Some Rare Phenomena of Nonlinear Parametric Oscillations of Flexible Elements

Vitaly Beresnevich[1], Alexey Klokov[2]
[1, 2] *Riga Technical University*

Abstract **− This paper is devoted to a study of nonlinear parametric oscillations of flexible elements in machines using three different methods of analysis: mathematical simulation on analogue-digital computer system, numerical solution with computer program *MATLAB*, experimental investigations. Mathematically the problem is presented as a partial differential equation describing lateral oscillations of flexible element under the periodic pulsation of its tension force with due account of geometrical, static and physical nonlinearities. It is shown that spectrum of resonance lateral oscillations of flexible elements may be sufficiently dense. The evaluation of danger extent of different resonant regimes which can occur in the system is made. Under some conditions continuous resonant zones occur in wide frequency range, which defines the scientific novelty of these investigations. Besides, rare phenomena of abrupt changes of amplitudes and modes of oscillations become possible within these zones. In such conditions tuning away of system from dangerous resonant regimes remains problematic. The possible ways for practical application are considered.**

Keywords **− Excitation, flexible element, nonlinear oscillations, parametric, resonant regime.**

I. Introduction

Flexible elements (belts, cables, guy ropes, taut threads, strings, etc.) are widely used in mechanical engineering for various practical purposes [1], [2]. In some cases lateral parametric vibrations of flexible elements, which can occur during the operation of machine, may be extremely detrimental (belt and chain transmissions, threads in weaving machines, etc.). In some other cases special excitation of lateral oscillations of flexible element is needed to ensure effective operation of vibration device (for example, flexible belt of vibration mixer, vibration setup for seismic ground probing, vibrating-element viscometer, elastic tail of robotic fish, etc.). Therefore, it is very important in the preliminary stage of designing to compare different resonant regimes which can occur in the system under periodic pulsation of axial tension force, using significant criteria (peak values of resonant displacements and tensile forces in the flexible element, width of frequency intervals of resonant regimes, etc.).

There is no complete theoretical basis for such criteria in scientific literature. Most of the known works on non-linear oscillations of flexible elements are concerned with analysis of free vibrations (e.g., [3]−[5]). But cases of parametric excitation are usually considered in application only to the first lower mode of string's lateral oscillations [6]−[8]. This paper seeks to study non-linear properties on different lateral modes of parametric oscillations. On the base of this study new practical applications are possible.

II. Dynamic Model and Methods of Analysis

Transverse oscillations of flexible element under parametric excitation are considered (Fig. 1). Parametric excitation is caused by periodic axial tension force of the flexible element. In forming of differential equation of oscillations some assumptions are made. It is supposed, that stiffness in bending of flexible element is negligible in comparison with its stiffness in tension, but the weight of flexible element is ignorable in comparison to axial prestressing force T_0. Besides, it is considered that oscillations are performed in *z*-*y* plane, which runs along the centre line of a non-deformed flexible element (see Fig. 1).

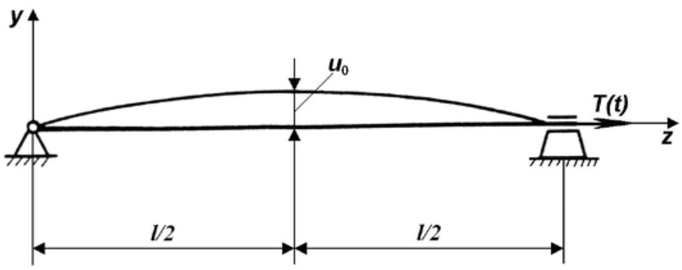

Fig. 1. Model of flexible element considered for dynamic analysis.

