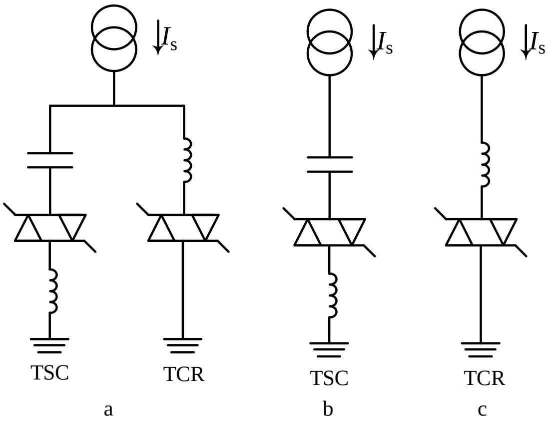

Fig. 1. A design of SVC: a) TSC+TCR; b) TSC; c) TCR.

In most cases, the static compensator consists of a TSC and a TCR (Fig. 1a). There can also be other combinations of the devices, for example, a separately designed TSC (Fig. 1b) or TCR (Fig. 1a). According to [3], SVC can consume or generate reactive power in order to control several specified parameters (normally voltage at some point of the network).

The working characteristic of SVC is presented in Fig. 2.

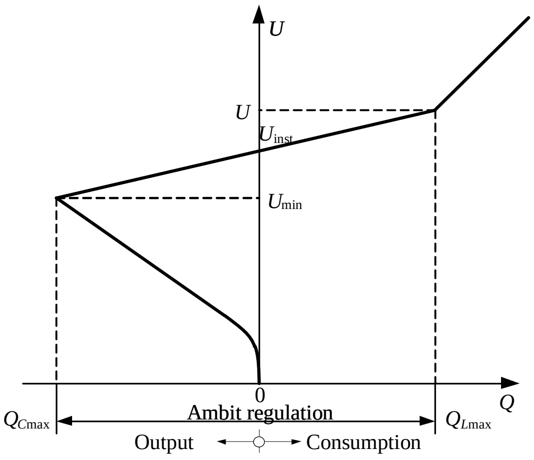

Fig. 2. Working characteristic of SVC.

Within the control range from $Q_{C\max}$ to $Q_{L\max}$, the characteristic has a slope (2 % to5 %) determined by the frequency droop. Beyond the control range, the characteristic changes linearly depending on the values of voltage U and capacitive X_C and inductive X_L reactances of SVC. Gradual control of reactive power in SVC is carried out by changing the firing angle of the thyristors in the reactor. In order to specify the admittance and maintain the specified voltage at node, it is necessary to determine the angle α.

When solving the state estimation problem and calculating steady state, the reactive power of the shunt at a node i is determined by the equation:

$$Q_i^{\mathrm{H}} = U_i^2 b_{i\mathrm{H}} \tag{1}$$

and is used in the equation of nodal balance for reactive power:

$$Q_i = Q_{i\mathrm{G}} + Q_{i\mathrm{H}} + \sum_{j\in\omega_i} Q_{ij} + Q_i^{\mathrm{H}}, \tag{2}$$

where: U_i – voltage at node i; $b_{i\mathrm{H}}$ – the nodal shunt admittance; $Q_{i\mathrm{G}}$, $Q_{i\mathrm{H}}$ – reactive power generated and consumed at node i; $\sum_{j\in\omega_i} Q_{ij}$ a sum of reactive power flows along the lines incident to node i; ω_i – a set of nodes incident to the i-th one.

The authors of [3], [4], and [7] show that the SVC admittance, depending on angle α, is determined by the expression:

$$b_{\mathrm{SVC}}(\alpha) = \frac{1}{X_C X_L}\left\{X_L - \frac{X_C}{\pi}\left[2(\pi-\alpha)+\sin 2\alpha\right]\right\}. \tag{3}$$

By substituting (3) to (1), we obtain an expression for the calculation of SVC reactive power:

$$Q_i^{\mathrm{SVC}} = -\frac{U_i^2}{X_C X_L}\left\{X_L - \frac{X_C}{\pi}\left[2(\pi-\alpha)+\sin 2\alpha\right]\right\}. \tag{4}$$

The authors of [3] and [4] considered the SVC model, which in an explicit form includes the control angle α. In these studies, the expression for the SVC reactive power is represented as (4) (in our notations).

