problem Solving in one respondent confirms the hypothesis that children have certain skills and abilities that are not made use by the school. VIT gives results of mechanical work and drill, not of everything the children are able to do. Traditional teaching strategies are not sufficient to fight school stereotypes. We believe children can be "awaken" if teachers work with them in new ways.

Interviews with the teachers confirm that higher (or lower) performance in solving of mathematical problems is significantly correlated to better (or worse) pupil's evaluation in the following three components of Culture of Problem Solving: VIT, RC, AUK. We are convinced that the proposed Culture of Problem Solving is a suitable tool for description of a pupil's disposition to successful solving of mathematical problems.

The research reported in this paper continues by a longitudinal experiment. In the whole period of the experiment the teacher works with his/her pupils as follows: When solving problems, he/she is trying to give his/her pupils insight into some selected heuristic problem solving strategies. The selected strategies are those that pupils rarely or never come across at

school but are very useful. The goal of the teachers is to teach their pupils to use these strategies. The experiment began in the beginning of October 2012. At the end of the experiment the pupils are expected to use selected heuristic problem solving strategies actively and in greater extent than in the beginning. The optimistic hypothesis is: The pupils will show better results in components CGT, RC and AUK of Culture of Problem Solving at the end of the experiment.

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**STUDENTS' CREATIVITY IN THE PRACTICES
OF ADOTTA SCIENZA E ARTE NELLA TUA CLASSE**

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GIURGOLA Giliola (I), LORENZI Marcella Giulia (I), PAROLINI Giovanna (I),
SARTORI Renato (I), SOLARI Amerigo (I), TORRE Matteo (I)**

Abstract. The first edition of the project Adotta Scienza e Arte nella tua classe (Adopt Science and Art in your class) has been held in the Italian schools in the school year 2012-2013. This paper describes how the popularisation of science through the quotes of famous scientists, annotated and contextualized in the collection of 100 +1 frasi famose sulla scienza (100 +1 famous quotes on science), used by the students as an inspiration for their graphic works on science, may contribute to increasing the curiosity for science, to motivate them and to create a multidisciplinary and creative learning environment. After a brief presentation of some basic facts – where in Italy the project has been implemented, how many students and teachers it has involved – we make an initial assessment of how scientific thinking and the practice of science can help shape the way the new generations see the world. This is done by comparing the quotes chosen by the students with their interpretations - expressed in their original comments accompanying the graphic creation - and the artworks themselves. The analysis of the classroom practices, the original comments by the students and the graphic works highlight the strong value of the project to define the school as a place for the integration and transmission of knowledge through creativity and for the dissemination of science and scientific culture. Even beyond the school itself, through exhibitions of works, public lectures, etc. Finally, we draw a balance of this first year work and describe the lines of development of the project in the second edition.

Key words. Science, arts, mathematics, physics, science communication, science popularisation.

Mathematics Subject Classification: Primary 00A66; Secondary 00A79.

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DYNAMIC MODEL IN ADVERTISING

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Key words. Marketing communication, Advertising, System of differential equations, Stability of system, Hopf bifurcation, Limit cycle.

Mathematics Subject Classification: 34C23, 34C25, 91B55.

The goal of article was to show to the reader how we can create dynamic model with two equations, two variables and one parameter and analyse it. It was created only from theoretical knowledge from marketing and economics. We have one product. The variables are number of customers of one company as a focus and other companies selling the same product. We tried to find out, what causes change in the number of customers of our product in companies. We focused on market and product with similar price, access to information, technologies, costs and quality of good in companies. In equation there are also constants in general, and they say about migration, mortality, and increasing customers by increasing salaries. As only one competitive advantage in our model we take marketing communication and advertising.

In first part, we try to find dynamic system from marketing and economic theory. In the second part it is simple analysis of system. We find out equilibrium and we try to find out, in which market environments what happens in neighbourhood of equilibrium. If an equilibrium is stable or unstable. In the third part, we tried to find out, if Hopf bifurcation exists in equilibrium for some environment and for some values of our constants and our parameter. The last part is conclusion and interpretation.

We found only one equilibrium in our dynamic system, so we tried to analyse it. We found that it can be stable and also unstable. It depends on values of constants and on our parameter – amount of money in advertising. Finally we also found that Hopf bifurcation can exists that in our system but only in market with decreasing number of customers and this is satisfied only for products in the end of his life time cycle. So we can see if it is better to increase or to decrease advertising for our product. But it is only supercritical Hopf bifurcation.

All condition of Hopf bifurcation was satisfied and we found that for our model Hopf bifurcation exists but only in his supercritical form and only for market environment mentioned above. It is older product, which is sold on market, because it has his segment of customers which slowly decrease in time. From Hopf bifurcation we found that we can have enough big this segment by good advertising, but not in every condition. In the beginning, when the company is small and it do not have enough orders, it can't have big investments into advertising and by effecting advertising it can attract some customers and create a segment of customers. It increases but product also goes to the second half of his lifetime cycle. We can have bigger investments but we can't cross border of value of parameter (14 in our article), in which we found the bifurcation, because then by increasing advertisement we don't achieve our goal and the increasing of customers will not happen. istic and creative sensibilities: drawings, paintings, engravings. The students' works were gathered, accompanied by short sentences associated with the selected quotes of Inferno and displayed on the exhibition that was held at Politecnico di Milano (May 2012). After the works were exhibited at the Museo Dantesco of Ravenna (September 2013) and at the Bergamo Science Festival (XI Edition, October 2013).

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13th Conference on Applied Mathematics **APLIMAT 2014**

BOOK OF ABSTRACTS

**DIDACTIC DIAGNOSTICS OF THE EDUCATIONAL PROCESS
CONCERNING BASIC OF MATHEMATICS**

FERDIÁNOVÁ Věra (CZ), KONEČNÁ Petra (CZ)

Abstract. Article introduces form, aims and results of the didactic diagnostics of first-grade students' performance in the cognitive field focusing on the basics of mathematics.

Key words. Didactic diagnostics, tertiary education, mathematics.

Mathematics Subject Classification: Primary 97A10, 97D40; Secondary 97E10.

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SIMULATION OF OPTION PRICES USING GARCH PROCESSES FOR AUTOCORRELATED STOCK RETURNS

FJODOROVŠ Jegors (LV)

Abstract. In this paper we describe practical example of our studies for an analytical solution for a problem of pricing financial actives with autocorrelated returns. This example is based on option price for Tesla Motors Inc stock. In previous studies we developed a continuous diffusion model for the case of serially correlated stock returns, obtain European call option pricing formula and show that even small levels of predictability due to serial correlation can give substantial deviation from results obtained by Black-Sholes formula and made Monte Carlo simulation for stock volatility. Finally, we showed recalculated option price, based on the simulation result.

Key words. Discrete time stochastic difference equation systems, GARCH models, Markov chain, Black Scholes option pricing formula.

Mathematics Subject Classification: Primary 39A50; Secondary 60J20.

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**OBSERVATION OF THE GOLDEN RATIO
 IN SPIRALS OF TRIANGLES AND SQUARES**

GELATTI Gabriele (I)

Key words. Golden ratio, gnomon, spiral, square, triangle.

Mathematics Subject Classification: 51M15.

In this paper it is introduced the notion of *L golden gnomon*, and some properties of it are observed as applied to the growth of spirals. Spirals of squares and of triangles are especially observed, with relevance of the *golden ratio* as a geometrical link between the two polygons.

1. The *L golden gnomon*

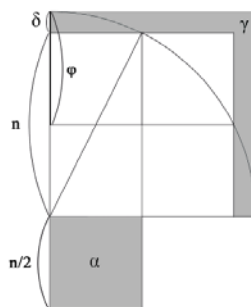


Figure: construction of the *L golden gnomon*

The area of the *L golden gnomon* is equivalent to 1/4 of the square, so that $\alpha = \gamma$. This can also be expressed in a formula:

$$\sqrt{n^2 + (n/2)^2} + (n/2) = \Phi_n \quad \text{and} \quad \sqrt{n^2 + (n/2)^2} - (n/2) = \Phi_n$$

2. Spirals of squares

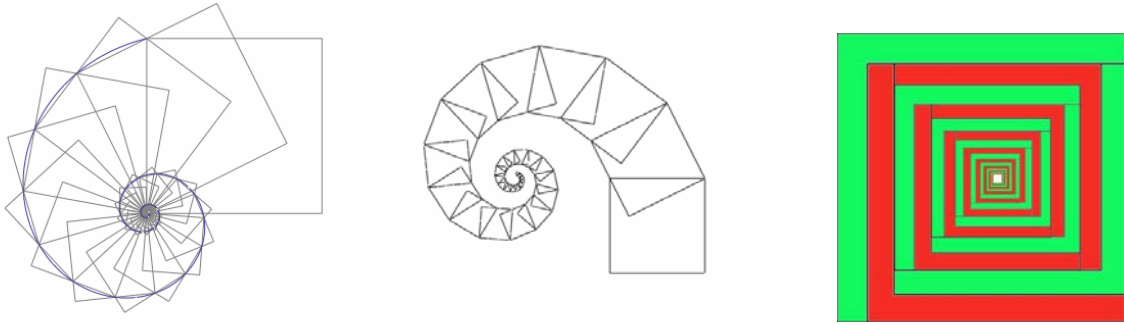


Figure: spirals growing the square $5/4$ from the square $4/4$

3. Spiral of triangles

A spiral of triangles can be produced where the side of every triangle is naturally equal to the height of the previous one. The side of the bigger triangle is the diameter of the circle inscribing the smaller one, so that the areas grow of one-third.