Taking the direction of the co-ordinate axis *z* along this centre line, the differential equation for transverse vibrations of flexible element can be stated as follows [2], [5]:

$$T_0\left(1+\mu\sin\Omega t\right)\left[1+f(\varepsilon)\right]\left(\frac{\partial^2 y}{\partial z^2}+b_1\frac{\partial^3 y}{\partial z^2\partial t}\right)-$$
$$-b_2\frac{\partial y}{\partial t}-\rho\left[1+\frac{1}{2}\left(\frac{\partial y}{\partial z}\right)^2\right]\frac{\partial^2 y}{\partial t^2}=0, \qquad (1)$$

where T_0 is the prestressing force; μ and Ω are the non-dimensional amplitude and the frequency of parametric excitation; accordingly b_1 and b_2 are the coefficients of internal and external frictions; *y* is the lateral displacement of the flexible element.

The functional $f(\varepsilon)$ in equation (1) takes into account additional tension force caused by elastic deformation of flexible element during its lateral oscillations (physical nonlinearity). The elongation ε of flexible element can be determined by [5]:

$$\varepsilon = \frac{1}{2l}\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 \mathrm{dz}\ , \tag{2}$$

where l is the length of flexible element. The relationship between axial stress σ in flexible element and its elongation ε can be approximately described by the expression:

$$\sigma = E\varepsilon - \beta\varepsilon^3 , \tag{3}$$

where E is the elasticity modulus of material; β is the coefficient of non-linearity. In this case the functional $f(\varepsilon)$ can be expressed in the following form:

$$f(\varepsilon) = \frac{EA}{2T_0 l}\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 \mathrm{dz} - \frac{\beta A}{8T_0 l^3}\left[\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 \mathrm{dz}\right]^3 , \tag{4}$$

where A is the cross-section area of flexible element.

Therefore an increment in tension is caused by integral elongation of flexible element and is independent of co-ordinate z. Non-linear term $\left[1+\frac{1}{2}\left(\frac{\partial y}{\partial z}\right)^2\right]$ of equation (1) takes into account geometrical non-linearity of flexible element [5]. In the case studied here the end boundary conditions are as follows:

$$y(z=0,t)=0\ ; \qquad y(z=l,t)=0\ , \tag{5}$$

Equation (1), subject to the conditions (4)–(5), was solved in *MATLAB* environment using *pdepe* solver [9]. For this purpose initial differential equation (1) was transformed into equivalent set of first-order partial differential equations, and special-purpose *MATLAB* functions describing nonlinear characteristics have been created:

$$\begin{cases} \dfrac{\partial y_1}{\partial t} = y_2, \\ \rho\left[1+\dfrac{1}{2}\left(\dfrac{\partial y_1}{\partial z}\right)^2\right]\dfrac{\partial y_2}{\partial t} = T_0\left(1+\mu\sin\Omega t\right), \\ \qquad \cdot\left[1+f(\varepsilon)\right]\left(\dfrac{\partial^2 y_1}{\partial z^2} + b_1\dfrac{\partial^3 y_1}{\partial z^2\partial t}\right) - b_2\dfrac{\partial y_1}{\partial t} \end{cases} \tag{6}$$

where

$$f(\varepsilon) = \frac{EA}{2T_0 l}\int_0^l \left(\frac{\partial y_1}{\partial z}\right)^2 \mathrm{dz} - \frac{\beta A}{8T_0 l^3}\left[\int_0^l \left(\frac{\partial y_1}{\partial z}\right)^2 \mathrm{dz}\right]^3 . \tag{7}$$

If:

$$\frac{\partial}{\partial t}\left(\frac{\partial y_1}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial y_1}{\partial t}\right) = \frac{\partial y_2}{\partial z}\ , \tag{8}$$

the system takes the form:

$$\begin{cases} \dfrac{\partial y_1}{\partial t} = y_2, \\ \rho\left[1+\dfrac{1}{2}\left(\dfrac{\partial y_1}{\partial z}\right)^2\right]\dfrac{\partial y_2}{\partial t} = \dfrac{\partial}{\partial z}T_0\left(1+\mu\sin\Omega t\right)\cdot \\ \qquad \cdot\left[1+f(\varepsilon)\right]\left(\dfrac{\partial y_1}{\partial z} + b_1\dfrac{\partial y_2}{\partial z}\right) - b_2 y_2 . \end{cases} \tag{9}$$