Linearized equations for SVC in the steady-state calculation in the k-th iteration are written as follows:

$$\begin{bmatrix}\Delta P_i\\ \Delta Q_i\end{bmatrix}^{(k)} = \begin{bmatrix}0 & 0\\ 0 & \dfrac{2U_i^2}{\pi X_L}\left[\cos 2\alpha - 1\right]\end{bmatrix}^{(k)} \begin{bmatrix}\Delta\delta_i\\ \Delta\alpha\end{bmatrix}^{(k)}. \tag{5}$$

The same as most of the FACTS devices, STATCOM represents a thyristor-controlled reactive power source that maintains a specified voltage value by consuming or generating reactive power at the connection point without using additional external reactors or high-power capacitor banks.

STATCOMs involve either gate turnoff thyristors (GTOT) or integrated gate-commutated thyristors (IGCT), or insulated-gate bipolar transistors (IGBT).

STATCOM can include either a voltage converter or a current converter. Fig. 3 presents a simplified scheme of STATCOM with a voltage converter.

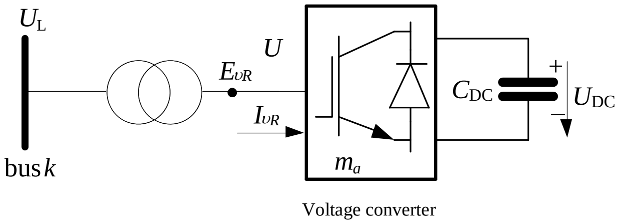

Fig. 3. STATCOM design with a voltage converter.

The reactive power flow Q between the voltage converter and the AC electric power system is controlled by changing the value of the output voltage of the voltage converter, U. When the value U exceeds the value of voltage in line, U_L, the STATCOM operates in a capacitive mode and reactive power is generated. As the value U declines below the value of voltage in line U_L, STATCOM operates in an inductive mode. In this case, the reactive power is consumed. When the voltages are equal $U_L = U$, $Q = 0$.

The parameters specified in STATCOM include the voltage setting U_{ref}, voltage droop X_{ST}, and ranges of change in the current I_{min}, I_{max}. Figure 4 demonstrates the STATCOM volt-ampere characteristics.

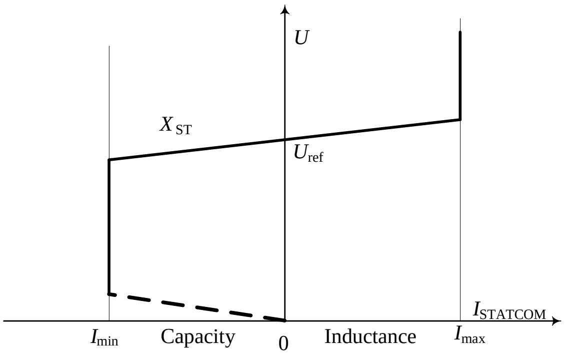

Fig. 4. STATCOM volt-ampere characteristics.

STATCOM devices use gate-turnoff and insulated-gate bipolar technologies with pulse-duration modulation, which makes it possible to regulate the amplitude and phase of voltage thanks to fast switchings of GTO or IGBT elements. High modulation frequencies allow a considerable reduction in the level of harmonics in the output signals.

Thyristor converter of STATCOM provides an exchange of reactive power between the network phases. Thus, STATCOM can both generate and consume reactive power.

STATCOM is characterized by fast operation and small size. It can regulate both the value and the phase of voltage in the electrical network it is connected to. In the case the DC section has a storage device, STATCOM can also regulate active power.

STATCOM can be represented as a voltage source connected to the network through shunt $Z_{\upsilon R}$ (Fig. 5) whose admittance p.u. equals droop.

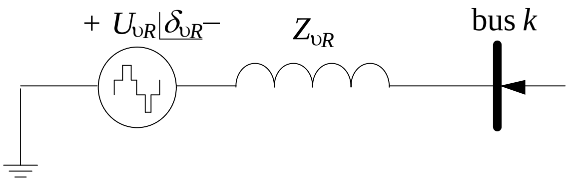

Fig. 5. Equivalence scheme of STATCOM design.

In this case, the controlled variables will be represented by voltage amplitude $U_{\upsilon R}$ and angle $\delta_{\upsilon R}$ of the voltage converter, which are state variables.

