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WHEN TECHNICAL COEFFICIENT CHANGES NEED TO BE ENDOGENOUS: THE CASE OF IMPORTS IN THE INFORUM ITALIAN MODEL

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Introduction

In contrast to the basic properties of the standard input-output (I-O) model (Miller & Blair, 2009) stated for example by Erik Dietzenbacher (IIOA Newsletter, 2015), INFORUM models have quantities and prices integrated. This distinctive feature of this class of multisectoral dynamic models designed for long-term policy simulation analyses poses peculiar and challenging modeling approaches (see Almon, 1991; 2016). This paper focuses on the interactions among imports, technical coefficients and price formation. First, the modeling approach to cope with the divergence between imports econometrically estimated and imports computed by means of account identities is shown. Second, the need to model technical coefficient changes for long-run forecasting is presented as empirical evidence from the model builder's data set. Third, even taking into account the Hawkins-Simon (Hawkins & Simon, 1949) conditions, modeling imports in an open economy may easily lead to negative outputs. A procedure to "update" input-output technical coefficients to fix a multisectoral model during the forecasting process is developed. Although a number of contributions are devoted to the technical coefficients (for example Hewings & Sonis, 1992; Jalili, 1999; Nishimura, 2002; Sonis & Hewings, 1996) no one tackles the problem of modeling them.

Finally, the algorithm to model technical coefficients in the INFORUM multisectoral models system is described.

1. Import shares in an INFORUM model

If an I-O table is available in total domestic flows and imported flows, an import shares matrix, which includes intermediate consumption and domestic final demand components can be computed, which we will call "MM". In an INFORUM type model, imports are *modeled* like other endogenous final demand components (i.e. personal consumption expenditures, investments, and exports, while other components are placed among scenario variables). In principle, the total output vector obtained from the solution of the model can be used to compute back the I-O table flows and, using the matrix MM, the associated I-O import flow matrix is obtained. The row sum of such a matrix equals to the imports vector m (imports by type of product). This imports vector will in general be different (except in the base year) from the imports vector obtained with a system of equations, $m^$; namely $m \neq m^$. This discrepancy is due to the different content of imports in both intermediate consumption and final demand components induced also by structural changes in the resources side (sectoral imports – total resources ratios).

In general, changes in resource composition (imports + output) do not necessarily imply a change of the size of the technical coefficients; but if imports of good *i* grow faster than the corresponding output, the imports share in intermediate consumption and final demand supplied by product *i* necessarily increases. If the technical coefficients remain constant, the MM coefficients must change.

The MM matrix does not play any role in solving the real side of the I-O model. Actually, it is central in the nominal side where the impact of the import prices in the price equations is related to the import contents of intermediate consumption. Therefore, changes of technical coefficients and of import shares affect the price formation, so that changes in MM coefficients turn out to interact with the real side of the I-O model.

In order to take into account such an interaction, INFORUM suggests and applies an algorithm to adjust the MM coefficients. Basically, the difference between imports estimated (m) and imports calculated ($m^$) of a product group *i* is used to modify the elements of the *i*-th row of matrix MM so that m_i is equal to $m_i^$. The adjustment cannot be simply proportional to the factor $m_i^ / m_i$ since import shares are not allowed to be greater than one. The algorithm provides increases as well decreases of import shares (the elements of matrix MM), greater for low shares and lower for great shares under the constraint $mm_{ij} < 1$ (see Meade, 1995).

On rare occasions this algorithm may be unsuitable. This may happen when product *i* imports obtained from econometrically estimated equations are larger than product i imports calculated (from the MM matrix) and the required increase of import shares does not cover the difference $m_i - m\hat{}_i$. For example, if non-zero import shares are all equal to one and imports econometrically estimated are larger than imports calculated by means of the matrix MM, their difference cannot be covered by changing MM coefficients. Such a case reveals that the I-O table flows must change in order to get $m_i = m\hat{}_i$ and, consequently, changes of I-O flows as well as technical coefficients should take place.