Then equation (1) takes the form:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \qquad \frac{\partial y}{\partial z} = \begin{bmatrix} \dfrac{\partial y_1}{\partial z} \\ \dfrac{\partial y_2}{\partial z} \end{bmatrix},$$
$$c\left(z,t,y,\frac{\partial y}{\partial z}\right)\frac{\partial y}{\partial t} = x^{-m}\frac{\partial}{\partial z}\left(x^m f\left(z,t,y,\frac{\partial y}{\partial z}\right)\right) + \tag{10}$$
$$+ s\left(z,t,y,\frac{\partial y}{\partial z}\right),$$

with boundary conditions

$$p\left(z,t,y\right) + q\left(z,t\right) f\left(z,t,y,\frac{\partial y}{\partial z}\right) = 0, \tag{11}$$

where

$$c\left(z,t,y,\frac{\partial y}{\partial z}\right) = \begin{bmatrix} 1 \\ \rho\left[1+\dfrac{1}{2}\left(\dfrac{\partial y_1}{\partial z}\right)^2\right] \end{bmatrix},$$
$$f\left(z,t,y,\frac{\partial y}{\partial z}\right) = \begin{bmatrix} 0 \\ T_0\left(1+\mu\sin\Omega t\right)\left[1+f(\varepsilon)\right]\cdot \\ \cdot\left(\dfrac{\partial y_1}{\partial z} + b_1\dfrac{\partial y_2}{\partial z}\right) \end{bmatrix},$$
$$s\left(z,t,y,\frac{\partial y}{\partial z}\right) = \begin{bmatrix} y_2 \\ -b_2 y_2 \end{bmatrix}, \tag{12}$$
$$m = 0,$$
$$p\left(z=0,t,y\right) = \begin{bmatrix} y_1\left(z=0,t\right) \\ y_2\left(z=0,t\right) \end{bmatrix}, \quad q\left(z=0,t\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$p\left(z=l,t,y\right) = \begin{bmatrix} y_1\left(z=l,t\right) \\ y_2\left(z=l,t\right) \end{bmatrix}, \quad q\left(z=l,t\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and the initial conditions, e.g.:

$$y\left(z,t_0=0\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{13}$$

Dynamics of the system also have been simulated on the specialized analogue-digital computer system developed in Riga Technical University [10] and some of these theoretical results have been verified by experiments with a uniform rubber cord. Some numerical results of nonlinear oscillations of flexible elements have been examined in works [11], [16]. The methods of mathematical simulation and the operational principle of the computer system are described in more detail in [2], [12].

III. Analysis of Parametric Oscillations of Flexible Element

As known [13],[14], parametric resonance of flexible element occur under periodic tensile force T with frequency Ω which fall in the vicinity of critical frequencies:

$$\Omega = \frac{2\omega_S}{e} \text{ or } \Omega = \frac{|\omega_S \pm \omega_k|}{e}, \tag{14}$$

where ω_S and ω_k are the natural frequencies with ordinal numbers S and k of lateral oscillations of flexible element; e = 1, 2, 3, … is the order of the parametric resonance. The first formula describes the condition of a simple parametric resonance, but the second formula – the condition of a combined parametric resonance.

As an example, Fig. 2 shows the diagram of parametric resonance zones on the plane of parameters μ and $\eta = \Omega/\omega_1$ (zones are section-lined and denoted with symbols $2\omega_S$ or $\omega_S+\omega_k$), which is obtained using the specialized analogue-digital computer system. Only main zones (e = 1) corresponding to the three first natural frequencies ω_1, ω_2 and ω_3 are shown. The diagram is constructed assuming $b_1\omega_1 = 0.003$ (it is the rough damping in real flexible elements). Combination resonances of difference type $(\omega_S - \omega_k)/e$ for the system were not observed.

