Stability of Multilayered Rubber-Metal Shock Absorbers

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Abstract **− Stability of shock absorbers with thin-multilayered rubber-metal elements (TRME) of flat, circular shape is considered in this work. TRME packages that are used as vibration isolators usually work under heavy compressive loads, which may lead to buckling failure.**
Next, formulas for package design are derived: the dependence of the critical force on geometry of layer, on mechanical properties of material of elastomeric layers, on packages end-fixity conditions. The dependence of mechanical modules of elastomeric on the compressive load level is taken into account. The obtained solutions are compared to experimental data of other authors.

Keywords **− Buckling, elastomeric, multilayer devices, stiffness, variational method.**

I. Introduction

Rubber (natural and synthetic) as construction material has a number of valuable properties: high elasticity, resistance to environmental influences, good dynamic characteristics [1], [2]. Laminated elastomer is the anisotropic elastic element of alternating thin layers of rubber and metal (or hard plastic) assembled by gluing or vulcanizing in a package of three or more layers having a large load carrying capacity (more than 30 MPa) in normal to the layer direction and higher compliance (0–80 % of relative deformation) in the transversal direction. This allows to obtain structures, which have axial compression stiffness that is several orders greater than shear stiffness. Packages of thin-layered rubber-metal elements (hereinafter referred to as TRME) are successfully used as vibration isolators, shock absorbers, compensating devices, bearings, joints, etc. [1], [3]. In practice TRME packages of different geometrical form are used: flat of various shapes, cylindrical, conical, etc. Elastomeric layer is considered to be thin if its width/thickness ratio is much more than 10. Multilayered packets of TRME have extensive use in almost all spheres of engineering and construction (joints and bearings for various applications, support of engineering structures, vibration and shock absorbers etc.). In flat-type packages working under significant compressive loads, the buckling of the middle layers of packet is observed, i.e. package loses buckling stability, which leads to decrease of performance capabilities of packages and their failure. Buckling has shear instability form (the layers are shifted sidewise), rather than bending (as in the classical theory of rods stability). This occurs because of TRME stiffness under axial compression and bending stiffness are in several orders greater than the shear stiffness.

Gent A. N. considers the stability of structures with thick rubber layers (with the shape factor ≈1) based on the classical Timoshenko theory of rods [3]. This approach and the main position of Gent’s work was later used by many authors [5], [6], but further investigations show that application of these solutions to thin rubber-metal elements leads to significant errors [2], [4]. Many successive works deal with TRME package buckling stability [7]−[9], the method of bending stiffness calculation based on the assumption that middle surface of elastomeric layer remains flat under deformation was elaborated.

When designing TRME packages, it is necessary to have an analytical expression (preferably in a simple manner) to calculate the critical external load taking into account TRME geometric parameters, scheme of external load imposing and method of TRME packet fastening, as well as mechanical properties of materials to improve their operational characteristics and increase permissible rate of compressive loads.

In this study all factors mentioned above are taken into account for the example of critical force calculation for the flat-type TRME of circular shape (Fig. 1).

II. Ananalytical Decisions for the Mathematical Model of TRME Packet

In studies mentioned above it is assumed that: nonelastomeric layers are nondeformable, external forces are conservative, elastomeric and nonelastomeric layers are rigidly connected to each other, the deformation of each individual TRME is linear. Besides that, the assumptions are introduced that the elastomeric material layer is volumetrically incompressible and its mechanical properties are not dependent on the rate of external loading. But there is no argumentation for applicability domain of the assumptions listed above and estimation of their influence on numerical value of the critical forces. In this paper the methodology of calculation of the critical force for TRME package buckling taking into account the weak compressibility of elastomeric layers and shear modulus dependence on the load level (which were not considered in the works [2]), is discussed. For example, the stability problem of a circular-type, flat TRME package under axial compression between two flat parallel plates with force P is considered. In Fig. 3 the forms of loss of stability of TRME device under axial compression are shown; because the compression and tension stiffness are much greater than share stiffness, the consideration of buckled shape includes share deformation (Fig. 3c). TRME package (of thickness $H = h_c N$) consists of N individual identical sections. Each section (thickness $h_c = h_e + h_0$) consists of nondeformable metal plate (thickness – h_0) and is vulcanized to an elastomeric layer (thickness – h_e and sectional area – F), deformation of which is considered small. When calculating the shear stiffness of elastomeric element K_y of shear force P_y scheme of simple share is applied; for bending stiffness T calculation – scheme in which metal plates are rotated in respect to each other, relative to the axis of symmetry.