2. Changes of technical coefficients

Technical coefficients cannot be assumed constant over time. In fact, if $A_0q_0 + f_0 = q_0$ is the Leontief equation of the interindustry multisectoral model in the base year (t = 0), given outputs and final demand (components) time series, let us say, ...f_{-3}, f_{-2}, f_{-1}, f_0, f_1, f_2, f_3...and ....q_{-3}, q_{-2}, q_{-1}, q_0, q_1, q_2..., in general, out of the base year, $A_0q_t + f_t \neq q_t$. Therefore, the coefficient matrix A, must necessarily change over time.

However, in building input-output models for comparative static analysis, modeling a matrix of technical coefficients is not a priority; but it may be the cornerstone scenario variable when changes of technical coefficients are the crucial component of an experimental design. This is the case, for example, of those numerous research efforts investigating the impact of carbon oxide reduction policies that imply changes in production functions.

Suggestions for modeling the matrix of technical coefficients in dynamic multisectoral models come from accounting identities. In fact, if an I-O table time series is available, a coefficient matrix time series A_t (for t = ...–3, –2, –1, 0, 1, 2, ...) can be computed, and a balanced Leontief equation in real term is obtained up to the last available year. A time series of matrix A_t may easily help projections of the technical coefficient matrix up to the time horizon of a planned simulation. In the model builder's strategy for building an Interindustry Multisectoral model, the econometric estimation of final demand components, value added primary inputs, price formation, and sectoral labor productivity, as well as macrovariables such as disposable income come before modeling matrix A. Therefore, at the beginning of the construction of an Interindustry Multisectoral model, a technical coefficients matrix may be not modeled, but placed among the scenario variables.

However, the impact of growing imports on the solution of the Leontief equation requires appropriate changes in the coefficient matrix. Let us state the Hawkins-Simon conditions (Hawkins & Simon, 1949) quoting their corollary using the present notation: "A necessary and sufficient condition that the q_i satisfying $Aq + f = q$ are all positive for any set $f > 0$ is that all principal minors of matrix A are positive". Furthermore, they remind us that this corollary comes from a theorem where it is assumed that the elements of matrix A are independent of the elements of f.

Let us consider the Leontief equation for a two sector economy:

$$\begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \qquad (1)$$

from which the final demand vector f, can be represented as a linear combination of two vectors and two scalars, q_1 and q_2:

$$\begin{bmatrix} 1-a_{11} \\ -a_{21} \end{bmatrix} q_1 + \begin{bmatrix} -a_{12} \\ 1-a_{22} \end{bmatrix} q_2 = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}. \qquad (2)$$

The Hawkins-Simon conditions are conditions for assuring a strictly positive solution (namely, q_1 and q_2) of a linear system where the parameters a_{ij} are assumed greater than or equal to zero and less than one. In the Hawkins-Simon paper, the empirical source of these parameters is not stated. INFORUM Interindustry models and any other input-output models that refer to an observable economy are based on I-O tables. Since the Leontief equation is a transformation of the accounting system of the I-O table, its standard solution $q = (I - A)^{-1}f$ is a strictly positive vector: the output vector of the I-O table. However, such a solution is not necessarily due to a strictly positive vector f as stated by the Hawkins-Simon conditions. In fact, net exports is a vector with negative and positive elements and the negative elements may prevail over the other non-negative components of final demand; however, the solution is still productive because the Leontief equation is simply an analytical transformation of the I-O table.

The geometrical representation of the above equation is shown in Fig. 1; it gives evidence of the solutions of the Leontief equation with strictly positive f_1 and non-strictly positive f_2 final demands; following the parallelogram rule, the representation of these vectors with the vector basis (the column vectors of matrix I-A) is obtained with positive scalars: the outputs.

Let us consider the case of the final demand vector shown in Fig. 2; this vector basis fails to relate the final demand to a positive set of outputs. A vector basis giving a positive solution with vector f may be obtained by changing (increasing) the vector's second coordinates a_{21} and a_{22} to a^*_{21} and a^*_{22}.

