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Digest

Free–Form 3–D Current Loop Magnetic Field Calculations

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Summary

This paper is aimed to describe well known mathematical approach implementation for numerical calculations of magnetic field induced by free–form current loop.

Introduction

Despite the fact that the description of the magnetic field induced by the single wire element with current has long been known – Biot-Savart law, with use of contours of a more complex geometry, the problem becomes complicated and is still actively studied. In many cases, authors describe the geometry of having a regular arrangement - parallelism and symmetry of some facets of the circuit with current to coordinate axes [2-3]. Such approach is very useful for analytical solving geometry optimization problems.

Because of financial and technological constraints, engineers often have to use no more than one signal generator, which leads to an appreciable complications of the geometry of circuit. Complex contours are uncomfortably to calculate using algorithms linked to certain directions. Therefore, we propose an algorithm makes it easy to implement numerical calculation of the magnetic field of freeform circuit with current. The only condition for the application of the algorithm is necessity to divide the circuit into linear elements.

Magnetic field of wire element

We consider static and quasi–static fields and uniform environment around contour with current. For straight wire lying between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ magnetic field intensity can be written as:

$$\vec{H}(\vec{r}) = \frac{I}{4\cdot\pi}\cdot\int_{P_1P_2}\frac{d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} =$$

$$= \frac{I}{4\cdot\pi}\cdot\int_{P_1P_2}\frac{(\vec{e}_x\cdot dx'+\vec{e}_y\cdot dy'+\vec{e}_z\cdot dz')\times[\vec{e}_x\cdot(x-x')+\vec{e}_y\cdot(y-y')+\vec{e}_z\cdot(z-z')]}{|\vec{r}-\vec{r}'|^3} =$$

$$= \frac{I}{4\cdot\pi}\cdot\int_{P_1P_2}\frac{\begin{vmatrix}\vec{e}_x & \vec{e}_y & \vec{e}_z\\ dx' & dy' & dz'\\ x-x' & y-y' & z-z'\end{vmatrix}}{|\vec{e}_x\cdot(x-x')+\vec{e}_y\cdot(y-y')+\vec{e}_z\cdot(z-z')|^3} =$$

$$= \frac{I}{4\cdot\pi}\cdot\int_{P_1P_2}\frac{\vec{e}_x\cdot[(z-z')\cdot dy'-(y-y')\cdot dz']+\vec{e}_y\cdot[(x-x')\cdot dz'-(z-z')\cdot dx']+\vec{e}_z\cdot[(y-y')\cdot dx'-(x-x')\cdot dy']}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}$$

Then magnetic field intensity vector components can be written as:

$$H_x(\vec{r}) = \frac{I}{4\cdot\pi}\cdot\left[\int_{y_1}^{y_2}\frac{(z-z')\cdot dy'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} - \int_{z_1}^{z_2}\frac{(y-y')\cdot dz'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}\right]$$

$$H_y(\vec{r}) = \frac{I}{4\cdot\pi}\cdot\left[\int_{z_1}^{z_2}\frac{(x-x')\cdot dz'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} - \int_{x_1}^{x_2}\frac{(z-z')\cdot dx'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}\right]$$

$$H_z(\vec{r}) = \frac{I}{4\cdot\pi}\cdot\left[\int_{x_1}^{x_2}\frac{(y-y')\cdot dx'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}} - \int_{y_1}^{y_2}\frac{(x-x')\cdot dy'}{[(x-x')^2+(y-y')^2+(z-z')^2]^{\frac{3}{2}}}\right]$$

Parameterization of line segment

Line segment between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ can be parameterized in such way:

$$\vec{r}'(t) = (1-t)\cdot\langle x_1, y_1, z_1\rangle + t\cdot\langle x_2, y_2, z_2\rangle, 0\le t\le 1$$

or

$$x' = (1-t)\cdot x_1 + t\cdot x_2$$
$$y' = (1-t)\cdot y_1 + t\cdot y_2, 0\le t\le 1$$
$$z' = (1-t)\cdot z_1 + t\cdot z_2$$

or

$$\vec{r}'(t) = \vec{e}_x\cdot((1-t)\cdot x_1 + t\cdot x_2) + \vec{e}_y\cdot((1-t)\cdot y_1 + t\cdot y_2) + \vec{e}_z\cdot((1-t)\cdot z_1 + t\cdot z_2), 0\le t\le 1$$

Then derivative of parameterization can be written as:

$$dx' = (x_2 - x_1)\cdot dt, dy' = (y_2 - y_1)\cdot dt, dz' = (z_2 - z_1)\cdot dt$$

Conclusion

Obtained magnetic field intensity vector components with parameterization approach could easily be implemented in numerical calculations of magnetic field induced by wire element. In case of complicated contour it should be divided in to linear elements. Then described approach for every element and superposition principle can be used. Next step for calculation acceleration is obtaining of analytical solution of integrals.

References

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