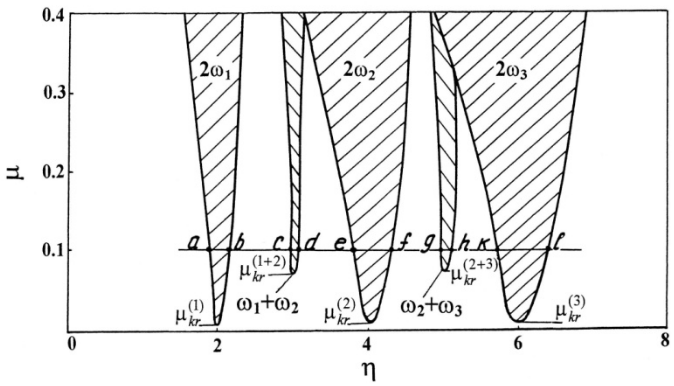

Fig. 2. Excitation zones of parametric lateral oscillations for flexible element obtained using the specialized analogue-digital computer system.

In accordance with the conditions (14), some critical frequencies Ω are possible, which theoretically allow excitation both simple parametric and combination parametric resonances. For example, such situation occurs under the condition $\Omega = 2\omega_2 = \omega_1+\omega_3$. In this case simple parametric oscillations dominate over the combination ones (see Fig. 2). Besides, critical frequencies Ω, which theoretically allow simultaneous excitation of some different combination resonant regimes, are possible, too. In this case, combination regime that resulted from the interaction of two adjacent vibration modes is excited. For example, at the excitation frequency $\Omega = \omega_1 + \omega_4 = \omega_2 + \omega_3$ combination oscillations corresponding to the zone $\omega_2 + \omega_3$ are realized.

On the base of diagram (see Fig. 2) simple and combination parametric regimes can be compared. As it is seen, zones of combination resonance are sufficiently more narrow (by excitation frequency Ω) than zones of simple parametric resonance. For example, under the $\mu = 0.2$ the width of zones $2\omega_1$ and $2\omega_2$ are equal to $\Delta\eta^{(1)} = 0.42$ and $\Delta\eta^{(2)} = 0.83$, correspondingly. But the width of combination parametric zone $(\omega_1 + \omega_2)$ is about only $\Delta\eta^{(1+2)} = 0.1$. Besides, excitation of combination parametric oscillations are possible only under higher pulsation μ of tensile force T than it is necessary for excitation of simple parametric resonances. For example, under the $T_0/EA = 2\cdot10^{-4}$ and $b_1\omega_1 = 0.003$ the threshold of parametric excitation for the zone $2\omega_1$ is equal to $\mu_{kr}^{(1)} = 0.0064$, but for the zone $(\omega_1 + \omega_2)$ it reaches $\mu_{kr}^{(1+2)} = 0.07$ (approximately ten times higher).

But objective conclusion on failure danger of flexible element in one or another parametric regime can be made by comparison of vibration amplitudes and corresponding tensile forces. Exponential rise of amplitudes of parametric oscillations during transient process is observed. For example, Fig. 3 shows the rise of parametric oscillations within the main zone $2\omega_1$ of parametric instability. A danger of failure of flexible element on the second and third parametric resonant regimes, as it is shown in [15], is slightly higher than on the first one (due to increase of tensile forces and dynamic stress in the flexible element).

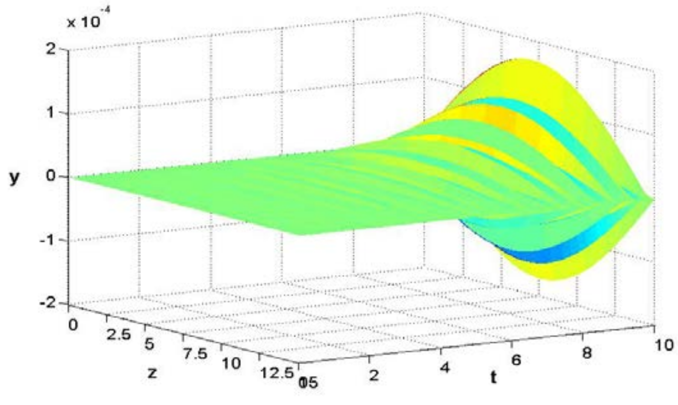

Fig. 3. Exponential rise of parametric oscillations within the main zone of parametric instability in flexible element (1) under parametric excitation. The diagram was obtained by numerical investigations using program *MATLAB*.