For active and reactive power components of the converter and bus k, we can write the following set of equations [3]:

$$\begin{aligned}
P_{\upsilon R} &= U_{\upsilon R}^2 G_{\upsilon R} + U_{\upsilon R} U_k \left[G_{\upsilon R} \cos\left(\delta_{\upsilon R} - \theta_k\right) + B_{\upsilon R} \sin\left(\delta_{\upsilon R} - \theta_k\right) \right];\\
Q_{\upsilon R} &= -U_{\upsilon R}^2 B_{\upsilon R} + U_{\upsilon R} U_k \left[G_{\upsilon R} \sin\left(\delta_{\upsilon R} - \theta_k\right) - B_{\upsilon R} \cos\left(\delta_{\upsilon R} - \theta_k\right) \right];\\
P_k &= U_k^2 G_{\upsilon R} + U_k U_{\upsilon R} \left[G_{\upsilon R} \cos\left(\theta_k - \delta_{\upsilon R}\right) + B_{\upsilon R} \sin\left(\theta_k - \delta_{\upsilon R}\right) \right];\\
Q_k &= -U_k^2 B_{\upsilon R} + U_k U_{\upsilon R} \left[G_{\upsilon R} \sin\left(\theta_k - \delta_{\upsilon R}\right) - B_{\upsilon R} \cos\left(\theta_k - \delta_{\upsilon R}\right) \right].
\end{aligned} \tag{6}$$

The fully written linearized model (6) is presented in [3].

Let us consider how the proposed approaches to the power system state estimation problem can be implemented.

III. THE PROBLEM OF POWER SYSTEM STATE ESTIMATION

The state estimation problem of an electric power system implies the calculation of state variables, which is based on the data of measurements [8], [14]. The measurements applied in the state estimation are mainly represented by remote measurements received from SCADA systems: magnitudes of nodal voltage U_i, generation of the active P_G and reactive Q_{ij} power at nodes, power flows in the transformers and lines P_{ij}, Q_{ij}, less often – currents at nodes and in lines I_i, I_{ij}. Accordingly, the vector of measurements looks like:

$$\bar{y} = \{P_i, Q_i, P_{ij}, Q_{ij}, U_i, I_i, I_{ij}\}. \tag{7}$$

When solving the state estimation problem, we introduce a notion of the state vector x with dimension $2n - 1$ (where n is the number of nodes of the calculation scheme). The state vector includes the voltage magnitudes U and the phase angles $\delta x = (\delta, U)$, except for the fixed phase of base node. Such a vector of state uniquely determines all the other state variables.

Mathematical statement of the state estimation problem is reduced to the minimization of the objective function

$$J(x) = \left(\bar{y} - y(x)\right)^{\mathrm{T}} R_y^{-1} \left(\bar{y} - y(x)\right), \tag{8}$$

i.e., to the search of the estimates of the state vector $\hat{x}$; here R_y^{-1} is a diagonal matrix of the weighting coefficients whose elements are inverse to the variances of the measurements that characterize their accuracy.

Due to the nonlinear dependence of $y(x)$, the problem is solved iteratively. In each iteration, a normalized system of equations is solved with respect to the correction vector:

$$\Delta x_k = \left[H_k^T R_y^{-1} H_k \right]^{-1} H_k^T R_y^{-1} \left[\bar{y} - y\left(x_k\right) \right] \tag{9}$$

where $H_k = \dfrac{\partial y}{\partial x}$ is the matrix of Jacobi, calculated in the k-th iteration.

IV. Modeling of SVC and STATCOM in State Estimation of Electric Power System

When modeling SVC, the angle α is specified as a state vector component and is determined directly in the process of solving the state estimation problem.

In the calculation of corrections to the state vector x by (9), the derivative of the injection measurement Q_i with respect to voltage U_i is taken equal to:

$$\frac{\partial Q_i}{\partial U_i} = \sum_{j \in \omega_i} \frac{\partial Q_{ij}}{\partial U_i} + 2U_i b_{i\text{H}}.$$

When using the *SVC* model with the control angle α in the state estimation problem, *SVC* is modeled by the variable of susceptance at node i at which the *SVC* is installed, and α is included in the state vector x instead of U_i, which is fixed. A derivative of the injection measurement at the node with *SVC* with respect to α_i will be equal to:

$$\frac{\partial Q_i}{\partial \alpha_i} = -\frac{U_i^2 b_L}{\pi}(-2 + 2\cos 2\alpha) = \frac{2U_i^2 b_L}{\pi}(1 - \cos 2\alpha). \qquad (10)$$

An algorithm for the calculation

1. Specify the admittance b_{SVC} at a necessary node i, using reference data $b_{\text{SVC}} = \frac{Q_{\text{SVC}}}{U_{\text{SVC}}^2}$.
2. Fix voltage at node i at a required value, by specifying it by the measurements with a zero (a very small) variance. The state vector x will include α_i for this node.
3. Specify the initial approximations of the vector $x = x_0$, at which specify $\alpha_{i0} = \pi/2$ for the node where the SVC is installed; $U_{i0} = U_{i\text{nom}}$; for the other nodes: $\delta_{i0} = 0$; $U_{i0} = U_{i\text{nom}}$.
4. Calculate the corrections in iterations by the method of Newton, using (9). In this case, there will be one nonzero element calculated by (9) in the column of derivatives $\frac{\partial Q_i}{\partial \alpha_i}$ in the matrix $\frac{\partial y}{\partial x}$. At node i, it is necessary to specify the measurement of the nodal injection Q_i.

