RIGA TECHNICAL UNIVERSITY
Faculty of Computer Science and Information Technology
Institute of Information Technology

Artūrs Stepčenko
Doctoral Student of the Study Programme “Information Technology”

FORECASTING SYSTEM DEVELOPMENT FOR NONLINEAR AND NONSTATIONARY TIME SERIES OF NORMALIZED DIFFERENCE VEGETATION INDEX

Summary of the Doctoral Thesis

Scientific supervisors
Professor Dr. habil. sc. comp.
ARKĀDIJS BORISOVS,
Professor Dr. sc. ing.
LUDMILA ALEKSEJEVA

Research consultant
Lead Researcher Dr. sc. ing.
JURIJS ČIŽOVS

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DOCTORAL THESIS PROPOSED TO RIGA TECHNICAL UNIVERSITY FOR THE PROMOTION TO THE SCIENTIFIC DEGREE OF DOCTOR OF ENGINEERING SCIENCES

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OFFICIAL REVIEWERS

Professor Dr. habil. sc. ing. Zigurds Markovičs
Riga Technical University

Professor Dr. sc. ing. Egils Stalidzāns
Latvia University of Life Sciences and Technologies, Latvia

Professor Dr. tech. sc. Vadim Romanuke
Polish Naval Academy, Poland

DECLARATION OF ACADEMIC INTEGRITY

I hereby declare that the Doctoral Thesis submitted for the review to Riga Technical University for the promotion to the scientific degree of Doctor of Engineering Sciences is my own. I confirm that this Doctoral Thesis had not been submitted to any other university for the promotion to a scientific degree.

Artūrs Stepčenko ……………………………. (signature)
Date: ………………………

The Doctoral Thesis has been written in Latvian. It consists of Introduction; 5 chapters; Conclusion; 36 figures; 21 tables; 4 appendices; the total number of pages is 171, including appendices. The Bibliography contains 121 titles.

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GENERAL DESCRIPTION OF THE THESIS

Introduction

Forecasting of vegetation is closely related to many important international problems such as global climate changes and monitoring of usage of energy, natural resource consumption management, forecasting of the prevalence of invasive plant species and protection of endangered plant species [7]. Collection of data on vegetation cover of Earth’s surface is usually done by using remote sensing. Remote sensing is remote surveillance of Earth’s surface by aircraft or satellite using different sensors [73]. Remote sensing usually provides satellite images where time series can be obtained from each smallest element (pixel) in the image [107].

Topicality

Analysis and forecasting of the life cycle of vegetation are essential in planning agricultural work as well as monitoring of agricultural crops and forecasting their productivity. In practice, vegetation indices are often used that are calculated from the values of satellite image pixels like normalized difference vegetation index (NDVI). Forecasting of this index in precision agriculture allows indicating problems which are related to agricultural crop growth on time and making timely decisions about necessary measures to fix these problems.

Research Aim and Tasks

The **aim** of the Doctoral Thesis is to develop a forecasting system of normalized difference vegetation index time series based on signal decomposition and sub-signal approximation approach, specialized data preprocessing methods and machine learning methods. The following **tasks** have been set to achieve the aim of the Doctoral Thesis.

1. Comparative research of methods and systems of time series of normalized difference vegetation index aimed at discovering its advantages and potential disadvantages that determine the accuracy of NDVI time series.
2. Comparative research of signal decomposition methods to be used for frequency analysis with the aim to identify methods that provide high prediction accuracy and identify possible gaps in forecasting tasks of different time series.
3. Development of a sub-signal approximation approach to forecasting time series of normalized difference vegetation index.
4. Development of a forecasting system that uses signal decomposition-based approach, data preprocessing and machine learning methods.
5. Evaluation of the developed forecasting system and comparison of its accuracy with other forecasting methods.