Under some conditions ($\mu > 0.15$ and $\eta > 11$) main zones of simple parametric regimes $2\omega_s$ are merging, and as a result a continuous resonant zone occurs in wide frequency range. For example, Fig. 4 shows a part of the diagram of parametric instability corresponding to the frequency range of excitation of regimes $2\omega_6$, $2\omega_7$ and $2\omega_8$. The diagram is constructed on the plane of parameters μ and $\eta = \Omega/\omega_1$, assuming $b_1\omega_1 = 0.003$. Above on the same figure AFC of lateral parametric oscillations of flexible element is constructed (for the case $\mu = 0.2$,

$T_0/EA = 2\cdot10^{-4}$ and $b_1\omega_1 = 0.003$). Dimensionless peak value of displacement u_0/l in antinodal point of corresponding resonant mode is projected as amplitude on this AFC.

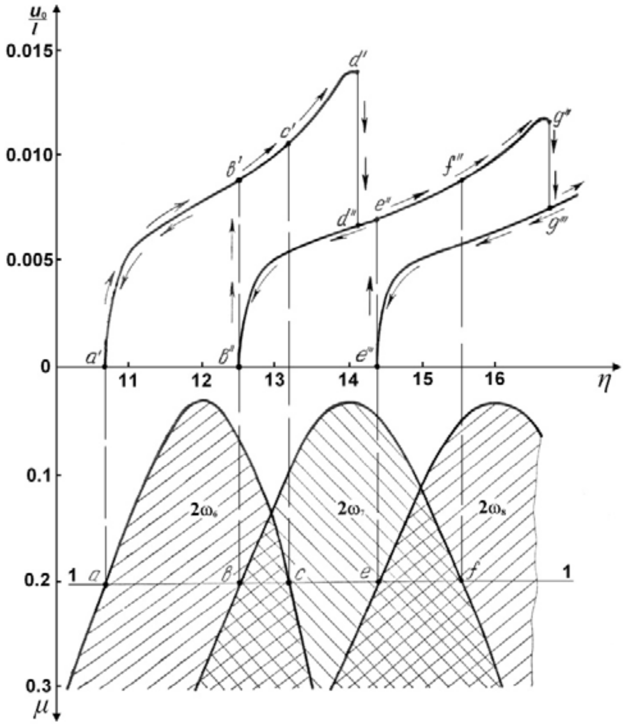

Fig. 4. Overlap of parametric instability zones under the high-frequency excitation obtained using the specialized analogue-digital computer system.

In accordance with the diagram presented, an ambiguity of exciting parametric regimes exists inside frequency ranges *bc* and *ef*. Dependent on realized initial conditions, regimes of orders $2\omega_6$ (6th mode) or $2\omega_7$ (7th mode) can be excited inside frequency range *bc*. But inside frequency range *ef*, it is possible to realize oscillations of flexible element by 7th or 8th mode. Besides, pulling of oscillations beyond the borders of instability zone occurs (frequency intervals *ce* and *fg*). Rare phenomena of abrupt changes of amplitudes and modes of oscillations are observed in bifurcation points of AFC (e.g. bifurcation jumps $b'' - b'$, $d' - d''$, $e''' - e''$, $g'' - g'''$).