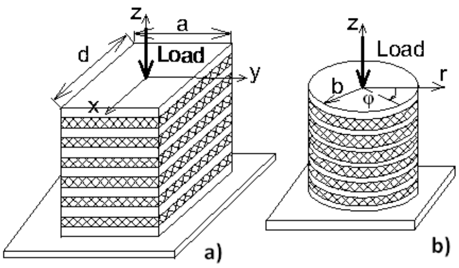

Fig. 1. Examples of multilayer elastomeric structures: a) flat, rectangular, b) flat, circular.

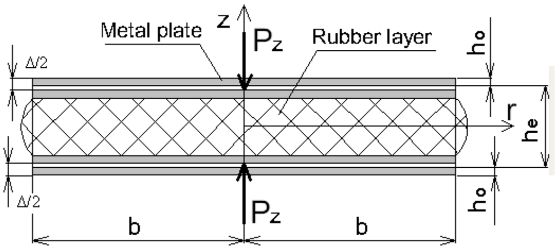

Fig. 2. Scheme of compression of a TRME section.

III. Models of TRME Package

Based on Timoshenko beam model, the critical force was derived in [2] for scheme a) and c). In paper [2] the loss of stability of TRME package with the square section of rubber layer and fixed end point is discussed; in particular, Euler buckling with share contribution. The condition of stability and the critical force for this case is:

$$\frac{P_{z,\mathrm{cr}}}{T h_c}\left(1+\frac{P_{z,\mathrm{cr}}}{K_y h_c}\right)=\frac{4\pi^2}{H^2},$$
$$P_{z,\mathrm{cr}} = 0.5 K_y h_c\left(\sqrt{1+\frac{16\pi^2 T}{K_y H^2}}-1\right). \tag{1}$$

Shear stiffness of the elastomeric layer K_y, determined from pure shear scheme, and bending stiffness T without accounting of elastomeric layer low compressibility is:

$$K_y = \frac{GF}{h_e},$$
$$T = \frac{2(1+\mu)GI_x\left(1+\gamma\ \Phi^2\right)}{h_e}, \tag{2}$$
$$\Phi = \frac{F_l}{F_f} = \frac{b}{2h_e},$$

where: G – shear modulus of the elastomer; I_x – axial moment of inertia of the cross section of element of the TRME package; Φ – shape factor; γ – empirical coefficient; F_l – loaded surface area of the block; F_f – free surface area.

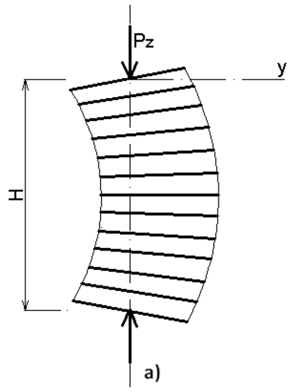

Fig. 3a. Loss of stability of TRME device under axial compression: Euler buckling.

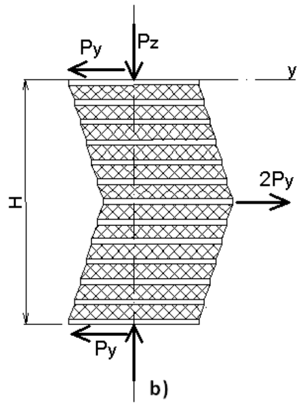

Fig. 3b. Loss of stability of TRME device under axial compression: pure shear buckling.

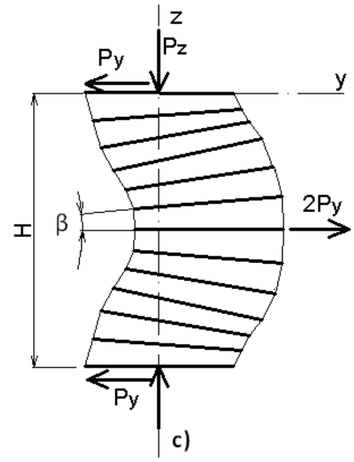

Fig. 3c. Loss of stability of TRME device under axial compression: Euler buckling with share contribution.

The dependence (1) gives acceptable results for elastomeric layers with a shape factor 1–2 (or $b/h_e < 5$), small deformations (< 10–15 %) and for specific axial load P_z/F till 5–10 MPa [3].