This geometrical representation has a rational economic base as shown in the following numerical example. Let us consider the product by product I-O table (Table 1) with three products, one domestic final

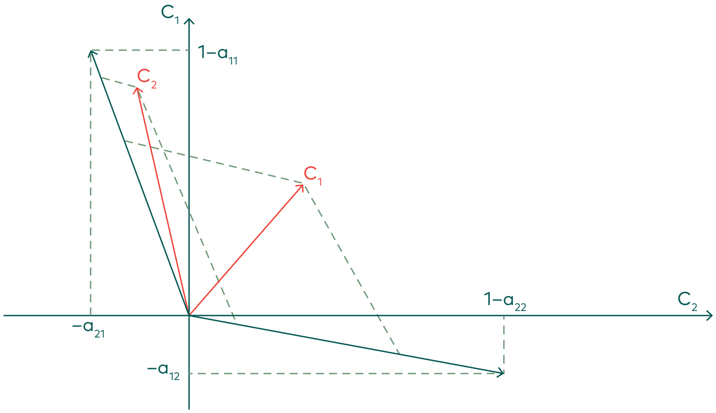

Fig. 1. Geometrical representation of the Leontief equation.

demand (DFD) vector on the USES side and imports and output on the RESOURCES side.

The coefficient matrix is

$$A = \begin{bmatrix} 0.30 & 0.29 & 0.56 \\ 0.21 & 0.22 & 0.31 \\ 0.16 & 0.15 & 0.22 \end{bmatrix}. \quad (3)$$

And its Leontief inverse is

$$(I - A)^{-1} = \begin{bmatrix} 2.30 & 1.26 & 2.16 \\ 0.89 & 1.87 & 1.39 \\ 0.65 & 0.61 & 1.99 \end{bmatrix}. \quad (4)$$

And, of course, multiplying the Leontief inverse by the final demand from the I-O table (Table 1), the total output in the table is replicated:

$$(I - A)^{-1} \cdot (\text{DFD} - \text{imports}) = (I - A)^{-1} \cdot \begin{bmatrix} 9 \\ 20 \\ 17 \end{bmatrix} = \text{output} = \begin{bmatrix} 61 \\ 55 \\ 32 \end{bmatrix}. \quad (5)$$

If imports of Product 3 increases from 8 to 18, the corresponding element of the final demand becomes –3 and the outputs from the Leontief equation are still positive:

$$(I - A)^{-1} \begin{bmatrix} 9 \\ 20 \\ -3 \end{bmatrix} = \begin{bmatrix} 39.4 \\ 41.2 \\ 12.1 \end{bmatrix}. \quad (6)$$

The Hawkins-Simon corollary assumes that final demand is strictly positive and clearly even if final demand has some negative element, the

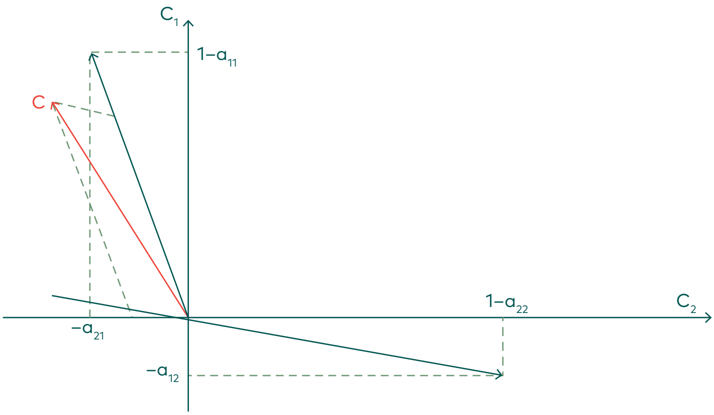

Fig. 2. Matrix A fails to match a positive set of outputs.

solution can still be productive (C_2 in Fig. 1). But if Product 3 imports change from 8 to 25, the corresponding element of final demand moves from 15 to –10 and

$$(I-A)^{-1}\begin{bmatrix} 9 \\ 20 \\ -10 \end{bmatrix} = \begin{bmatrix} 24.3 \\ 31.5 \\ -1.8 \end{bmatrix}. \tag{7}$$

The negative output of Product 3 reveals that the coefficient matrix does not match a “suitable” representation of Table 1 (C in Fig. 2). With the increase of imports from 8 to 25, resources of Product 3 change from 40 to 57 and the I-O table becomes unbalanced, because the uses of Product 3 remain 40. If the final demand of Product 3 is assumed to maintain the previous level (15) as well as the output of Product 3, the identities in the table imply that the total intermediate consumption of Product 3 needs to increase by the same amount that imports increase. This leads to tackling of the problem of ”updating” technical coefficients in matrix A.