Typical parametric regimes of oscillations realized within instability zones $2\omega_6$ and $2\omega_7$ are presented in Fig. 5 and Fig. 6, obtained by numerical investigations using *MATLAB*. As it is seen, during transient process shapes of vibration modes gradually approach to the shapes peculiar to 6th and 7th natural modes. Some distortions of vibration modes from the standard harmonic shapes can be caused by two different factors. First, the influences of geometric and static nonlinearities of flexible element have to be taken into consideration. Second, the interaction of different mechanisms of excitation of parametric regimes may be the reason for distortion of vibration modes. For example, due to the condition (6) at the excitation frequency $\Omega = 2\omega_6$ there is a theoretical possibility for additional exciting of combination parametric regimes of orders $\omega_5+\omega_7$, $\omega_4+\omega_8$, $\omega_3+\omega_9$, $\omega_1+\omega_{11}$ and others. Interaction of simple parametric and combination regimes can result in specific distortions of vibration modes and frequency spectrum. Further, more detailed analysis of these effects is needed.

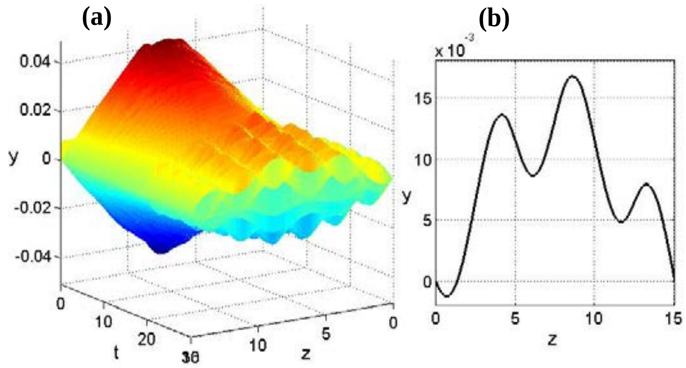

Fig. 5. Typical parametric regimes of oscillations within parametric instability zone $2\omega_6$: (a) transition to steady-state oscillations from non-zero initial conditions; (b) vibration mode at instant of time t = 30 s.

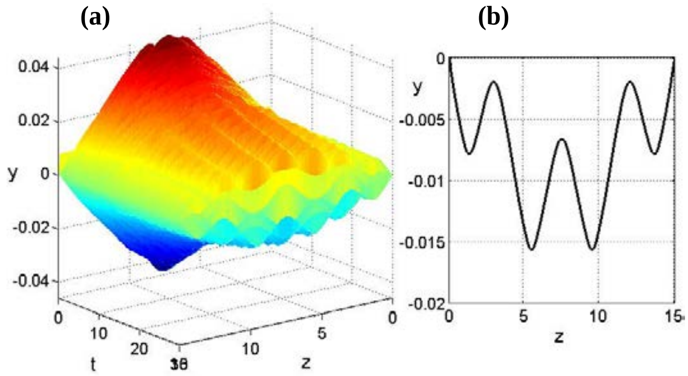

Fig. 6. Typical parametric regimes of oscillations within parametric instability zone $2\omega_7$: (a) transition to steady-state oscillations from non-zero initial conditions; (b) vibration mode at instant of time t = 30 s.

IV. Experimental Investigations

Results of the theoretical study have been verified by experiments with a uniform rubber cord having the length l = 0.66 m and linear density ρ = 0.0025 kg/m (in unloaded condition). The experimental setup (Fig. 7a) consists of uniform rubber cord, stationary base, DC power supply and block of parametric excitation (Fig. 7b). This block ensures periodic pulsation of axial tension force and consists of an electric motor, a slide-block of a crank gear and an eccentric drive. One end of the flexible element was fixed, but the other one was connected to the slide-block (Fig. 7b) of a crank gear (through a system of guiding rolls). During the reciprocation of slide-block a tension force of flexible element was periodically changed, and lateral parametric oscillations of the cord were excited.

The obtained experimental results demonstrate good agreement with numerical results. Specifically, experimental relationships between lateral displacements on three first vibration modes have shown close agreement with the numerical results. Besides, the existence of wide continuous resonant frequency ranges and occurrence of vibration regimes accompanied with abrupt changes of amplitudes and modes of oscillations is confirmed.