Research Object and Subject

The **object** of the Doctoral Thesis is the process of forecasting nonlinear and nonstationary time series of normalized difference vegetation index. The **subject** of the Thesis is methods of data preprocessing, signal decomposition and forecasting, which are suitable for forecasting time series of nonlinear and nonstationary normalized difference vegetation index.

Research Hypotheses

Two hypotheses have been put forward for verification during the development of the forecasting system and sub-signal approximation approach.

1. The forecasting accuracy of normalized difference vegetation index increases when the approximation approach based on signal decomposition method is applied.
2. The forecasting model trained on normalized difference vegetation index and obtained preprocessed data parameters can be used for preprocessing and forecasting of other NDVI time series obtained from other locations with similar accuracy.

Methods of the Research

The following methods are used for theoretical development of the Thesis: machine learning, linear algebra, digital signal processing, mathematical statistics, and probability theory. The MATLAB application, which is also a numerical analysis environment and high-level programming language, is used for the practical realization of the forecasting system.

Scientific Novelty and Value of the Thesis

The Doctoral Thesis presents the developed forecasting system which allows performing short-term forecasting of NDVI time series. Several new approaches have been designed in the system development process as well as a set of methods and approaches that are necessary for the system implementation and evaluation.

1. The approximation approach for a sub-signal obtained from a modified variational mode decomposition method is developed, which allows calculating approximate sub-signal values during every time step where historical observations for the appropriate time series of normalized difference vegetation index are available.
2. A set of methods and approaches is developed that provides high accuracy for short-term forecasting of time series of normalized difference vegetation index.
3. A transferring approach for data preprocessing parameters and forecasting model is developed that provides forecasting of other normalized vegetation index time series without new preprocessing and training if the Euclidean distance between the time series used in the training and the time series using the data processing parameters and the forecasting model is small enough.

The Practical Significance of the Thesis

A forecasting system is developed that allows performing short-term forecasting with good accuracy of time series of normalized difference vegetation index using a sub-signal approximation approach obtained from modified variational mode decomposition method. The system has two applications for precision agriculture.

1. It is necessary to be able to identify on time the number of nutrients needed to ensure that no additional costs are incurred.
2. It provides the opportunity to use forecasts of normalized difference vegetation index for crop yield models and to calculate expected income on time.

Approbation

The Thesis studies and results have been presented in nine international scientific conferences.

1. International Conference on Aerospace Engineering, Applied Sciences, Information Technology, Electrical & Mechanical Engineering, Amsterdam, Netherlands, April 27–28, 2019.
2. 2nd International Conference on Research in Engineering and Fundamental Applied Sciences, Barcelona, Spain, April 20–21, 2019.
3. RTU 58th International Scientific Conference, Riga, Latvia, October 12–15, 2017.
4. 11th International Scientific and Practical Conference "Environment. Technology. Resources", Rezekne, Latvia, June 15–17, 2017.
5. RTU 57th International Scientific Conference, Riga, Latvia, October 14–18, 2016.
6. 5th International Virtual Scientific Conference on Informatics and Management Sciences, Zilina, Slovakia, March 21–25, 2016.
7. 3rd Virtual Multidisciplinary Conference QUAESTI Zilina, Slovakia, December 7–11, 2015.
8. RTU 56th International Scientific Conference, Riga, Latvia, October 14-16, 2015.
9. 10th International Scientific and Practical Conference "Environment. Technology. Resources", Rezekne, Latvia, June 18–20, 2015.

The Thesis studies and results are reflected in eight publications in international scientific journals.