Table 1

Product I-O table

	RESOURCES			USES				
	Output	Imports	Total	Prod 1	Prod 2	Prod 3	DFD	Total
Prod 1	61	25	86	18	16	18	34	86
Prod 2	55	28	83	13	12	10	48	83
Prod 3	32	8	40	10	8	7	15	40

3. Updating the technical coefficients in matrix A

One of the primary uses of Leontief input-output economics is impact analysis based on output multipliers for measuring the effect of a change in final demand (Miller & Blair, 2009). Every input-output analysis is based on the Leontief equation deduced from I-O accounting identities from which the basic "identity" structure $q_0 = (1 - A_0)^{-1} \cdot f_0$ is computed. If f_1 is the final demand chosen according to a given research strategy, then the impact analysis is usually measured with $q_1 = (1 - A_0)^{-1} \cdot f_1$.

Vectors q_1 and f_1 are now a part of a new I-O table where intermediate consumption flows can be computed back from matrix A_0. In an I-O table built this way, row identities (resources versus uses) fail to match. Since final demand components are part of the scenario, variables chosen by the model builder and output is a dependent variable, the identities can be restored only by changing the technical coefficients matrix, so that $q_1 = (1 - A_1)^{-1} \cdot f_1$ where A_1 is the "updated" matrix. If the changes in the final demand are expected to occur in the short run, let's say one year, changes in matrix A may be considered a minor point. On the other hand, if the impact analysis refers to long run forecasting, matrix A may become seriously "outdated". Furthermore, an outdated matrix A cannot be compatible with a positive output vector.

An economic example may help understand why technical coefficients need to be updated. Let us consider the case of an increase in oil imports due to an energy policy addressed to increase the production of energy based on fossil fuel in a country with no oil fields. Oil is a pure intermediate consumption commodity; it is neither in the household basket nor an investment good. Therefore, the oil intermediate consumption flows necessarily increase, final demand does

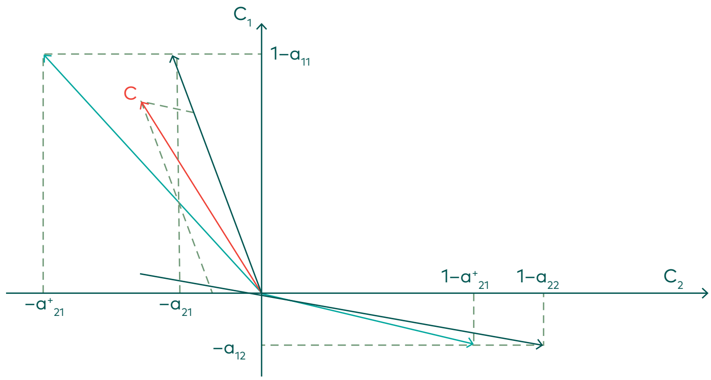

Fig. 3. Solution with positive outputs changing the second row of matrix A.

not change and, consequently, outputs remain unchanged while technical coefficients of oil increase.

A geometrical representation of this economic example shows the impact of a technical coefficient change for "restoring" a productive solution. In Fig. 3 there are two vector bases that differ from matrix A second row technical coefficients where a^*_{21} and a^*_{22} are greater than, respectively, a_{21} and a_{22}; thereafter, the representation of vector f is still a linear combination of matrix A column vectors with positive scalars: a solution with positive outputs.

However, matrix A_0 is the industry technology obtained from an I-O table and is necessarily assumed to be measured in real terms, and each column sum of matrix A_0 is less than one. Substituting elements a_{21} and a_{22} with a^*_{21} and a^*_{22}, column sums of matrix A_1 turn out to be greater than those of A_0. It is known that the A_1 output multiplier is greater than that of A_0. Figure 3 shows the geometrical impact on matrix multiplier of moving from the A_0 vector basis to the A_1 vector basis. The angle between the vector basis in A_1 is wider than that in A_0, so that the "scalars" – the outputs – of vector f represented in term of A_1 vector basis are greater than those in A_0. On the other hand, in a long run forecast, the above mentioned annual updating of matrix A increases progressively the column sum of matrix A, which, sooner or later, leads to a non-productive economy. However, a way to prevent such an outcome is scaling the column sum of matrix A_1 with respect to that of A_0.