Presented articles task is to determine the critical axial compression force for small and middle (15–60 %) deformations domain with thin layers ($b/h_e > 10$) and high specific axial loads ($P_z/F > 10$ MPa). In this case instead of stiffness (2) to substitute shear K_y and bending T, stiffness is calculated accounting to weak compressibility of elastomeric layer and the loading level effect on the shear modulus of the elastomeric material.

IV. Mathematical Models of Simple TRME Layer

Flat, circular TRME is considered in cylindrical coordinate system r, φ, z (Fig. 1).

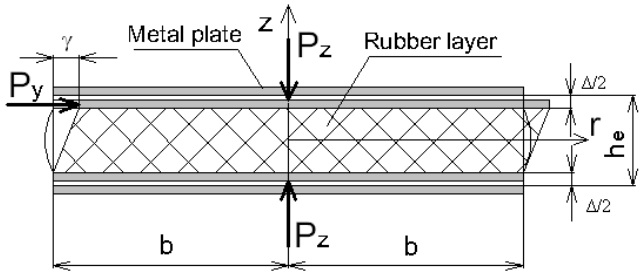

Fig. 4. Scheme of shear for TRME section.

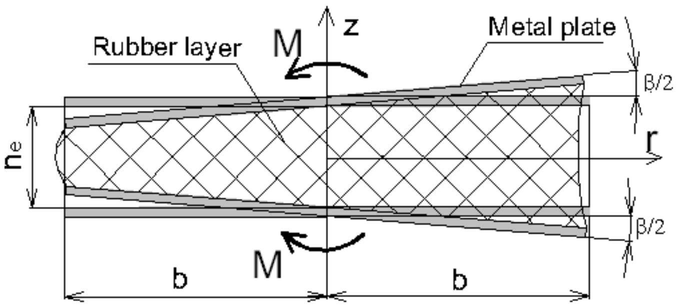

Fig. 5. Scheme of bending for TRME section.

Shear stiffness of the precompressed thin elastomeric layer K_y that is determined from pure shear scheme (Fig. 4) is calculated using the formula:

$$K_y = \frac{GF}{h_e \delta},$$

where $$\delta = 1 - \frac{\Delta}{h_e}, \qquad (3)$$

Δ − axial deformation of the elastomeric layer (Fig. 2).

This dependence is confirmed by shear deformation experiments on the preloaded TRME [7], [8]. Flat circular TRME is considered in cylindrical coordinates. Required dependences "P_z – Δ" and "M – β" ("axial force − displacement" and "bending moment – rotation angle") are defined by means of the Ritz's method, minimizing the additional strain energy [7], subject to the weak compressibility of elastomer, assuming that metal layers are nondeformable either (or may undergo only a plane tensile strain):

$$\Pi = \iiint_V W(\sigma_{ij})dv - \int_{F_u} \sigma_{ij} n_j u_i dF \qquad (4)$$

where:

$$W(\sigma_{ij}) = \frac{\sigma^2}{2K} + \frac{\sigma_r^2}{6G}, \quad K = G\frac{2(1+\mu)}{1-2\mu},$$

$$\sigma = \frac{1}{3}\left(\sigma_{rr} + \sigma_{\phi\phi} + \sigma_{zz}\right),$$

$$\sigma_r = 0.5\left[\left(\sigma_{rr} + \sigma_{\phi\phi}\right)^2 + \left(\sigma_{\phi\phi} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{rr}\right)^2 + 6\sigma_{r\phi}^{\ 2}\right].$$

Here: K and μ – accordingly, bulk modulus and Poisson ratio of elastomeric layer, σ – average stress (called specific hydrostatic pressure S). Stress state of the elastomeric layer is determined by the superposition of shear stress on the hydrostatic pressure [7], [8]. Stress components must satisfy the equilibrium equations and boundary conditions for the components of stress on the elastomeric layer, where loading is set.