STATCOM is modeled as a voltage source connected to the network through shunt $Z_{\upsilon R}$. The control parameters of STATCOM (voltage $U_{\upsilon R}$ and angle $\delta_{\upsilon R}$ of a voltage converter) represent the state vector components in state estimation. Therefore, such a model is easily included in the state estimation algorithm.

1. We will dwell on the calculation algorithm in more detail.
2. At node i, where voltage should be maintained constant, we specify a branch with a fictitious node.
3. Branch parameters: resistance $R = 0\ \Omega$, the value of reactance X is assumed equal to a droop of the STATCOM characteristic presented in Fig. 4. In our case study, $X = 0.086\ \Omega$.
4. Fix voltage at node i at a required value by specifying it with the measurement with a zero (very small) variance. Node i is assumed to be a transit one.
5. Fictitious node is specified by a node transit with respect to active power.
6. Make state estimation. In the process of calculation, we obtain the reactive power injection Q_{STATCOM} at the STATCOM connection point, which is necessary to maintain the required voltage value.
7. Check the condition: $Q_{\min} < Q_{\text{STATCOM}} < Q_{\max}$, into which, as is shown in [5], the condition $I_{\min} < I_{\text{STATCOM}} < I_{\max}$ can be transformed within the control range.

The condition $U_{\min} < U_{\text{STATCOM}} < U_{\max}$ is controlled when solving the problem of state estimation.

7.1. In the case condition (9) is met, we go to p. 6.

7.2. In the case condition (9) is not met, the algorithm of considering the inequality-constraints on unmeasured variables, which is implemented in the software “Otsenka”, operates [14].

8. Exit.

V. Calculation Results

A scheme for testing the suggested methods is represented by a 19-node scheme of the Irkutsk electric power system presented in Fig. 6.

For modeling, we used STATCOM with two inverter units, 50 MVar each, at voltage of 11 kV, and SVC with two inverter units, 40 MVar each, at voltage of 11 kV.

STATCOM and SVC are modeled at a low side of autotransformer of the “Irkutskaya” substation (nodes 17, 18, and 19). When modeling STATCOM, the scheme was supplemented with fictitious nodes 177, 188, 199 and branches 17–177, 18–188, 19–199.

The results of comparative calculations for STATCOM and SVC are presented in Table I.