1. Stepchenko, A. Land Cover Classification Based on MODIS Imagery Data Using Artificial Neural Networks. In: *Proceedings of the 11th International Scientific and Practical Conference "Environment. Technology. Resources", June 15–17, 2017, Rezekne, Latvia*. Rezekne: Rezekne Academy of Technologies, 2017, pp. 159–164. Indexed in: **Scopus**.
2. Stepchenko, A., Chizhov, J., Aleksejeva, L., Tolujew, J. Nonlinear, Non-stationary and Seasonal Time Series Forecasting Using Different Methods Coupled with Data

Preprocessing. *Procedia Computer Science*. 2016, vol. 104, pp. 578–585. Indexed in: **Scopus** and **Web of Science**. Cited: 2.
3. Stepchenko, A., Chizhov, J. Markov Chain Modelling for Short-Term NDVI Time Series Forecasting. *Information Technology and Management Science*. 2016, vol. 19, pp. 39–44. Indexed in: EBSCO, CSA/ProQuest and VINITI.
4. Stepchenko, A. NDVI Index Forecasting using a Layer Recurrent Neural Network Coupled with Stepwise Regression and the PCA. In: *Proceedings of the 5th Virtual International Conference of Informatics and Management Sciences, March 21–25, 2016, Zilina, Slovakia*. Zilina: EDIS-Publishing Institution of the University of Zilina, 2016, pp. 130–135. Indexed in: Google Scholar and Index Copernicus. Cited: 2.
5. Stepchenko, A. Normalized Difference Vegetation Index Forecasting using a Regularized Layer Recurrent Neural Network. In: *Proceedings of the 3rd Virtual Multidisciplinary Conference QUAESTI, December 7–11, 2015, Zilina, Slovakia*. Zilina: EDIS-Publishing Institution of the University of Zilina, 2015, pp. 261–266. Indexed in: Google Scholar.
6. Stepchenko, A., Chizhov, J. Applying Markov Chains for NDVI Time Series Forecasting of Latvian Regions. *Information Technology and Management Science*. 2015, vol. 18, pp. 57–61. Indexed in: EBSCO, CSA/ProQuest and VINITI. Cited: 2.
7. Stepchenko, A., Chizhov, J. NDVI Short-Term Forecasting Using Recurrent Neural Networks. In: *Proceedings of the 10th International Scientific and Practical Conference "Environment. Technology. Resources", June 18–20, 2015, Rezekne, Latvia*. Rezekne: Rezeknes Augstskola, 2015. Indexed in: **Scopus**. Cited: 2.
8. Stepchenko, A., Borisov, A. Methods of Forecasting Based on Artificial Neural Networks. *Information Technology and Management Science*. 2014, vol. 17, pp. 25–31. Indexed in: EBSCO, CSA/ProQuest and VINITI.

The results of the Doctoral Thesis have been developed in relation to the project "Estimation of forest inventory parameters for afforested agricultural lands and non-inventoried forest lands using remote sensing data" implemented by "Forest Sector Competence Centre of Latvia" Ltd., Ventspils University of Applied Sciences, Latvian State Forest Research Institute "Silava" and "Microcode" Ltd. (14.04.2014–30.09.2015).

This Doctoral Thesis has been developed with the support from Ventspils City Council in accordance with the regulation of granting a scholarship "Support for Ph.D. candidates at Ventspils University of Applied Sciences".

Structure and Content of the Thesis

The Thesis contains an introduction, five chapters, conclusions, bibliography, and appendices. The Thesis is written in Latvian.

The introduction describes the topicality of the chosen topic and the aim and tasks of the research, puts forward hypotheses, lists the scientific methods used in the development of the Doctoral Thesis, demonstrates the scientific novelty of the research and the practical value of the results obtained, as well as provides the characterization of the work.

Chapter 1 describes the theoretical basis of a normalized difference vegetation index including the basics of remote sensing and satellite image preprocessing, gives the current situation analysis dedicated to the normalized vegetation index forecasting, as well as analyses to the use of signal decomposition methods in forecasting tasks of time series.

Chapter 2 presents data preprocessing methods and time series forecasting methods used in the development of normalized difference vegetation index forecasting system NVDI FS.

Chapter 3 experimentally tests the applicability of the original variational mode decomposition method in the forecasting task of time series of normalized difference vegetation index. Based on the analysis of experimental results, a modified variational mode decomposition method is developed, which is experimentally tested. The approximation approach is developed for the sub-signal that is obtained from the modified variational mode decomposition method.