In the case of bending (load scheme is given in Fig. 5) for the stress distribution in the middle layer of flat TRME, the hypothesis is applied [8]:

$$\sigma_{rr} \approx \sigma_{\phi\phi} = S(r,\phi),$$
$$\sigma_{zz} = \sigma_{zz}(r,\varphi), \quad \sigma_{r\varphi} = 0.$$

With regard to this, the equation of equilibrium in the volume of the elastomeric layer will be satisfied if:

$$\sigma_{rz} = -z\frac{\partial S}{\partial r}, \quad \sigma_{z\varphi} = -\frac{z}{r}\frac{\partial S}{\partial \varphi}.$$

Boundary conditions for stress on the free surface of the elastomeric layer are:

$$\sigma_{rr}(b,\varphi) = 0,$$

$$\int_{-0.5h_e}^{0.5h_e} \sigma_{rz}\Big|_{r=b} dz = \int_{-0.5h_e}^{0.5h_r} \sigma_{\kappa z}\Big|_{r=b} dz = 0.$$

Taking into account the boundary conditions, equations for stress functions are chosen:

$$\sigma(r,\varphi) = C_1 \cos\varphi\left(\frac{r^3}{b^3} - \frac{r}{b}\right),$$

$$\sigma_{zz}(r,\phi) = C_2 S(r,\phi) + C_3 \frac{r}{h}\cos\phi.$$

Constants C_1, C_2, C_3 are found by minimizing of the functional (4) of additional energy.

$\Pi = \Pi(C_1, C_2, C_3)$, from system of functional minimum $$\frac{\partial \Pi}{\partial(C_1, C_2, C_3)} = 0. \qquad (5)$$

The dependence "$M - \beta$" and, hence, bending stiffness is defined from the equation:

$$M = -\int_0^{2\pi}\int_0^b \left(\sigma_{zz}\cos\phi - \sigma_{rr}\sin\phi\right) r^2 \mathrm{d}r\mathrm{d}\phi = -\int_0^{2\pi}\int_0^b \sigma_{zz}\cos\phi \cdot r^2 \mathrm{d}r\mathrm{d}\phi.$$

Bending stiffness of elastomeric layer with accounting of elastomeric layer weak compressibility is:

$$T = \frac{M}{\beta} = \frac{G\pi b^4}{h_e\left(1+\frac{G}{3K}\right)}\left(0.75 + 0.125\frac{b^2}{h_e^2}\frac{\left(1-\frac{2}{3}\frac{G}{K}\right)^2}{\left(1+\frac{G}{3K}\right)}\left(1+\frac{3}{4}\frac{b^2}{h^2}\frac{G}{\left(1+\frac{G}{3K}\right)K}\right)^{-1}\right) \quad (6)$$

Bending stiffness T_1 without accounting of elastomeric layer low compressibility ($K \to \infty$) is:

$$T_1 = \frac{M}{\beta} = \frac{G\pi b^4}{h_e}\left(0.75 + 0.125\frac{b^2}{h_e^2}\right). \quad (7)$$

Dependences (6) and (7) may be used in the case of the precompressed TRME at low axial deformation under preloading. In case of average deformation the dependence may be obtained from (6) using the delta method of integration [8],[10]:

$$T = \frac{M}{\beta} = \frac{G\pi b^4}{h_e\left(1+\frac{G}{3K}\right)\delta}\left(0.75 + 0.125\frac{b^2}{h_e^2\delta^2}\frac{\left(1-\frac{2}{3}\frac{G}{K}\right)^2}{\left(1+\frac{G}{3K}\right)}\left(1+\frac{3}{4}\frac{b^2}{h^2\delta^2}\frac{G}{\left(1+\frac{G}{3K}\right)K}\right)^{-1}\right) \quad (8)$$

$$\delta = 1 - \frac{\Delta}{h_e}.$$

Bending stiffness without accounting of elastomeric layer low compressibility is:

$$T_1 = \frac{M}{\beta} = \frac{G\pi b^4}{4h_e\delta}\left(3 + \frac{1}{2}\frac{b^2}{h_e^2\delta^2}\right), \qquad \delta = 1 - \frac{\Delta}{h_e}. \quad (9)$$

In the case of axial compression (load scheme is given in Fig. 2) "$P_z - \Delta$" dependence is defined the same way. "$P_z - \Delta$" dependence accounting low compressibility of elastomer is:

$$P_z = \frac{3FG}{\left(1+\frac{G}{3K}\right)}\frac{\Delta}{h_e}\left(1 + \frac{1}{2}\frac{b^2}{h_e^2}\frac{\left(1-\frac{2}{3}\frac{G}{K}\right)^2}{1+\frac{G}{3K}}\left(1+\frac{2b^2G}{\left(1+\frac{G}{3K}\right)h_e^2K}\right)^{-1}\right). \quad (10)$$