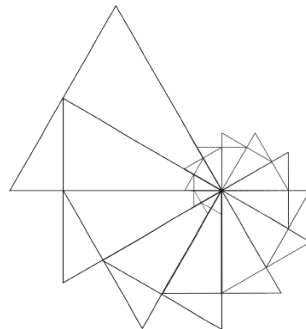


Figure: growth of $1/3$ in a spiral of triangles

In figure below be $CE = 1$, then $AE = \sqrt{5/4} + 1/2 = \Phi$ [Odom, 1983] then $AB = \sqrt{5/4} - 1$ and $AS = \varphi\sqrt{2}$ and $AF = \sqrt{2}$. By the theorem of Thales on proportional line segments, it is demonstrated that point S cuts in *golden section* also segment EF^1 , so that $ES = \varphi\sqrt{3}$.

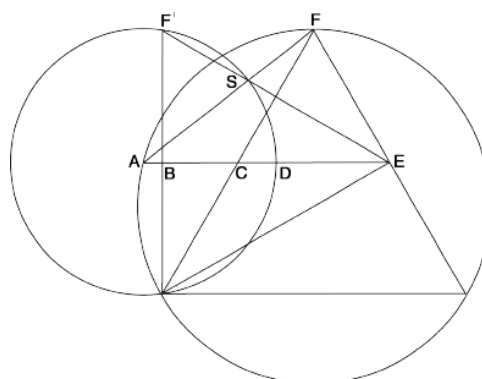


Figure: growth of one-third in the triangle with *golden ratio* of $\sqrt{2}$ and $\sqrt{3}$

The *golden ratio* appears in the growth of the triangle of one-third, and connects to the growth of the square of one-quarter. In figure below it is now constructed the square with side $BC = 1$, then AB will be the measure of a *L golden gnomon* with area $1/4$.



**GARCH MODEL APPLICATIONS.
COMPARISON OF STATIONARY SOLUTIONS OF STOCHASTIC
DIFFERENCE AND DIFFERENTIAL EQUATIONS.**

GEHSBARGS Aleksandrs (LV)

Abstract. Main results for stochastic differential equations apply to their stationary solutions or to the solution for time tending to infinity. For obvious reasons underlying processes are simulated with stochastic difference equations. Such equations also have stationary solutions that depend on the time step h. In the current paper relation between statistical characteristics of stationary solutions (first three moments and p.d.f.) of stochastic difference and differential equations are investigated. Moreover, possible approaches to the choice of time step h are discussed.

Key words. GARCH models, stochastic difference equations, stochastic differential equations, stationary solutions.

Mathematics Subject Classification: 60H30, 60H35

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**ON THE SYNERGY OF ART, TECHNOLOGY, RITUAL
AND MATHEMATICS. THE BÉZIER CURVES REVISITED**

GHEORGHIU Dragoș (RO)

Abstract. This paper, dedicated to the memory of Professor Mauro Francaviglia, pleads for a synthesis between art and mathematics, by presenting a set of new data on the Bézier curves, identified on 18th century azulejo from Portugal.

As the Portuguese artifacts show, the ritual action of tracing the curves can be considered a cultural universal, its presence being identified after thousands of years span and thousands of kilometers distance in the prehistoric Balkan objects discussed in a previous Aplimat issue. The existence of a mathematical function that copies a vegetal shape across different cultures also infers that Bézier curves are a cultural archetype, neglected until now by the history of art, and whose study would contribute to the understanding of certain mental processes.

Key words. Bézier curves, control points, azulejo, ritual.

Mathematics Subject Classification: 33; 33E99

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**A VIRTUAL MUSEUM ON ART AND SCIENCE PROJET “ADOPT”
A SCIENTIST IN CLASSROOM**

GIURGOLA Giliola (I), TRICARICO Michelangelo (I), LORENZI Marcella Giulia (I)

Abstract. In a virtual 3D world we have set up an exhibition using drawings made by students as results of the project “Adotta Scienza ed Arte nella tua classe”(Adopt Science and Art in your classroom). The exhibition is a tool to visualize and communicate secondary school student's creations on art and science. The virtual art and science museum completes the design of the project allowing remote access to an immersible environment in synchronous and asynchronous time. The platforms have a cultural/educational characteristics that stimulate creative works, improve teaching effectiveness and at the same time recover scientific and cultural values of great social importance.

Key words. Mathematics and Art, Teaching 3D, education, science, virtual worlds, OpenSim, museum.

Mathematics Subject Classification: 00A66, 97U80, 97U50.

Acknowledgement

Special thanks to all CraftWorld's builders, to expert Claudio Pacchiega, to Raffaele Macis.

This paper is dedicated to the memory of Prof. Mauro Francaviglia, a great scientist on Physics of relativity and a pioneer of the contemporary studies on Science and Art and their connection to science popularization in Italy and Europe.

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**ARTISTIC ASPECTS OF COMPLEX FUNCTIONS
IN ALGEBRAIC SURFACES**

IORFIDA Vincenzo (I)

Abstract. The mathematics of the Nineteenth century was characterized by the development of complex analysis, that has also had a significant role in the study of physics. This process was more profitable, in many aspects, than the real analysis. The study of complex functions has been carried out independently, since, according to most of the mathematicians, the problems with complex numbers have emerged from their use in the theory of functions and not in algebra. We will examine some artistic aspects, related to the glamour of modeling of complex numbers whose shapes pass through the cryptography.

Key words. Algebraic Curves, Conformal Maps, Geometry, Art.

Mathematics Subject Classification: AMS_01A99.

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NETRADIČNÍ PŘÍSTUPY K MATEMATICKÉ ANALÝZE I NEKONEČNĚ MALÉ VELIČINY

JANČAŘÍK Antonín (CZ), PILOUS Derek (CZ)

Key words. matematická analýza, nekonečně malé veličiny, alternativní přístupy.

Mathematics Subject Classification: Primary 97U50.

Matematická analýza měla své nezastupitelné místo v učivu matematiky na gymnáziích po více než sto let. První snahy o rozšíření učiva matematiky o tuto oblast jsou však již mnohem starší, v českých zemích se datují až do poloviny 19. století. Roku 1864 vychází česky psaný text prof. Šimerky. Příklad k Algebře pro vyšší gymnázia, který přehledným způsobem seznamuje čtenáře s touto oblastí i se způsobem, jak ji do učiva zařadit. Systematická snaha o zařazení diferenciálního a integrálního počtu do učiva středních škol byla odstartována slavným projevem Felixe Kleina na setkání německých přírodovědců v Meranu roku 1905. Dle Kleina musí osu vyučování matematice tvořit funkční myšlení, jehož nedílnou součástí je i diferenciální a integrální počet.

Politické požadavky na to, aby stále větší část populace měla úplné středoškolské, resp. vysokoškolské vzdělání si vybírají svou daň na rozsahu a kvalitě probíraného učiva. Již pouhý pohled na Gaussovu křivku rozdělení inteligence v populaci nás přesvědčí o tom, že maturitu musí být schopni složit i studenti průměrní až podprůměrní. Tomu jsou přizpůsobovány i nároky na ně. Je smutným faktem, že po sto letech od počátku Kleinových snah o zavedení kalkulu na střední školy, roku 2007, vstoupil v České republice v platnost Rámcový vzdělávací program pro gymnázia, který diferenciální a integrální počet zcela ignoruje. V rámci obsahu učiva se vracíme o více jak 100 let zpět a pojmy jako limita, derivace či integrál se opět stávají učivem nepovinným, kterému může přikládat pozornost učitel pouze v rámci volitelné nadstavby.

Změna obsahu učiva způsobuje, že na vysoké školy přicházejí žáci, jejichž porozumění matematice je o mnoho menší, než v předchozích letech. Vypuštění kalkulu z učiva středních

škol postihuje především školy přírodovědného a technického zaměření, které nejenom kalkulus, ale především funkční myšlení přímo potřebují pro praktické aplikace. Již v předchozích letech si zástupci těchto škol stěžovali na stále klesající úroveň znalostí nastupujících středoškoláků. Nyní přichází generace středoškoláků, jejichž znalosti v této oblasti jsou z naprosto objektivních důvodů nulové.

V rámci pedagogické fakulty univerzity Karlovy v Praze se snažíme problémům žáků a studentů předcházet a to jak v přípravě budoucích učitelů, tak v rámci přípravy budoucích učitelů, tak i v rámci vzdělávacích akcí pro učitele z praxe a aktivit pro středoškolské studenty. Příkladem může být aktuálně řešený projekt Podpora vědy a přechodu na VŠ (reg. č. CZ.2.17/3.1.00/36215), který se zaměřuje na středoškolské studenty a jejich učitele. V rámci projektu byl zvolen méně obvyklý přístup k matematické analýze, který se vrací k původním historickým přístupům tohoto oboru. Výklad základních pojmů a jejich aplikace v rámci geometrie a fyziky je prováděn prostřednictvím pojmu nekonečně malé. Při tomto přístupu sice není dosahováno absolutní korektnosti při definování pojmů i formulování hypotéz, je však v žácích podporován intuitivní přístup a porozumění pojmům, včetně schopnosti je aplikovat.

V rámci projektu byl vytvořen kurz pro učitele nazvaný Alternativní přístup k matematické analýze a on-line kurz matematické analýzy pro středoškolské studenty. Cílem kurzu kurzu pro učitele je seznámit je s možností intuitivního přístupu k matematické analýze, jeho výhodami a nevýhodami. Kurz pro studenty si klade za cíl seznámit studenty se základními pojmy a postupy v matematické analýze, porozumění významu jednotlivých pojmů a zvládnutí základní výpočtů. Článek představuje metody, které byly pro výklad v rámci vytvořených kurzů použity.