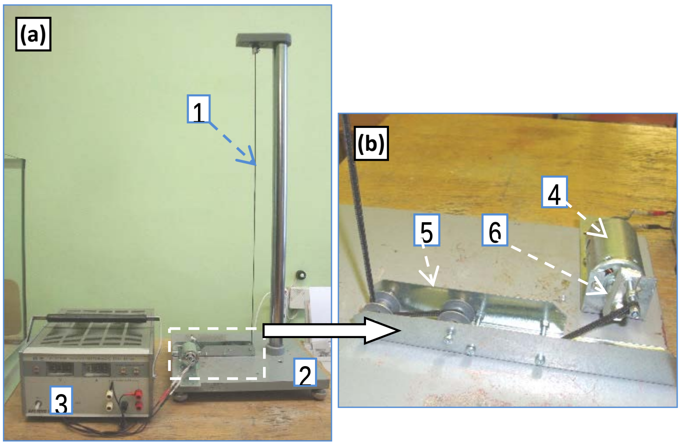

Fig. 7. (a) General view of the experimental setup: 1 – uniform rubber cord, 2 – stationary base, 3 – DC power supply; (b) block of parametric excitation: 4 – motor, 5 – slide-block of a crank gear, 6 – eccentric drive.

V. Conclusion

Existence of resonant parametric oscillations of flexible element within wide continuous frequency range, realization of nonlinear effects of abrupt changes in amplitudes and modes of oscillations (Fig. 4) cause difficulties in vibration protection. In such cases tuning away of the system from dangerous resonant regimes remains problematic (e.g., threads in weaving machines). On the other hand, special excitation of these regimes can be useful in vibration devices intended for executing of some technological processes with the aid of vibrating flexible element (flexible belt of vibration mixer, elastic tail of robotic fish, emptying of loading hoppers with the aid of vibrating belt, etc.).

The obtained theoretical results were confirmed experimentally.

Acknowledgment

Authors want to express gratitude to their colleague V. Yakushevich from research laboratory „Nonlinear Phenomena of Vibrating Systems" of RTU for his help in creating the experimental setup.

This work has been supported by Latvian Council of Science under the project Nr. 09.1267 and European Social Fund within the project „Support for the implementation of doctoral studies at Riga Technical University" 2012–2013.

References

1. Svetlitsky, V. A. *Mechanics of Flexible Rods and Filaments*. Moscow: Mashinostroeniye, 1978, 222 p. (in Russian).
2. Tsyfansky, S. L., Beresnevich, V. I., Oks, A. B. *Nonlinear and Parametric Oscillations of Technological Vibromachines*. Riga: Zinatne, 1991, 231 p. (in Russian).
3. Gottlieb, H. P. W. Non-linear vibration of a constant-tension string. In: *Journal of Sound and Vibration*, 1990, vol. 143, p. 455–460.
4. Han, S. M., Grosenbaugh, M. A. Non-linear free vibration of a cable against a straight obstacle. In: *Journal of Sound and Vibration*, 2004, vol. 273, p. 337–361.
5. Bondar, N. *Nonlinear Autonomous Problems in Mechanics of Elastic Systems*. Kiev: Budivelnik, 1971, 140 p. (in Russian).
6. Tagata, G. A parametrically driven harmonic analysis of a non-linear stretched string with time-varying length. In: *Journal of Sound and Vibration*, 1983, vol. 87, p. 493–511.
7. Tagata, G. Parametric oscillations of a non-linear string. In: *Journal of Sound and Vibration*, 1995, vol. 185, p. 51–78.
8. O'Reilly, O., Holmes, P. J. Non-linear, non-planar and non-periodic vibrations of a string. In: *Journal of Sound and Vibration*, 1992, vol. 153, p. 413–435.
9. Dukkipati, R. V. *MATLAB for Mechanical Engineers*. New Age Science Limited, 2009.
10. Cifanskis, S., Beresnevich, V. Specialized analogue-digital computer system. In: *High Tech in Latvia 2004*, Riga: AGB Publishing House, 2004, p. 30.
11. Beresnevich, V., Klokov, A. Some characteristic features of nonlinear parametric oscillations of flexible elements. In: *Proceedings of the 2nd International Symposium RA'11 on "Rare Attractors and Rare Phenomena in Nonlinear Dynamics"*, Riga – Jurmala, Latvia, May 17–20, 2011. Riga: RTU Publishing House, 2011, p. 57–64.
12. Tsyfansky, S. L. *Electric Simulation of Oscillations in Complex Nonlinear Mechanical Systems*. Riga: Zinatne, 1979, 180 p. (in Russian).
13. Bolotin, V. V. *The Dynamic Stability of Elastic Systems*. San-Francisco: Holden-Day, 1964.
14. Cartmell, M. *Introduction to Linear, Parametric and Nonlinear Vibrations*. London: Chapman & Hall, 1990.
15. Beresnevich, V. Characteristic properties of nonlinear parametric oscillations of flexible elements. In: *Scientific Proceedings of Riga Technical University*, Series 6: Transport and Engineering. Mechanics, 2008, 28, p. 39–47.
16. Beresnevich, V., Tsyfansky, S., Klokov, A. Nonlinear Lateral Oscillations of Flexible Elements: Characteristic Features and Practical Applications // *Springer Proceedings in Physics: Vibration Problems ICOVP 2011*. – 2011, vol. 139, p. 75–80.