"$P_z - \Delta$" dependence without elastomeric layer low compressibility taken into account is:

$$P_z = 3FG\frac{\Delta}{h_e}\left(1 + 0.5\frac{b^2}{h_e^2}\right). \quad (11)$$

Dependences (10), (11) are used in the case of low axial deformation. In case of average deformation "$P_z - \Delta$", dependence taken into account of elastomeric layer low compressibility, is obtained using the delta – method of integration [7], [8]:

$$P_z = \frac{3FG}{\left(1+\frac{G}{3K}\right)}\frac{\Delta}{h_e}\bullet \quad (12)$$

$$\bullet\left(-\ln\delta + \frac{1}{4}\frac{b^2}{h_e^2}\frac{\left(1-\frac{2}{3}\frac{G}{K}\right)^2}{1+\frac{G}{3K}}\left(\delta^{-2}-1\right)\left(1+\frac{2b^2G}{\left(1+\frac{G}{3K}\right)h_e^2K}\left(\delta^{-2}-1\right)\right)^{-1}\right)$$

$$\delta = 1 - \frac{\Delta}{h_e}.$$

"$P_z - \Delta$" dependences without accounting of elastomeric layer low compressibility are:

$$P_z = 3FG\left(-\ln\delta + 0.5\frac{b^2}{h_e^2}\left(\delta^{-2}-1\right)\right), \qquad \delta = 1 - \frac{\Delta}{h_e}. \quad (13)$$

V. Material Model of Elastomer

Results of experiments on thin TRME compression [2], [9] show that at relatively small strains (up to 10–15 %), specific loading (P_z/F) may reach 200 MPa. The dependence of the "force-displacement" has highly nonlinear character, indicating that the mechanical modules of elastomer depend on the level of specific compressive strength even in small deformations area. In experimental studies it is shown that shear and bulk modulus of elastomeric layer G and K depend on the intensity of the specific loading if $S = P_z/F$ is more than 5 MPa [9].

In order to take into account load intensity influence on "force − displacement" dependence and on the critical force, it is proposed to substitute the values $G(S)$ and $K(S)$ instead of modules G and K. For thin, flat elastomeric layers it can be assumed with sufficient accuracy that $S = P_z/F$ (where F − the area of plane layer). This approach allows calculation of $G(S)$ and $K(S)$ from the volumetric "tension – compression" experiments with accuracy up to the assumption of small deformations. Due to the lack of experimental data, it is proposed in [9] at first approximation to assume that the dependence of $G(S)$ and $K(S)$ has the same type:

$$G(S)/G \approx K(S)/K = 1 + S\beta, \quad (14)$$

where β − empirical coefficient is defined from experiment on pure volumetric compression.

These equations (6)–(14) using (1) allow to estimate critical force for large values of specific external compressive load. The order of critical buckling force calculation: in accordance with (1), (2) the preliminary force is defined; for this P_z hydrostatic pressure S and displacement Δ in accordance with (10), (12), (14) are defined; depending on Δ value bending and shear stiffness (T, K_y) are defined using equation (6), (8), (3); (14); received T and K_y are substituted in (1) and P_{cr} is calculated. If necessary, calculation is repeated till the coincidence of received and previous critical forces.

VI. Numerical examples

The results of critical force calculation for flat rectangular TRME package with typical in industrial application dimensions are presented below. Plots of buckling force dependence on the number of sections in packet are given for TRME, which is presented in [2]. For thin, layered packages experimental critical force is much greater than the calculated in accordance with conventional equations (1) and (2).

In Fig. 6 the critical buckling force plots for sample with brass bonded layers are given; layer dimensions: b = 27.5 mm, h_e = 1.0 mm, h_o = 0.1 mm, section height $h_c = h_e + h_o$ = 1.10 mm, Φ = 27.5, TRME package height $H = h_cN + h_o$, where N – number of sections. Mechanical properties of elastomers: G = 1.17 MPa, K = 2760 MPa, μ = 0.49936, β = 0.001.