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**BLENDED LEARNING IN TEACHING MATHEMATICS
AT PRIMARY AND SECONDARY SCHOOL**

KOREŇOVÁ Lilla (SK)

Abstract. In the recent years the usage of electronic materials among math-teachers has been steadily increasing. E-materials are part of a new trend, the so called blended learning, i.e. a new teaching method combining physical and virtual sources. In this presentation we would like to show the possibilities lying in the application of "blended learning" in the teaching of mathematics and some examples of e-materials(in GeoGebra). We will also show the potential of e-materials to develop the creativity of already gifted students.

Key words. blended learning, e-learning, constructivism, GeoGebra.

Mathematics Subject Classification: Primary 97U50.

E-materials are part of a new trend, the so called blended learning, i.e. a new teaching method combining physical and virtual sources. Besides the classical presentation of the material, the students have access to electronic materials such as videos, presentations, electronic worksheets, java applets, e-tests and educational software's like GeoGebra. These materials can be attained through web-applications accessible not only through a computer but also through tablets and smartphones. The presence of tablets, smartphones and netbooks grows in direct proportion to their price. Now days, almost every student owns a smartphone with an internet connection which allows an individual approach to electronic materials. Teachers can thus exploit the potential of ICT in the teaching of mathematics.

- In the context of teaching mathematics in an ICT environment there are several ways of approach:
- Direct teaching (face to face) - both teachers and students have access to ICT and use it actively
- E-learning - learning and teaching only in electronic form

- blended learning - a combination of contact teaching and integrating ICT means while actively using e-learning. "(Žilková, 2010)
- To make the teaching more effective teachers can now use electronic materials:
- videos, presentations (created in MS PowerPoint, Prezi, software's for interactive boards), concept maps, electronic texts - these materials are used for self-study or as course material for students who need an individual approach (physically, mentally or socially disadvantaged groups of students or vice versa, gifted students) or can be also used in new methods of learning (constructivism in the digital environment, the Flipped Classroom method)
- Electronic worksheets, e-tests that are used to practice the curriculum, mathematical Java applets, educational software such as GeoGebra and the like.
- Electronic tests to evaluate

Acknowledgement

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MATH(ML) IN EDUCATION DIGITAL CONTENT

KOVÁČOVÁ Monika (SK)

Abstract. Publishers and authors want math to be at least as straight-forward to produce as a print book with high-quality equation presentation. This paper explains the various options to create MathML and deliver it to all platforms and devices.

We will speak about MathML, SVG, MathJax, PNG in combination with xml and ePub.

Key words. MathML, MathJax, SVG, PNG, ePub.

Mathematics Subject Classification: 97R70, 97R10.

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FAULT DETECTION & ISOLATION ALGORITHMS AND MODELING

KVAPIL David (CZ), WEISMAN Andrej (SK)

Abstract. Fault detection and isolation (FDI) is very important for enhancing the safety and reliability of aircraft systems. Several mathematical methods and algorithms useful for processing and operation evaluating in engine control systems are presented in this article. The role of FDI in System Health Monitoring (SHM) issues and Data-Driven approach is reminded. Residual-based modeling and analysis is considered. Control chart approach and state estimator using Kalman filtering is presented and modelled. Perturbed system with implemented dummy fault is simulated and analyzed, all simulations are created in MATLAB.

Key words. Control charts, estimation, detection, reliability and life testing.

Mathematics Subject Classification: Primary 90B25; Secondary 93E10, 62N05, 62P30.

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**PARAMETRIC HYBRID WALL:
A RESPONSIVE SURFACE FOR EXHIBITION DESIGN**

LALATTA COSTERBOSA Cecilia (I)

Abstract. A Responsive Surface is an architecture able to model its own shape in relation to contextual specific impulses. The surface shapes itself differently depending on the stimuli which are related to a spectrum of variables classified as sensors. The deformation of the surface happens subsequently to the received impulse: the dynamism of these structures is then the result of the interaction between the end-user, who acts upon the sensors, and the surface, which modifies its own shape after having been subjected to the stimuli of the sensors. Parametric Hybrid Wall not only presents these preliminary characteristics, hence resulting to be a responsive surface, but also it outlines a further process which combines the planning parametric approach with specific instruments for Open Source and Low Cost modelling and prototyping.

Key words. Parametric geometries, responsive surfaces, Open Source.

Mathematics Subject Classification: 74M05.

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**PROFESSOR MAURO FRANCAVIGLIA:
MATHEMATICIAN BY PROFESSION, ARTIST IN HIS HEART.
HIS ROLE IN TEACHING, RESEARCH AND COMMUNITY
CREATION IN MATHEMATICS AND ART.**

LORENZI Marcella Giulia (I)

Abstract. A brief retrospect on Professor Mauro Francaviglia's biography and incredibly prolific scientific carrier, focused on his role in teaching, research and community creation in Mathematics and Art. In particular, he was the founder of the Mathematics and Art section within Aplimat Conference: from this year, it will be dedicated to his memory, with gratitude for the great ideas he left here behind him and the community he created.

Key words. architecture, art, geometrical inspiration.

Mathematics Subject Classification: Primary 00A67.



Fig. Mauro Francaviglia. National Conference on Relativity and Physics of Gravitation at Scuola Normale di Pisa, Italy (2010) – Photo copyright Marcella Giulia Lorenzi

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I wish to thank all the Aplimat organizers and those who contributed to the session on Mathematics and Art dedicated to Mauro, and those who have present after his departure.

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**A TONELLI-TYPE FUNCTIONAL-INTEGRAL
EQUATION, VIA WEAKLY PICARD OPERATORS TECHNIQUE**

MUREȘAN Viorica (RO)

Abstract. In this paper we consider a Tonelli-type functional – integral equation with linear modification of the argument in a Banach space. By using weakly Picard operators' technique we obtain existence, data dependence and comparison results for the solutions.

Key words. Picard operator, weakly Picard operator, functional-integral equations, solutions set, data dependence, comparison theorems.

Mathematics Subject Classification: 34K05, 34K15, 47H10.

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PARTITION PROBLEM AND ANALOG METHODS

MYCKA Jerzy (PL), PIEKARZ Monika (PL), ROSA Wojciech (PL)

Key words. Partition Problem, NP-problems, Analog Computation.

Mathematics Subject Classification: 65Y05, 65Y06, 68R04.

In this paper we test non-standard methods of dealing with one of the most famous NP-problems - namely with the partition problem. We can present the partition problem as follows: is it possible to divide a given finite set S of integer numbers into two subsets S_1 and $S \setminus S_1$ such that the sum of the elements of the set S_1 is equal to the sum of the elements of $S \setminus S_1$?

We will look at technical details of analog integration suited for our problem and partially based on the *GPAC* model. We employ elements of analog computation in two ways: first by use of the specific analog device, second by use of numerical analysis.

The above mentioned analog device is a part of a hybrid construction consisting of an analog unit, a digital computer and some set of communication devices (D/A, A/D converters and relays). Numerical method used for our problem is simply the composite trapezoidal rule.

For chosen examples we compare times of computations and reliability of obtained answers. Results of experiments are presented in 4 tables with details of computations: ranges, sums and elements of every used set, answers, time and safety of the result for the proper method. We were testing problems varying in the number of elements (5,10,30,50), the range of elements (from 10^2 to 10^8) and the final answer to the partition problem (yes, no). We provide some estimations of bounds guaranteeing the proper answer of the mentioned methods. In particular the following estimation

$$h \leq \sqrt{\frac{3}{2^{n+1}}} \frac{1}{\sum_{j=1}^n |x_j|}$$

gives us such a length of subintervals of numerical integration which is necessary for the partition problem.

The article ends with some discussion of conclusions: how important are ranges of sets for complexity of the solution; how inherent is exponential character of the partition problem in the case of analog computation. Finally, we suggest that despite their theoretical attractiveness analog methods are for the moment no real alternatives for digital, discrete NP-complete algorithms. This remark could be further tested by use of more sophisticated analog devices and stronger numerical methods.

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**PLASTIC DEFORMATION OF Cd-Zn ALLOYS SINGLE
CRYSTALS – SOME EXPERIMENTAL PROBLEMS**

NAVRÁTIL Vladislav (CZ), NOVOTNÁ Jiřina (CZ)

Abstract. The character of plastic deformation is a sensitive function of such variables as temperature, the strain rate of deformation, the past history of the sample, crystal size and other parameters. It is purpose of the present work to examine the reproducibility of measurements of some basic parameters, characterized plastic flow of metals, depending on stress, temperature and concentration of impurities. Our investigations proved that the stress dependence of activation area and stress sensitivity parameter does not depend on such subjective and objective conditions as adding or removing stress increment, or the strain rate at which the increment or decreasing stress is realized.

Key words. Creep, activation area, stress sensitivity parameter, stress change, concentration of solutes, reproducibility of measurements.

Mathematics Subject Classification: 74Cxx .

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**A NEW METHODOLOGICAL APPROACH TO UPGRADING
THE STATEMENT OF GROSS DOMESTIC PRODUCT (GDP)
GROWTH RATES AND IMPLICIT GDP DEFLATORS**

NJAVRO Mato (HR), RAGUŽ Andrija (HR), ŠUTALO Ivan (HR).

Abstract. The purpose of this paper is to present a new methodological approach to upgrading the statement of Gross domestic product (GDP) growth rates and implicit GDP deflators. We construct Lloyd-Moulton-Törnqvist-Fisher (LMTF) model which improves GDP price-volume decomposition. LMTF supported Fisher index is also built. Our approach preserves the product test identity. We obtain some empirical results which show that LMTF supported Fisher index can be considered as "ideal" in the practical applications.