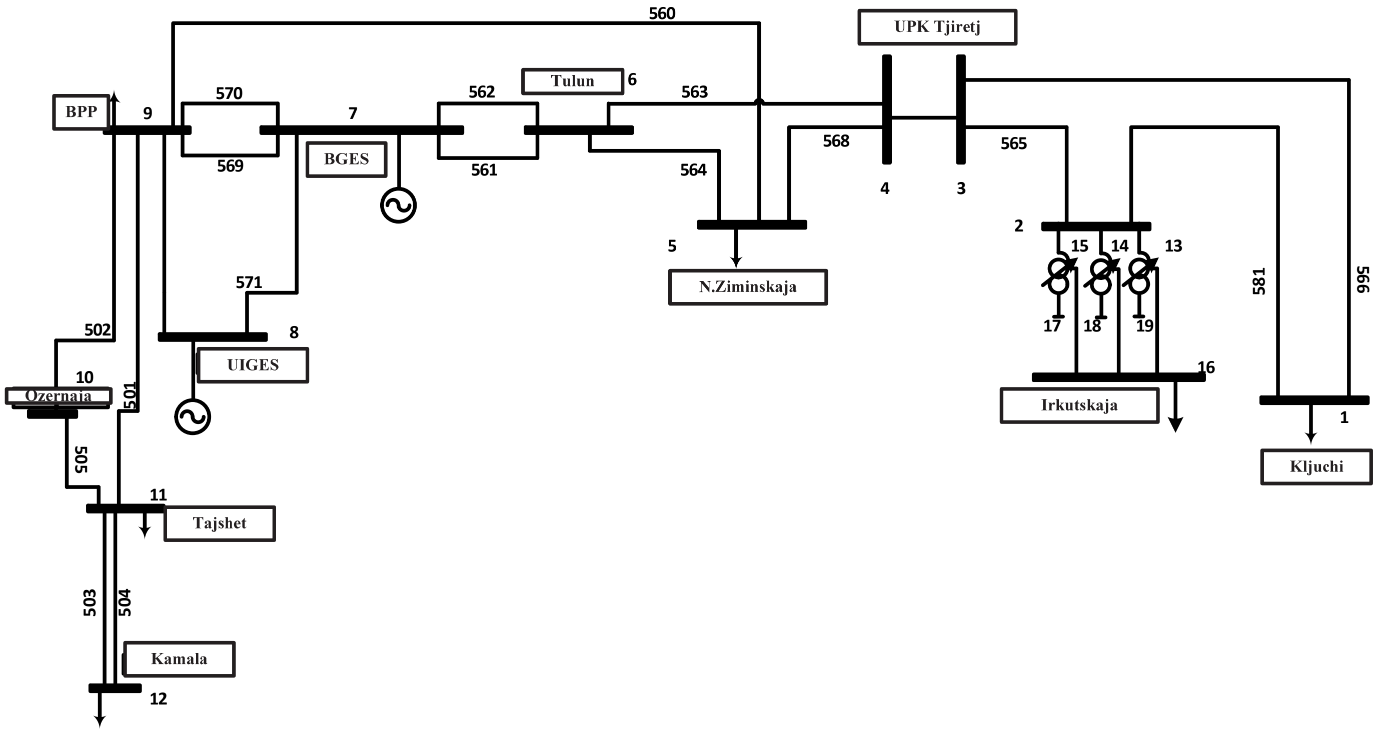

Fig. 6. A scheme of the Irkutsk electric power system.

TABLE I

CALCULATION RESULTS

Node	U_{meas}	U_{se}	δ	P_{meas}	P_{se}	Q_{meas}	Q_{se}	b_{SVC}	α
No.	kV	kV	degree	MW	MW	MVar	MVar	mcSim	
Calculation with STATCOM									
16	220	224	–7.9	–381	–382	–402	–402		
17	10.5	10.5	–8.0	–	0.0				
18	10.5	10.5	–8.0	–	0.00				
19	10.5	10.5	–8.0	–	0.00				
177	–	11.4	–8.0	–	0.00		11		
188	–	11.4	–8.0	–	0.00		11		
199	–	11.4	–8.0	–	0.00		11		
Calculation with SVC									
16	220	223.64	–7.9	–382	–394	–402	–396.67		
17	10.5	10.55	–8.0	–	0.00		10	0.090	–0.934
18	10.5	10.55	–8.0	–	0.00		10	0.090	–0.934
19	10.5	10.55	–8.0	–	0.00		10	0.090	–0.934

The data of Table I demonstrate the calculation results for the cases of connecting two different reactive power control devices (SVC and STATCOM) that virtually coincide (within a specified accuracy of calculation).

The analysis of the algorithms shows that for modeling SVC, it is necessary to include additional components in the vector of state, but this makes it possible to determine the angle of control α in real time [13]. The algorithm allows us to form a mathematical model of an energy system with embedded SVC model, which does not require additional preparatory or subsequent calculations and can be used for off-line operation in various software packages. The main flaw of the algorithm is the fact that it is hard to implement and should be adjusted to a specific program of state estimation, considering its specific features and constraints of the applied methods.

STATCOM is modeled by an additional branch with a voltage source at the node where STATCOM is installed. The algorithm is easily implemented with the minimum labor efforts and can be implemented virtually in any state estimation software (Cosmos, ANARES, Otsenka, etc.) without changes in programming code and introduction of new components in the vector of state as in the case of including the SVC model in the power system state estimation problem. This algorithm does not require direct participation of an operator in the calculations and can operate in real time. The necessary condition for the implementation of the STATOM model in the state estimation software is availability of a module for considering inequality-constraints on measured and unmeasured variables. It should also be noted that compared to SVC STATCOM can be applied when voltage declines greatly. Moreover, the STATCOM response to changes in operating conditions is faster and the device does not generate harmonic oscillations.

VI. CONCLUSION

The paper shows the relevance of including the models of FACTS devices in the equivalent circuit when solving the power system state estimation problem. Consideration is given to the models of SVC and STATCOM, which are used in the steady state calculation. The modified state estimation algorithms developed for modeling the SVC and STATCOM devices provide voltage stabilization and gradual or stage-by-stage change in the consumed and (or) supplied reactive power. The calculations made for a fragment of the 500 kV Irkutsk power system demonstrated high-speed operation and good convergence of the developed algorithms, which makes it possible to use them for obtaining the estimates in real time.

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