Vitaly Beresnevich, *Dr. sc. ing.*, associate professor, leading researcher, Riga Technical University, Institute of Mechanics. Scientific interests: nonlinear oscillations, machine dynamics and vibration diagnostics. Vitaly Beresnevich has authored 189 scientific publications, including 4 monographs, 1 book and 56 patents (11 of them are adopted in industry). Address: 6 Ezermalas iela, Riga, LV-1006, Latvia. Phone: +37167089469.
E-mail: vitalijs.beresnevics@rtu.lv.

Alexey Klokov, PhD student, Riga Technical University, Institute of Mechanics. Scientific interests: applied nonlinear dynamics, chaos, complexity and control. Alexey Klokov has authored of 38 scientific publications, including 1 monograph. Address: 6 Ezermalas Street, Riga, LV-1006, Latvia.
E-mail: alex_klokov@inbox.lv.

Vitālijs Beresņevičs, Aleksejs Klokovs. Lokano elementu nelineāro parametrisko svārstību dažas retas parādības
Raksts ir veltīts lokano elementu nelineāro parametrisko svārstību izpētei mašīnās, lietojot trīs dažādas analīzes metodes: matemātiskā modelēšana analogu-ciparu datorizētā kompleksā, skaitliskā analīze ar datora programmu *MatLab* un eksperimentālie pētījumi. Matemātiski uzdevums ir formulēts kā lokana elementa parametrisko svārstību parciālais diferenciālvienādojums, kas raksturo šī elementa parametriskās svārstības, turklāt tiek ņemta vērā lokanā elementa ģeometriskā, statiskā un fiziskā nelinearitāte. Parādīts, ka lokano elementu rezonanses šķērssvārstību frekvenču spektrs varētu būt būtiski blīvs (dažādu kārtu parastie parametriskie rezonanses režīmi, kombinacionālās rezonanses). Apskatāmajā sistēmā tiek novērtēta dažādu potenciāli iespējamu rezonanses režīmu bīstamības pakāpe. Konstatēts, ka pie dažiem nosacījumiem var eksistēt nepārtrauktas rezonanses zonas plašā frekvenču diapazonā. Turklāt šo nepārtraukto zonu ietvaros novērotas retas parādības, kas realizējas lokana elementa svārstību amplitūdu un formu lēcienveida izmaiņās. Šīs īpatnības nosaka izpildīto pētījumu zinātnisko novitāti. Nepārtraukto rezonanses zonu eksistēšana apgrūtina sistēmas noregulēšanu pēc frekvences no bīstamiem rezonanses režīmiem. Rakstā apskatīti iespējamie varianti šo nelineāro efektu praktiskai izmantošanai.