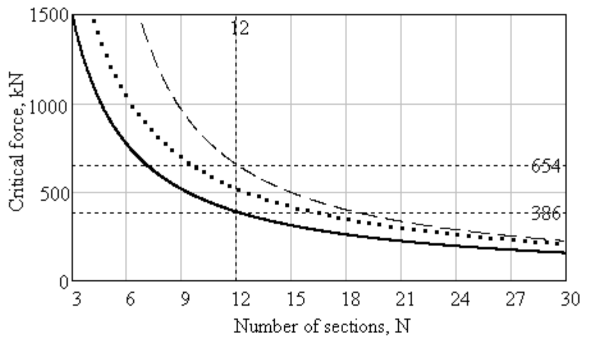

Fig. 6. Plots of critical force dependence on the number of sections with h_e = 1 mm: —— − in accordance with equations (1) and (2), ▪▪▪▪ − in accordance with (8), (12), (3), (1), — — − in accordance with (8), (12), (3), (1), taking into account (13).

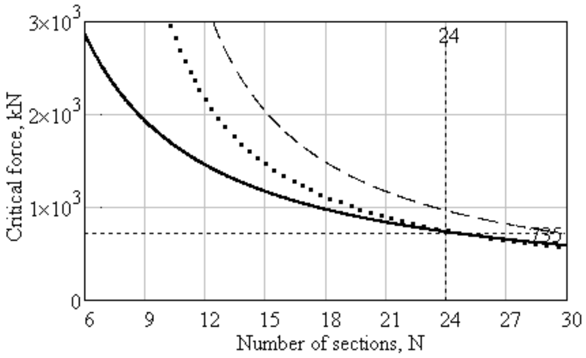

Fig. 7. Plots of critical force dependence on the number of sections with h_e = 0.5 mm: —— − in accordance with equations (1) and (2), ▪▪▪▪ − in accordance with (7), (11), (3), (1), — — − in accordance with (6), (10), (3), (1) and (13).

In Fig. 7 critical buckling force plots are presented for the sample of the same material with the same b and h_o, but h_e = 0.5 mm, h_c = 0.6 mm, shape factor Φ = 55.

Specific load in all cases is more than 100 MPa. It is seen from the plots how critical force value depends on the thickness of elastomeric layers and on the number of sections.

VII. Conclusion

This work presents the methodology of the buckling force calculation for thin-layered rubber-metal packages widely used as vibroisolators, shock absorbers, and compensation devices. Such devices usually carry a very large load and should be checked for buckling. Three approaches are discussed: conventional, taking into account the thickness of elastomeric layers, taking into account the thickness of elastomeric layers and changing of the elastomeric mechanical properties (shear and bulk modules) depending on pressure. The results of numerical examples show, that when the number of layers in package increases, the critical force value becomes closer. Each type of TRME demands an individual approach.

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Vladimirs Gonca, Egons Lavendelis, Māris Eiduks. Multislāņu gumijas-metāla amortizatora stabilitāte
Šajā darbā apskatītas daudzslāņu gumijas – metāla plakanu elementu paketes (TRME), kas tiek pielietotas kā vibrāciju izolatori pie bīdes un kompresijas slogojumiem dažādās mašīnbūves nozarēs, kā arī celtniecības konstrukcijās. Vibrāciju izolatoriem paredzētajās TRME paketēs pie ass virziena spiedes slogojuma galvenā bīstamība saistās ar iespējamību zaudēt garenvirziena stabilitāti. Pielietojot Timošenko matemātisko modeli, ņemot vērā šķērsspēku, tiek sastādīts matemātiskais modelis TRME pakešu garenvirziena stabilitātes pētīšanai ass virziena spiedē. Ņemot vērā: slāņu skaitu, ģeometriskās īpašības, iestiprināšanas veidu, mazās un vidējās elastomēru slāņu garenvirziena deformācijas − iegūti analītiskie vienādojumi TRME pakešu kritiskā spēka noteikšanai. Pielietojot dažādu autoru eksperimentālos rezultātus, tiek ņemta vērā elastomēra fiziskā nelinearitāte – atkarībā no bīdes moduļa un elastomēra tilpuma moduļa atkarības no hidrostatiskā spiediena elastomērā. Iegūtās analītiskās atkarības sniedz iespēju projektēt TRME paketes ar to dažāda veida iespējām stiprināšanai, nodrošinot garenvirziena stabilitāti nepieciešamajā darba slodžu diapazonā. Analītiski izteiktie TRME pakešu stinguma raksturlielumi pie bīdes, izlieces un garenvirziena spiedes iegūti ar Ritca metodi, pielietojot pievienoto deformāciju potenciālās enerģijas minimuma principu. Pētījuma rezultāti tiek salīdzināti ar teorētiskiem un eksperimentāli iegūtiem rezultātiem no citu autoru darbiem.