Key words. Törnqvist, Fisher and Lloyd-Moulton (LMTF) model, Fisher index supported by LMTF model, Gross domestic product (GDP) decomposition, superlative indices, elasticity of substitution, additive GDP consistency.

*Mathematics Subject Classification: Primary 91B82, 91B02; Secondary 00A06.
JEL Subject Classification: Primary E01, C02; Secondary C02*

The main goal of this paper is to establish a new methodological approach to upgrading the statement of Gross domestic product (GDP) growth rates and implicit GDP deflators – on annual and quarterly bases. The traditional methodological approach in the practice of National statistical agencies around the world is the chain-linking methodology. The main mathematical apparatus of the chain-linking methodology are two indices. The first one, Laypeyres index with fixed base, substantially overestimates the second, Paasche index, which is the most appropriate GDP deflator due to statistical (Cauchy theorem) and economic (substitution-transformation effect) reasons. By means of chain linking, index number drift has been resolved partially in the sense of the second best solution. But index number mathematics provides a solution. By it's theoretical considerations Törnqvist and Fisher

indices have been chosen among so called “superlative indices” as superior ones in terms of GDP compilation. According econometric estimations Lloyd-Moulton index has been also calculated as the best estimator of elasticity of substitution. By putting together Lloyd-Moulton with Törnqvist and Fisher indices, we construct Lloyd-Moulton-Törnqvist-Fisher (LMTF) model. LMTF model improves GDP price-volume decomposition due to more precise substitution measurement. Fisher index supported by LMTF model is also built as it resolves the problem of additive (absolute and relative) inconsistency in GDP data. The whole estimation procedure is implemented on the case study of Croatia (the data base dealing with Croatian Quarterly GDP data relates to the period from q1 2000 to q4 2007). Thanks to the approach proposed in this paper, ex-post smoothing of the preliminary raw-data driven by original (price and volume) indicators not only preserves indicator content of GDP data, but also rectifies “maturity” of GDP data. An integral part of the survey are test results which prove that Fisher index supported by LMTF model can be considered as "ideal" in the practical applications. To conclude, the new methodological approach proposed in this paper has at least three advantages : a) it better decomposes “mature” GDP data on price and volume, b) it assures additive consistent GDPs for publication and c) it preserves (by means of Fisher index supported by LMTF) product test identity (value = volume times price). This is the reason why we recommend it.

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**CREATING NEW “MATHEMATICALLY-SUSTAINABLE”
WORLD: FROM THE SPIROGRAPH, A REVERSE PATH**

PALLADINO Nicla (I), PASTENA Nicolina (I)

Abstract. The approach to the subject of mathematic learning, represents, in the pedagogical-educational field, a problematic situation. Nowadays we attend to an heated discussion on the character of the basic mathematical concepts, on the analytical-critical succession between processes and objects. The purpose of these notes is to recover and to value the contribution of the autopoiesis theory in the characterization of the mathematical domain going beyond the mere reiteration, inspecting and testing new and generative paths of ideas. In this mechanism we would insert the use of the spirograph as a disturbing, uncertain element able to nourish the “mind-system”.

Key words. Reiteration, spirograph, regular polygons.

Mathematics Subject Classification: Primary 97G40; Secondary 00A08.

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BOOK OF ABSTRACTS

NUMBERS

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Abstract. Pythagoras saw numbers in everything and he was right. Our Universe lays in 80 octaves of vibrations and countless secrets. Every vibration is a number.

Key words. Digits, numbers, prime numbers, messages.

Mathematics Subject Classification: 91F99.

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POSLOUPNOSTNÍ A FUNKČNÍ PŘÍSTUP K VÝUCE LIMIT

PILOUS Derek (CZ)

Key words. limita posloupnosti, limita funkce, proces a koncept.

Mathematics Subject Classification: 97I30, 97I40.

Výuka standardní analýzy na vysokých školách začíná limitami. Existují dva přístupy: posloupnostní, který zavádí nejprve limitu posloupnosti a pak buduje limitu funkce (pomocí okolí či v návaznosti na posloupnosti podle Heineho), a funkční, který začíná rovnou definicí limity funkce (pomocí okolí) a limity posloupností, jsou-li zařazeny, jsou brány jako její speciální případ. Motivací pro funkční přístup bývá především rychlé zavedení metod diferenciálního a integrálního počtu. Postup přes limity posloupností má ovšem některé didaktické výhody.

Především je limita posloupnosti chápána jako proces, zatímco $\varepsilon - \delta$ definice limity funkce je obtížněji pochopitelným a variabilnějším konceptem. Definici posloupnosti také lze také jednodušeji virtuálně redukovat o dva kvantifikátory, což podstatně napomáhá jejímu porozumění. Procesuální chápání je pak díky Heineho definici možno přenést i na limitu funkce.

Jako proces je model limity posloupnosti zpravidla budován na pojmu blížení. To umožňuje přiblížit koncept velmi rychle, ale způsobuje některé miskoncepce, nejčastěji předpoklad, že posloupnost nemůže dosáhnout limitní hodnoty. Možným řešením je zavedení pomocí jiného prekonceptu, atraktoru.

Zatímco definice limity je v teorii posloupností jednodušší než v teorii funkcí, u běžných typů úloh je tomu obecně naopak. V úlohách úloh konstrukčních, tedy nalézání objektů daných vlastností, jsou posloupnosti omezeny existencí jediného hromadného bodu jejich definičního oboru. Proto se hůře konstruje např. posloupnost neomezená shora ani zdola než funkce se stejnými vlastnostmi. Ještě větší rozdíl je v úlohách na limity dané středoškolskými výrazy – v posloupnostech je např. možno na rozdíl od funkcí používat faktoriál, především ale

diskrétní povaha posloupností mění někdy úlohy, které jsou pro spojité funkce triviální, v těžké problémy z teorie čísel. Příkladem je $\lim | \sin n |^n$.

Základy teorie posloupností a limit jsou shodné, avšak každá z teorií má i svoje speciální nástroje, vyplývající z diskrétního či naopak spojitého charakteru jí zkoumaných objektů. U posloupností jsou to např. věty nazývané podílové a odmocninové kritérium podle vět z teorie řad, s nimiž sdílejí předpoklady. Speciální nástroje teorie funkcí jsou bohatší: spojitost, věta o limitě složené funkce (VLS) a důsledky derivací (Taylorovy polynomy).

Bez VLS či její obdoby lze určovat limity superpozic (jako $\lim \sin \frac{1}{n}$) jen obtížně ad hoc, a také bez ní nelze používat při výpočtu limit substitucí. Ukážeme, jak lze tyto dvě aplikace věty o limitě složené funkce (VLS) provést v teorii posloupností bez použití limit funkcí.

Ze zjevných důvodů lze obecnou reálnou funkci s posloupností skládat pouze jedním způsobem, vnější funkci s vnitřní posloupností.

Věta (O limitě složení funkce a posloupnosti)

Nechť je reálná funkce f definovaná na intervalu $\langle a, b \rangle$ a nechť $\lim_{x \rightarrow \infty} a_n = L \in (a, b)$. Nechť je dále f monotónní na $\langle a, L \rangle$ a $\langle L, b \rangle$ a obraz každého z těchto intervalů nechť je buď interval nebo jednoprvková množina. Pak $\lim f(a_n) = f(L)$.

Důkaz této věty je jednoduchý, byť poněkud zdlouhavý, neboť rozebírá různé případy monotonií zleva a zprava. Jednouše jde odvodit jednostranné analogie a případ, kdy je f definována pouze na prstencovém okolí L , díky čemuž může být L i nevlastní. V tomto případě je třeba nahradit funkční hodnotu $f(L)$ supremem či infimem funkce okolí L (podle monotonie). Lokální monotonie je silným předpokladem, který je ale snadno ověřitelný, u základních funkcí je ve vlastních bodech splněn, a umožňuje místo limity používat jednodušších pojmů suprema a infima. Příklad užití: fce sinus splňuje předpoklady v okolí každého vlastního L , proto $\lim \sin \frac{1}{n} = \sin \lim \frac{1}{n} = \sin 0 = 0$.

Typickým příkladem na užití substituce v limitách funkcí je $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$, který se převádí substitucí $y = \ln x$ na již známou limitu $\lim_{y \rightarrow \infty} \frac{y}{e^y}$. Takto jednoduše u posloupností postupovat nelze.

Věta [O substituci]

Nechť (m_n) je posloupnost celých čísel divergující k $+\infty$ a $\lim a_n = L \in \mathbb{R}^$. Pak $\lim a_{m_n} = L$.*

Větu lze chápat jako generalizaci věty o limitě vybrané posloupnosti (což je stejné tvrzení, ale s předpokladem, že (m_n) je rostoucí). Vzhledem k tomu, že vnitřní posloupnost (m_n) musí být celočíselná, je třeba zpravidla provádět zaokrouhlení a sevřít vyšetřovanou posloupnost mezi dvě posloupnosti, na něž již lze větu použít. To lze dostatečně jednoduše obvykle jen tehdy, jsou-li zúčastněné posloupnosti od nějakého členu monotónní. Ukažme si to výše uvedeném příkladě:

$$0 \leq \lim \frac{\ln n}{n} = m \frac{\ln n}{e^{\ln n}} \leq \lim \frac{\lfloor \ln n \rfloor + 1}{e^{\lfloor \ln n \rfloor}} \stackrel{(*)}{=} \lim \frac{n+1}{e^n} = 0$$

Rovnost označená hvězdičkou plyne z věty o substituci, přičemž $m_n = \lfloor \ln n \rfloor$, závěrečná rovnost z výše zmíněného podílového kritéria.