4. Modeling technical coefficient changes in an INFORUM type model

Outputs, investments, imports and exports are the main endogenous variables of the real side of the INFORUM model that is a member of a system of country models linked through a Bilateral Trade Model (BTM); this (truly bilateral) model generates country exports based on country imports so that exports turn out to be endogenous in the system of models. In particular, the generation of country exports takes into account endogenous variables from the price side of the models, specifically the prices themselves. Household consumption depends on prices and disposable income; disposable income comes from the primary and the secondary income distribution and is computed in the process of aggregation (bottom-up) of sectoral variables to compute endogenous macrovariables.

However, the solution of the model implies the solution of the standard Leontief equation that is one cornerstone of the model real side. To tackle the problem of displaying a functioning economy, the relations described above between output and imports have to be properly modeled.

Remarks

INFORUM country models are designed to run together with the Bilateral Trade Model (BTM). BTM (Ma, 1996; Bardazzi & Ghezzi, 2018) is a model designed to take the sectoral imports from each country model and allocate them to the exporting countries within the system; this allocation is done by means of import share matrices computed from trade flow matrices built for several commodities (the number of commodities in BTM is larger than a country sectoral imports detail). For each commodity, the sum of imports demanded by each country in the system to a given country turns out to be equal to its exports; then BTM ensures that for each commodity in the world market total imports are equal to total exports.

The key task of the model is to calculate the movement in import-share matrices. Each import share in each import-share matrix is assumed to be influenced by price and technology competitiveness. Price competitiveness is measured with domestic price versus world price, and technology competitiveness is related to industry capital stock with special attention to the weight of new investment.

First, imports by product, prices by product, and capital investment by industry are taken from the national models. Then the model allocates the imports of each country among supplying countries by means of the import share matrices mentioned above. BTM takes prices, imports and investments from country models and gets back import prices and exports to them.

To take advantage of being a part of the INFORUM system of models, each country model needs to properly supply the BTM with its domestic prices, imports and investments, as well as to receive import prices and exports. In this respect, modeling imports described in the present paper is not an end in itself, but a cornerstone of the INFORUM system of models.

REFERENCES

Almon, C. (1991). The INFORUM approach to interindustry modeling, *Economic Systems Research*, *3*, 1–7.

Almon, C. (2017). *The craft of economic modeling. Part III, Multisectoral models*, http://www.inforum.umd.edu/papers/publishedwork/books/craft_all_091017.pdf.

Almon, C. (2016). Inforum models: origin, evolution and byways avoided, *Studies in Russian Economic Development, 27*, 119–126.

Bardazzi, R., & Ghezzi, L. (2018). Trade, competitiveness and investment: an empirical assessment, *Economic Systems Research*, DOI: 10.1080/09535314.2018.1446913

IIOA Newsletter, (2015). *Fellows corner: Erik Dietzenbacher, 29 (Feb)*, 8–11. https://www.iioa.org/news/newsletters.html

Hawkins, D., & Simon, H.A. (1949). Some Conditions of Macroeconomic Stability, *Econometrica, 17*, 245–248.

Hewings, G. J. D., Sonis, M., & Jensen, R. C. (1988). Fields of influence of technological change in input-output models, *Regional Sciences Association, 64*, 25–36.

Hewings, G. J. D., & Sonis, M. (1992). Coefficient change in input-output models: theory and applications, *Economic Systems Research*, *4*, 143–160.

Jalili, A. R. (1999). Comparison of two methods of identifying input-output coefficients for exogenous estimation, *Economic Systems Research*, *12*, 114–129.

Ma Q. (1996). A Multisectoral Bilateral World Trade Model, Ph.D. Thesis, University of Maryland.

Meade, D. (1995). *Interdyme report #1: import group fixes*, http://www.inforum.umd.edu/papers/wp/wp/1995/impfix.pdf

Miller, R. E., & Blair, P. D. (2009). *Input-Output Analysis. Foundations and Extensions.* Cambridge: Cambridge University Press.

Nishimura, K. (2002). Introduction of new technology with sound transition to the input-output structure, *Economic Systems Research*, *14*, 89–94.

Sonis, M., Hewings, G. J. D., & Guo, J. (1996). Source of structural change in input-output systems: a field of influence approach, *Economic Systems Research, 8*, 15–32.