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TWO GENERAL FORMULAS FOR THE SUM OF THE REDUCED HARMONIC SERIES GENERATED BY n PRIMES

POTŮČEK Radovan (CZ)

Abstract. This paper is a follow-up to author's papers dealing with sums of the reduced harmonic series generated by 2, 3, and 4 primes. These sums were determined analytically and computed by using CAS Maple. In this paper two general formulas for the sum of the reduced harmonic series generated by n primes are derived and proved by mathematical induction. These series so belong to such special types of convergent series, as geometric and telescoping series, which sums can be easily calculated by means of a simple formula.

Key words. harmonic series, geometric series, reduced harmonic series, sum of the series.

Mathematics Subject Classification: Primary 40A05; Secondary 65B10.

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**'NORTHEAST VOLATILITY WIND' EFFECT
AND IT'S FORECASTING**

PUCKOVŠ Andrejs (LV), MATVEJEVS Andrejs (LV)

Abstract. This paper describes volatility forecasting approach applicable for stock indexes, allowing to reveal the instability of financial time series initially. This approach is based on time series (signal) decomposition into components by using wavelet filtering with subsequent volatility evolution research in time of each signal component. According to research, a slight increase in volatility in the low-frequency components of the signal leads to significant disturbances in high-frequency components destine entire signal volatility growth.

Key words. 'North-East Volatility Wind' Effect, Wavelet filtering, Direct Continuous wavelet transform (Direct CWT), Inverse Continuous wavelet transform (Inverse CWT), Signal Decomposition, Volatility evolution, Disturbances transmission, Time Series, Stock Index.

Mathematics Subject Classification: Primary 42C40, 65T60; Secondary 46N30.

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NUMERICAL SOLUTION OF A SINGULAR FREDHOLM INTEGRAL EQUATION OF THE SECOND KIND DESCRIBING INDUCTION HEATING

RAK Josef (CZ)

Key words. integral equation of the second kind, induction heating, collocation method, Nyström method.

Mathematics Subject Classification: Primary 65R20, 45B05; Secondary 65Z05.

A bounded metal body Ω_1 with a Lipschitz-continuous boundary is heated by external electromagnetic field produced by inductor Ω_2 .

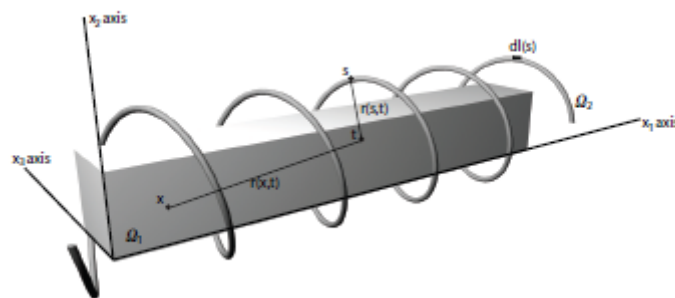


Figure 1. Heated body and coil.

For simplicity let us assume that Ω_1 is a cuboid. The inductor is formed by a conductor of general shape and position that carries harmonic current I_{ext} . The main task is to compute the eddy current of density J_{eddy} . It is described by integral equation of the second kind of the form

$$iJ_{eddy}(x) - \kappa(x) \int_{\Omega_1} \frac{J_{eddy}(t)}{r(x,t)} dt_1 dt_2 dt_3 = \kappa(x) I_{ext} \int_{\Omega_2} \frac{dl(s)}{r(x,s)} \quad (1)$$

where

$$\kappa(x) = \frac{\omega \gamma (T(x)) \mu_0}{4\pi}, \quad (2)$$

$r(x,t)$ is the Euclidean distance, $x = (x_1, x_2, x_3)$ and $t = (t_1, t_2, t_3)$ are points in the metal body, $s = (s_1, s_2, s_3)$ is a point at the inductor, ω is angular frequency, γ electrical conductivity, μ_0 permeability of vacuum and i complex unit. For each bounded and continuous temperature distribution T , $\kappa(x)$ is a real, positive, bounded and continuous function. Since $J_{eddy} = (J_{eddy,x_1}, J_{eddy,x_2}, J_{eddy,x_3})$ is a phasor we can rewrite (1) into three complex integral equations for J_{eddy,x_1} , J_{eddy,x_2} and J_{eddy,x_3} . With the notation $J_R(x) = \text{Re } J_{eddy,x_1}(x)$, $J_I(x) = \text{Im } J_{eddy,x_1}(x)$, $I_R = \text{Re}(I_{ext})$ and $I_I = \text{Im}(I_{ext})$ the equation for J_{eddy,x_1} can be split into equivalent system of two real equations

$$J_R(x) - \kappa(x) \int_{\Omega_1} \frac{J_I(t)}{r(x,t)} dt = I_I F(x), \quad -J_I(x) - \kappa(x) \int_{\Omega_1} \frac{J_R(t)}{r(x,t)} dt = I_R F(x). \quad (3)$$

Formulas for the remaining components J_{eddy,x_2} and J_{eddy,x_3} can be obtained by mere interchanging of indexes. The specific Joule losses which are needed to compute temperature distribution are given by

$$\omega_j(x) = \frac{1}{\gamma} J_e(x) \overline{J_e(x)} \quad (4)$$

where

$$J_e(x) = \sqrt{[\text{Re} J_{eddy,x_1}(x)]^2 + [\text{Re} J_{eddy,x_2}(x)]^2 + [\text{Re} J_{eddy,x_3}(x)]^2} + i \sqrt{[\text{Im} J_{eddy,x_1}(x)]^2 + [\text{Im} J_{eddy,x_2}(x)]^2 + [\text{Im} J_{eddy,x_3}(x)]^2}$$

and $\overline{J_e(x)}$ is complex conjugate to $J_e(x)$.

For solution of (1) was used piecewise constant collocation method and Nystöm method. Both method were applied for a brass cuboid body with the measures $0,15 \times 0,01 \times 0,01$ m which is heated with a stationary inductor starting at the room temperature 20°C . The inductor has the form of a coil which turns around the heated body in the x_1 -direction in 6 loops. Radius of the coil is $0,015$ m, exciting current 500 A, frequency 150 kHz. The length of the coil is $0,15$ m. The cuboid is partitioned by 75 elements in x_1 direction, 5 elements in x_2 and x_3 direction. First two graphs on figure 2 show the specific Joule losses distribution calculated by piecewise constant collocation (left picture) and Nyström method with compound mid-cuboid integration rule (right picture) on the x_1 axes with $x_2 = -0.004$ where the blue color matches $x_3 = -0.004$, the red color matches $x_3 = 0$ and the black color matches $x_3 = 0.004$. The same situation with $x_2 = 0$ show last two graphs. As we can see both pictures have similar results.

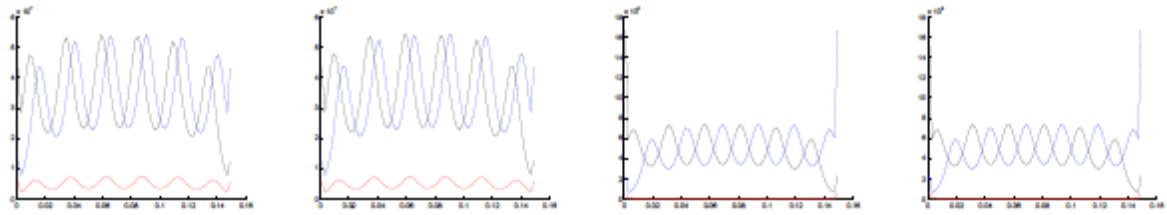


Figure 2. $x_2 = -0.004$

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THE ROLE OF BASIC GEOMETRICAL PATTERNS

RIČHTÁRIKOVÁ Daniela (SK)

Key words. mathematics and art, the role of basic geometrical patterns in history, abstraction, geometrical design, prehistoric pottery on the territory of Slovakia, folkwear, modern art.

Mathematics Subject Classification: 00A66.

Findings, documenting the oldest occurrence of elementary planar geometric patterns created by a man, date back even to the Palaeolithic era. The palaeolithic art artefacts, e.g. a double spiral with clockwise rotation, engraved on an amulet of mammoth tooth (Ukraine), or a triangle instead of a heart in the rock carving of bull (Africa), both cca 20,000 years old, indicate that these patterns play crucial role in development of human abstraction ability. Neolithic pottery artefacts (found also on territory of Slovakia) represent a reliable source for cultural phylogeny study. Patterns like circles, spirals, undulating curves, or line segments arranged into triangles, squares or rectangles, as well as zig-zags, meanders, crosses, or svastikas have been present on artworks since that time till present days reflecting the human spiritual life. Rhythm, regularity, and harmony always fascinate people. The outstanding invariant properties of patterns determine their outstanding position within the history time. They have always had their own meaningful value symbolizing various qualities and powers of nature, humans, or divinities; although their meaning slightly differed with respect to topical culture, religion or other field of application. Through centuries they have achieved a top in process of abstraction, they become to be a symbol of purity in modern art. Beside symbolics, regular patterns had an important function also in other aspects of living. The first dwellings were of circular or oval shape. The shape reflects the option possibilities of treatment with available natural materials, and what was very important, saves the energy at maximum volume. The later rectangular dwellings allowed to be enlarged in easy manner simply to be added on. Looking at art decoration from practical point of view, regular shapes allow to place the figures next to each other and form more complex patterns - ornaments. All

7 types of frieze ornaments, even signs of conformal and similarity symmetry were used in middle and late Palaeolithic. All 17 wallpaper patterns occurred in wall decorations of Neolithic and early Middle Ages.

The simple geometrical figures became the basis for the formation of ancient European script. The regularity of triangle or square stood at the beginning of natural numbers theory. Properties of basic geometrical figures, open the path to the studying of the invariants, “the fundamental organising principle in nature and culture“.

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**FIBONACCI NUMBERS OF GRAPHS CORRESPONDING
TO A TYPE OF HEXAGONAL CHAINS**

SEIBERT Jaroslav (CZ), ZAHRÁDKA Jaromír (CZ)

Abstract. The concept of the Fibonacci number of an undirected graph G , refers to the number of subsets S of $V(G)$ such that no two vertices in S are adjacent. In this contribution a variant of the decomposition theorem is derived. The Fibonacci numbers of graphs corresponding to one type of hexagonal chains are calculated by using the decomposition formula. Searching of the Fibonacci numbers of certain classes of graphs leads to difference equations or their systems. The Fibonacci number of hexagonal chain with linearly annelated hexagons is found as a function of the number of hexagons in the chain.

Key words. Fibonacci number, simple graph, recurrence, decomposition theorem, hexagonal chain, Fibonacene.

Mathematics Subject Classification: Primary 12H10, 39A10; Secondary 11B39.

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SIMPLE AUTOMATION OF SYNTACTIC ANALYSIS

SCHENK Jiří (CZ), ROMBOVÁ Zuzana (CZ)

Abstract. This article is focused first on the lexical analysis and then on the syntactic analysis and the processes connected to both of them. Also the way of creating simple generators for the analyzers with a given grammar is shown.

Key words. Syntactic analysis, syntactic analyzer, syntactic generator, lexical analysis, lexical analyzer, lexical generator, Backus-Naur form, lexem, grammar, token, terminal, nonterminal

Mathematics Subject Classification: 68Q45

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TWO METHODS OF QUADRATIC CALIBRATION

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Mathematics Subject Classification: Primary 62F25, 62F10; Secondary 62P30.

Calibration is a set of tasks which gives relationship between the reference device and the calibrated device (if some special conditions are realized). This relationship is described by the calibration function and represented by the calibration curve. In the contribution we assume that the calibration curve is a parabola. So we focus on the quadratic calibration. We use the maximum likelihood method and method based on linearized errors-in-variables model and Kenward-Rogers type approximation for obtaining the calibration curve. Both method will be presented and compared. Calibration process can be divided into two parts: 1) creation of the calibration model (calibration of the device), 2) application (use) of the calibration model (measuring by calibrated device). In the contribution we estimate parameters of the calibration function and their confidence region by both above mentioned methods and we also show the use of the calibration curve in the measuring process. At the conclusion both methods will be compared in a small simulation study. Programs used for the simulation study are programmed in software Matlab. We assume that we measure m different objects by two different devices with different precision (we assume that device A is less precise than device B). Each measurement is n -times repeated. It is assumed that measured values on both devices are realizations of independent random variables with normal distribution and the calibration curve is a quadratic function.

Maximum likelihood method

We know the distributions of random variables which represent the measurements and we know that they are independent, and therefore we can derive the likelihood function and the logarithmic likelihood function of the whole measurement model (of all measurements):

$$l(x_{11}, \dots, y_{mn}, \Theta) = \ln L(x_{11}, \dots, y_{mn}, \Theta) = -mn \ln(2\pi) - \frac{mn}{2} \ln \sigma_x^2 - \frac{mn}{2} \ln \sigma_y^2 - \frac{1}{2\sigma_x^2} \sum_{i=1}^m \sum_{j=1}^n (x_{ij} - \mu_i)^2 - \frac{1}{2\sigma_y^2} \sum_{i=1}^m \sum_{j=1}^n (y_{ij} - a - b\mu_i - c\mu_i^2)^2.$$

We easily obtain the system of likelihood equations. If we solve these equations we obtain estimators of calibrating function parameters. According to [4] this estimator has normal distribution

$$\hat{\Theta}^{abc} \sim N \left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \frac{1}{n} J(\Theta^{abc})^{-1} \right)$$

where $J(\Theta^{abc})$ is the Fisher Information Matrix. Consequently we calculate an asymptotic $(1-\alpha)$.100% confidence region for the vector of parameters (a, b, c) ,

$$*C_{(1-\alpha)}^1 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : \begin{pmatrix} \hat{a} - a \\ \hat{b} - b \\ \hat{c} - c \end{pmatrix} (\hat{\Sigma}^{abc})^{-1} \begin{pmatrix} \hat{a} - a \\ \hat{b} - b \\ \hat{c} - c \end{pmatrix} \leq \chi_3^2(1-\alpha) \right\}$$

which would be used for comparing the empirical coverage with the nominal coverage in simulated

measurements. We also derive the confidence region for the calibration function if the errorless (ideal) value measured by the less precise device is x ($\mathbf{L}_x' = (1, x, x^2)$)

$$P \left\{ \mathbf{L}_x' \hat{\Theta}^{abc} - \sqrt{\mathbf{L}_x' \hat{\Sigma}^{abc} \mathbf{L}_x} \cdot \chi_1^2(1-\alpha) < \mathbf{L}_x' \Theta^{abc} < \mathbf{L}_x' \hat{\Theta}^{abc} + \sqrt{\mathbf{L}_x' \hat{\Sigma}^{abc} \mathbf{L}_x} \cdot \chi_1^2(1-\alpha) \right\} = 1 - \alpha.$$

Further we focus on measuring with calibrated device, i.e. on deriving the confidence region for the true (errorless) value measured by the more precise measuring device B if on the less precise measuring device A the value x is read.

Method based on linearized errors-in-variables model and Kenward- Rogers type approximation

Firstly we assume that we make only one measurement of each object (see [2]). So we obtain the model

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N \left[\begin{pmatrix} \mu \\ v \end{pmatrix}, \begin{pmatrix} \sigma_x^2 \mathbf{I}_m & \mathbf{0}_m \\ \mathbf{0}_m & \sigma_y^2 \mathbf{I}_m \end{pmatrix} \right]$$

with nonlinear constraints on parameters $v_i = c\mu_i^2 + b\mu_i + a$. We linearize these constraints using Taylor series and obtain linearized constraints

$$\left(\text{diag}(b_0 \mathbf{1}_m + 2c_0 \mu_0), -\mathbf{I}_m \right) \begin{pmatrix} \delta \mu \\ v \end{pmatrix} + \left(\mathbf{1}_m, \mu_0, \mu_0^2 \right) \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

The measurements are repeated n times and we receive the replicated model

$$\begin{pmatrix} X^1 - \mu_0 \\ Y^1 \\ \vdots \\ X^n - \mu_0 \\ Y^n \end{pmatrix} \sim N \left[I_n \otimes \begin{pmatrix} \delta\mu \\ v \end{pmatrix}, I_n \otimes \begin{pmatrix} \sigma_x^2 I_m & 0_m \\ 0_m & \sigma_y^2 I_m \end{pmatrix} \right]$$

with the same constraints. This model is a special case of the incomplete indirect measurement model with condition of II. type to parameters of 1. order (see [1]). We estimate parameters of the mean by procedure described in [3] and parameters σ_x^2 , σ_y^2 are estimated by MINQUE method (see [1]). So we obtain an approximately normal distributed estimator of $(a, b, c)'$ (matrices \mathbf{A} and \mathbf{W} are computed according to [3])

$$\begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{pmatrix} \sim N \left[\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \left(\mathbf{A}' \left(\frac{1}{n} \mathbf{W} \right)^{-1} \mathbf{A} \right)^{-1} \right]$$

and also an approximated $(1 - \alpha) \cdot 100\%$ confidence region for the vector of parameters $(a, b, c)'$ (estimators of Σ_A , λ , u are computed according to [3])

$$C_{(1-\alpha)}^2 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : \begin{pmatrix} \hat{a} - a \\ \hat{b} - b \\ \hat{c} - c \end{pmatrix} \left(\hat{\Sigma}_A \right)^{-1} \begin{pmatrix} \hat{a} - a \\ \hat{b} - b \\ \hat{c} - c \end{pmatrix} \leq \frac{3F_{3,u}(1-\alpha)}{\lambda} \right\}$$

which would be used for comparing the empirical coverage with the nominal coverage in simulated measurements. We also show the confidence region for the calibration function if the errorless (ideal) value measured by the less precise device is x and we present how to measure with the calibrated device, i.e. we derive the confidence region for the true (errorless) value measured by the more precise measuring device B if on the less precise measuring device A the value x is read. Finally both methods are compared in a small simulation study.

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AUTOMATED THEOREM PROVING SYSTEMS

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Abstract. This article describes the process of theorem proving and number of independent experiments of theorem provers that were run in order to evaluate the most appropriate implementation for automated theorem proving.

Key words. automated theorem proving, prover, resolution, Prover9, SPASS, GERDS, Vampire, E, TPTP.

Mathematics Subject Classification: 68T15.

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CONSTRUCTION AND STATISTICAL ANALYSIS OF ZDENĚK SÝKORA'S LINES

SVOBODA Martin (CZ), MAREK Jaroslav (CZ)

Key words. Painter Zdeněk Sýkora, Nonlinear regression model with constraints, Linearization, BLUE, Least Squares Method.

Mathematics Subject Classification: Primary 00A66; Secondary 62J02, 62J05.

Czech painter Zdeněk Sýkora (1920 – 2011) is one of first painters who started using random variables as a tool for artistic production. He used a die for choosing parameters of curves. Later he used a computer for the simulation of those parameters.

In 1974 Zdeněk Sýkora created new line-based pictures with consistent use of randomness as the principle for all decisive data: the number of Lines (arcs), their starting points, width, lengths, colour, course and point of intersection. The main goal was achieve the maximum movement of individual lines on the surface of the canvas. Firstly he used a die for choosing parameters of his arcs, later he simulated those characteristics with help of a computer. Several interesting views on this kind of art we can find in [2].

In [7] Zdeněk Sýkora wrote:

The aim of all this painstaking work is to make something that's absolutely independent, free, something that inspires a feeling of freedom.

In this article a construction of his Lines (arcs) based on the solution of one geometric problem is shown. All steps of his algorithm are not given at Zdeněk Sýkora's books. The main idea follows from one mathematical problem. But it is not a suprising observation. Zdeněk Sýkora graduated from the Pedagogical Faculty of Charles University with degrees in art teaching, descriptive geometry and modelling. On his construction of Lines he collaborated with a mathematician, Jaroslav Blažek.

Then we propose regression models for detection of arcs. The main goal is to find estimators of unknown parameters of arcs and determine the random numbers formulating Zdeněk

Sýkora's painting Lines No. 145. We use the model of incomplete measurement with constraints and linearization of nonlinear models.

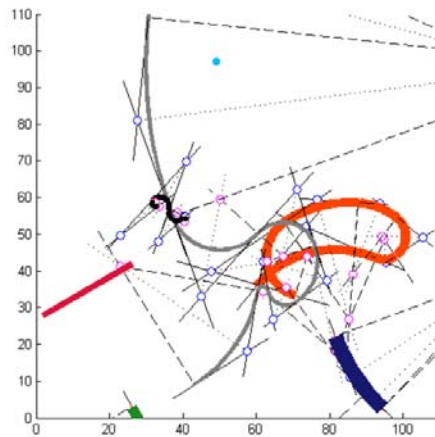


Fig. 1. Construction of Lines 145

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**OPUŠTĚNÍ DIMENZE
DIMENZE JAKO ROZMĚR MATEMATICKÉHO
A VÝTVARNÉHO PROSTORU**

ŠMÍD Jan (CZ), JANČAŘÍK Antonín (CZ)

„Narušení fyzikálních zákonů zázraky, o nichž mluví Písmo, netrápilo filosofy věnující se náboženství tolik jako narušení matematických zákonů. Matematika se tak nachází ve zvláštním postavení, kdy možnost narušení jejích věčných pravd, byť i ze strany všemohoucího Boha, je hluboce znepokojivá.“

Philip Davis a Reuben Hersch ([1], str. 218)

Abstract. Příspěvek na téma „Opuštění dimenze“ řeší téma dimenze a rozměru jako fenoménu prostoru a času v matematické a umělecké (výtvarné) oblasti, zabývá se možnostmi a způsoby „opouštění dimenzí“ v různém chápání zvolené problematiky a navrhuje způsoby, kterými danou problematiku definovat pro možnosti aplikace do didaktiky i živé výuky matematiky a výtvarné výchovy

Key words. matematika, výtvarná výchova, výtvarné umění, transformace, digitální fotografie, didaktická aplikace, znázornění, představivost, komparace.

Mathematics Subject Classification: Primary 97U50.

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SOME PROPERTIES OF MINKOWSKI OPERATORS

VELICHOVÁ Daniela, (SK)

Key words. Minkowski set operations, Minkowski combinations, Minkowski operators, multidimensional visualisation, orthographic projections in more dimensional spaces.

Mathematics Subject Classification: Primary 51N25, Secondary 53A056.

Few results of investigations concerning properties of various Minkowski set operators are discussed in the paper. Based on Minkowski point set operation of sum and on concept of Minkowski linear or matrix combinations of two point sets in \mathbf{E}^n , Minkowski set operators can be defined as smooth mappings of the pairs of point sets from the potential set of the basic Euclidean space \mathbf{E}^n . There exists more than one possible applicable forms of such definitions, while the most natural way will be used to define a simple Minkowski sum linear operator.

Definition 1. Minkowski sum linear operator $L_{k,l}$ is a mapping defined on $P(\mathbf{E}^n) \times P(\mathbf{E}^n)$, in which any ordered pair of point sets A, B in \mathbf{E}^n is attached a point set C in \mathbf{E}^n in the following way

$$L_{k,l} : P(\mathbf{E}^n) \times P(\mathbf{E}^n) \rightarrow P(\mathbf{E}^n)$$
$$(A, B) \mapsto C = L_{k,l}(A, B) = C = k.A \oplus l.B, k, l \in R$$

Defined mappings can be used for determination and modelling of differentiable manifolds that are generated as images of pairs of smooth manifolds in \mathbf{E}^n .

Operation of a multiple of a point set in the space \mathbf{E}^n by a suitable matrix can be defined by matrix multiplication. Position vectors of points in the space \mathbf{E}^n are matrices of type $1 \times n$, operation of multiplication therefore will be determined for square matrices of type $n \times n$. Concept of Minkowski matrix sum operator can be then introduced.

Definition 2. Minkowski sum matrix operator $L_{M,N}$ is a mapping defined on $P(\mathbf{E}^n) \times P(\mathbf{E}^n)$, in which any pair of point sets A, B in \mathbf{E}^n is attached a point set C in \mathbf{E}^n in the following way

$$L_{\mathbf{M},\mathbf{N}} : P(\mathbf{E}^n) \times P(\mathbf{E}^n) \rightarrow P(\mathbf{E}^n)$$

$$(A, B) \mapsto C = L_{\mathbf{M},\mathbf{N}}(A, B) = C = A \cdot \mathbf{M} \oplus B \cdot \mathbf{N}$$

where \mathbf{M} and \mathbf{N} are regular square matrices of rank n with real entries.

Consequence 1. Minkowski linear combination of point sets is a special case of Minkowski matrix combination, in which both matrices \mathbf{M} and \mathbf{N} are diagonal matrices with all zero entries, except the entries on the main diagonals. These are equal real numbers in respective matrices, k in matrix \mathbf{M} and l in matrix \mathbf{N} .

Some of the intrinsic geometric properties of resulting manifolds are discussed, that are inherited from the original manifolds appearing as operands in the Minkowski operators, or that appear due to the properties of the Minkowski operators as generating principles. In case of equal parameterisation of generating manifolds, the resulting manifold is a smooth curve segment, whose curvatures are determined from vector maps of generating curves. For not-equal parameterisation a surface patch is generated, whose fundamental forms and Gaussian curvatures are derived. Several examples of resulting mosaics, curves and surfaces are presented in Fig. 1.

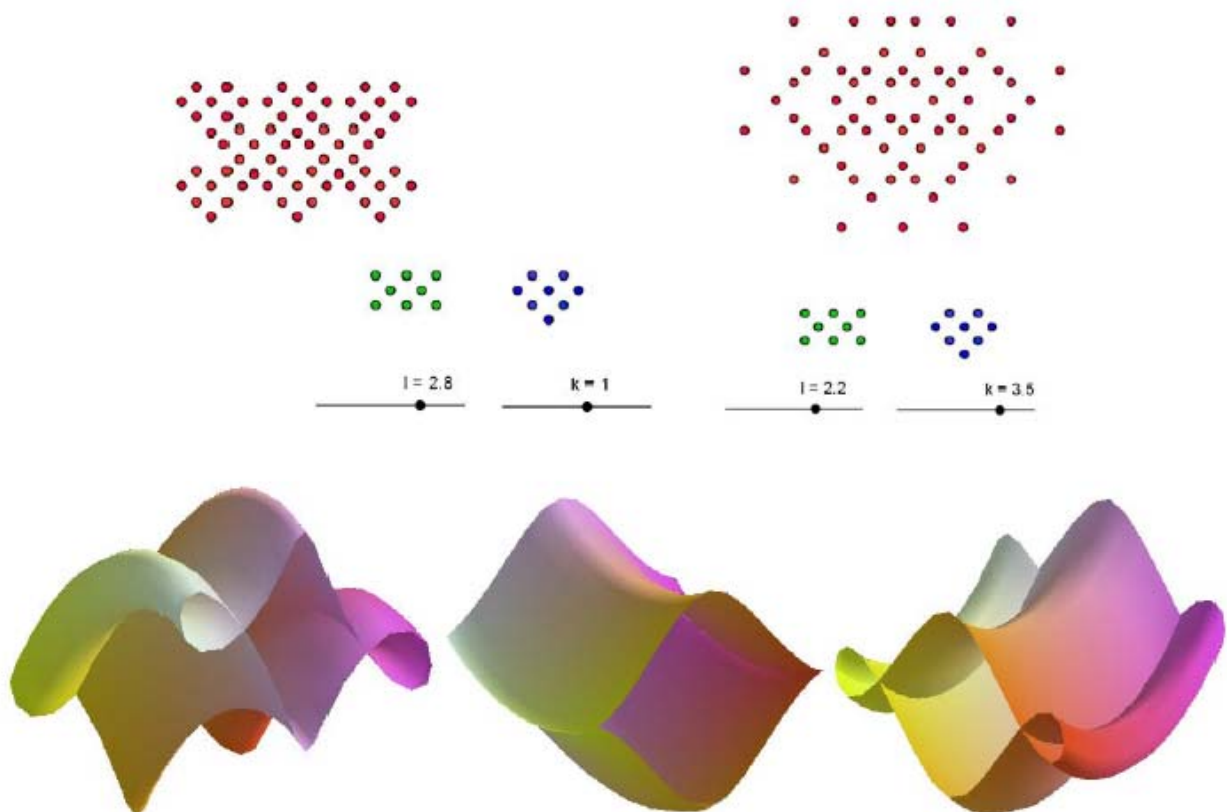


Fig 1. Minkowski sum combinations of discrete point sets, and two curve segments.

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INTERAKTIVNÍ TABULE V KURZU DIDAKTIKY MATEMATIKY PRO BUDOUCÍ UČITELE MATEMATIKY

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Key words. interaktivní tabule, příprava učitelů matematiky, didaktika matematiky, elektronické učebnice, aplety, Moodle.

Mathematics Subject Classification: Primary 97U50; Secondary 97B50.

V roce 2013 byl na Pedagogické fakultě Univerzity Karlovy v Praze řešen projekt FRVŠ 211/2013 *Podpora informačních technologií v přípravě budoucích učitelů matematiky a 1. stupně základní školy*, jehož cílem bylo vybudovat a rozšířit stávající technologické zázemí pro využívání moderních technologií v přípravě budoucích učitelů matematiky, seznámit studenty s nejnovějšími trendy v oblasti využití ICT ve výuce a rozšířit kompetence absolventů pedagogické fakulty tak, aby byli schopni informační a komunikační technologie využívat ve své budoucí praxi. Předkládaný článek seznamuje s východisky, které byly pro řešení projektu zvoleny, a se současným stavem řešení. Jeho jádro tvoří pojednání o interaktivní tabuli a jejím konkrétním využití v přípravě učitelů.

Interaktivní tabule se stává běžnou součástí učeben. V současnosti si už odborná veřejnost uvědomuje, že interaktivní tabule není samospasitelná, že její motivační účinek u žáků postupně upadá a je nutné hledat účinné didaktické způsoby jejího využití. Výsledky výzkumů jsou smíšené ([2]), někde se interaktivní tabule ukázala jako přínosná, někde byl její vliv zanedbatelný. Objevují se však i rizika (např. tendence k transmisivnímu přístupu). V případě matematiky vidíme možnosti interaktivní tabule zejména v oblasti různých interaktivních apletů a použití specializovaného softwaru (např. GeoGebry), případně učebních pomůcek vytvořených přímo pro interaktivní tabule (např. elektronických učebnic s dynamickými prvky).

Předmět didaktika matematiky patří mezi klíčové předměty navazujícího magisterského studia pro budoucí učitele matematiky na 2. a 3. stupni školy. Třisemestrální kurz není veden formou přednáška – seminář, ale spíše interaktivní formou, kdy je kladen velký důraz na participaci studentů. Studenti dostávají prostřednictvím kurzu v Moodle materiály z didaktiky matematiky, které mají předem prostudovat a případně k nim vypracovat nějaký úkol. Při

vlastní výuce pak o získaných poznacích diskutují. Při práci nám výrazně pomáhá přítomnost interaktivní tabule v učebně. Níže alespoň stručně popíšeme ty prvky kurzu didaktiky matematiky, kde se její přínos projevuje nejvíce (v článku jsou podány ilustrace jednotlivých činností).

Společná kontrola úkolů v Moodle: Velkou část kurzu tvoří domácí práce, pro niž je kurz v Moodle klíčový. Studenti dostanou úkol, který mají předem splnit. Zkontrolovat jejich řešení však není zcela triviální a je časově náročné. Úkoly jsou didakticko-matematické povahy, nelze zpravidla říci jednoznačně, co je dobře a co špatně. Navíc studenti mohou profitovat z konfrontace svého řešení s pohledem ostatních. Proto se někdy ukazuje jako účelné provést společnou kontrolu splnění úkolu; odpovědi studentů jsou přítomné online v Moodle, a nemusí se tedy do počítače přepisovat (či zobrazovat vizualizérem, který je ale v učebně též k dispozici).

Analýza videozáznamů z hodin matematiky: Jednou z klíčových součástí kurzu didaktiky matematiky je rozvoj didaktických znalostí obsahu u budoucích učitelů, a to prostřednictvím analýz videozáznamů z hodin matematiky. Studenti jsou prostřednictvím Moodle kurzu požádáni o zhlédnutí videozáznamu celé hodiny matematiky nebo její části (videa jsou buď volně na internetu, nebo nahrána na uzavřený server) a zpravidla mají zodpovědět několik otázek či napsat nestrukturovanou reflexi ukázky. I zde se ukazuje výhoda interaktivní tabule pro práci v semináři.

Elektronické učebnice: V dnešní době existují první elektronické učebnice pro výuku matematiky (např. nakladatelství Fraus či Prodos), s nimiž by se měli budoucí učitelé aktivní formou seznámit. Přítomnost interaktivní tabule v učebně je nutným předpokladem pro to, aby se tak dělo ne během jednoho, předem určeného semináře, ale zcela přirozeně v průběhu celého kurzu. Elektronické učebnice zatím nejsou podle našich zkušeností ve výuce příliš používané, lze ale předpokládat, že se to v budoucnosti (i v souvislosti s nástupem tabletů do škol) změní.

Didaktická analýza apletů: Matematika má tu výhodu, že řada pojmů a tvrzení se dá zpracovat názorným grafickým či dynamickým způsobem. K tomu mohou sloužit tzv. aplety, jejichž počet na internetu neustále roste. Lze snad s jistou dávkou jistoty říci, že většina z nich slouží k upevnění či procvičení učiva. Řada z nich obsahuje sadu úloh, které žák řeší, a tím postupuje v nějaké hře. Pro nás jsou zajímavější aplety, kdy se žák něco sám učí nebo objevuje. Ne všechny aplety jsou však didakticky propracované. Přítomnost interaktivní tabule umožnila, že studenti mohli přímo na místě prozkoumat vybrané aplety a diskutovat o jejich didaktickém potenciálu.

Hodnocení existujících prezentací: Na internetu najdeme v současné době velké množství již hotových prezentací pro interaktivní tabule. Jejich kvalita je však kolísavá (viz také [6]). Učitel matematiky by měl být schopen kriticky zhodnotit nejen matematickou ale i didaktickou kvalitu prezentace. To by mu následně mělo pomoci i při přípravě vlastních materiálů pro interaktivní tabuli.

Příprava vlastní prezentace pro interaktivní tabuli: V ilustraci k předchozímu bodu se studenti nejen učili kriticky hodnotit hotové materiály pro interaktivní tabuli, ale také se seznámili s možnostmi jejich tvorby. Tuto tvorbu jsme zakomponovali do jejich závěrečné ročníkové práce.

V článku jsme na příkladu z kurzu didaktiky matematiky ilustrovali, jakým způsobem lze účelně využít interaktivní tabuli v přípravě budoucích učitelů matematiky tak, aby se seznámili s jejími přínosy i riziky a naučili se základy práce s ní. Ilustrace samozřejmě

ukazují jen malou část kurzu didaktiky matematiky, zmíněné typy aktivit nepředstavují náplň celého kurzu.

Díky překotnému vývoji ICT prostředků se budou učitelé muset vyrovnávat s dalšími výzvami na tomto poli. Při přípravě učitelů tak není nejdůležitější to, aby se naučili tyto prostředky technicky ovládat (i když ani to není zanedbatelné), ale aby byli flexibilní a kreativní v případě, kdy se s novým prostředkem setkají. Současně nesmějí dopustit, aby jejich uvažování ovládla představa, že tento nový prostředek vyřeší veškeré problémy s výukou. Naopak měli být mít na paměti, že při používání jakýchkoli ICT prostředků je třeba sledovat matematické i didaktické zřetele a současně hledat to, co přináší do výuky matematiky nového a přínosného z hlediska kvality získávání a upevňování matematických poznatků jejich žáků. Právě tak se snažíme pojmut výuku v didaktice matematiky, což snad ukazují i představené ilustrace.

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GENERALIZATION OF THE METALLIC MEANS FAMILY (MMF)

Vera W. de SPINADEL (Ar)

Abstract. As it is well known, the MMF is divided into two subfamilies:

[1] the PPMF (Purely Periodic Continued Fraction Expansion;

[2] the PMMF.(Periodic Continued Fraction Expansion)

In the first case we have all the positive quadratic irrationals, that are solutions of equations of the type $x^2 - px - 1 = 0$, where p is a natural number. All of them have a purely periodic continued fraction expansion of the type $x = [n, n, \dots]$ $n \in \mathbb{N}$. If $n = 1$ the solution is the well known Golden Mean $\phi = 1,618\dots$

In the second case, we consider the solutions of equations of the type $x^2 - x - q = 0$, where $q \in \mathbb{N}$. And all of them have a periodic (not purely) continued fraction expansion. Applying George Odom,s idea of looking for the Golden Mean in n-polygons and polytopes, we begun analyzing the sequence of ratios obtained in considering spaces of higher dimensions and taking p and q as real numbers. In this way we were able to generalize the MMF, getting beautiful geometric figures, some of them we are going to show in this presentation.

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**FRACTIONALLY INTEGRATED GARCH APPROACH TO
ESTIMATING FINANCIAL TIME SERIES**

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Abstract. This paper investigates application of the fractionally integrated generalized autoregressive conditional heteroscedastic (FIGARCH) models. Fractionally integrated models are used mainly for describing the observed persistence in the volatility of a time series. FIGARCH models are long memory models for volatility and allow to be a better candidate than other conditional heteroscedastic models for modeling volatility in exchange rates, stock market returns and option prices. The application of a FIGARCH model to foreign exchange rate data is discussed in experimental part. Finally, the results and further possibilities of analysis are discussed.

Key words. Fractionally integrated volatility models, financial time series, long memory models.

Mathematics Subject Classification: Primary 62M10; Secondary 62E15.

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