Chapter 4 presents forecasting system NDVI FS. The system architecture has been represented which consists of the user interface, data preprocessing module, and machine learning module and data store, as well as the flow of data store operations and computing flow in this system. The computing flow of each block in modules is shown, and the description of each block is given.

Chapter 5 describes the experiments with the developed system and the results obtained. The system with the use of approximation approach and without the use of this approach is evaluated in forecasting of time series of normalized difference vegetation index. The accuracy of the forecasting system is compared with the accuracy achieved using classical forecasting methods. Experiments on the transfer of preprocessed parameters and forecasting models are performed.

The last chapter contains the results and conclusions of the work based on the performed experiments and development and application of the proposed system NDVI FS.

1. TIME SERIES OF NORMALIZED DIFFERENCE VEGETATION INDEX AND ITS FEATURES

The chapter provides an insight into the theoretical basis in normalized difference vegetation index, abbreviated NDVI, and forecasting of time series of this index. The formal statement of the forecasting task is defined. Problems are identified that are related to forecasting of time series of normalized difference vegetation index and application of decomposition methods in forecasting tasks of different time series. A scheme with a time series decomposition, phase space reconstruction, feature selection, feature extraction, and forecasting method is provided.

1.1. Analysis of NDVI Time Series and Forecasting Research

Normalized difference vegetation index (NDVI) is a numerical indicator of photosynthetic active biomass [108], calculated by Equation (1.1):

$$NDVI = \frac{NIR - RED}{NIR + RED}, \tag{1.1}$$

where NIR – reflectance in the near infrared band;
RED – reflectance in the red band.

The research area used in the Thesis is Ventspils municipality of the Republic of Latvia. Agricultural land in Ventspils municipality covers an area of 511 000 ha or 20.9 % of the area of the municipality, while forest land occupies 64 % of the total area of the municipality [104]. The data set of the Doctoral Thesis consists of 814 smoothed NDVI images obtained from Moderate Resolution Imaging Spectroradiometer (MODIS) Terra satellite images with 250 m spatial resolution, the temporal resolution of seven days and 16-bit radiometric resolution. These images cover the territory of Ventspils municipality.

NDVI images were downloaded from data service platform for MODIS vegetation indices time series processing at Vienna University of Natural Resources and Life Sciences, which also performs preparing of MODIS Terra NDVI images that includes smoothing pixel values and filling missing values due to cloud cover or bad weather conditions [106].

In the Thesis, nonlinear and nonstationary univariate NDVI time series with elements of additive noise and seasonal components are used [37]. Each element of time series is obtained with a period of one week. The number of observations for each NDVI time series is 814 and an interval of values [–1; 1].

The Doctoral Thesis solves a forecasting task where a short-term forecast of the NDVI time series has been calculated for one week, using data preprocessing, signal decomposition, linear algebra, and machine learning methods. For estimating accuracy, the following loss functions have been used: root mean square error RMSE, directional symmetry DS and adjusted coefficient of determination R^2_{adj}.

The comparison of used methods [5], [13], [24], [38], [40], [60], [61] of forecasting NDVI time series according to their suitability for forecasting nonlinear, nonstationary and noisy time series is given in Table 1.1.

Table 1.1

Comparison of Methods Used in NDVI Time Series Forecasting

Features \ Methods and algorithms	ARIMA model	Multiple linear regression	Feedforward neural network
Suitability for modelling nonlinear processes	–	–	+
Suitability for modelling nonstationary processes	+	–	+
Robustness against noise	–	–	+

The studies [5], [13], [24], [38], [40], [60], [61] do not always achieve high forecasting accuracy, but it depends on various factors, e.g. the use of additional data. Several studies used additional input data such as data of temperature or rainfall data. Obtaining additional data may be difficult, because:

- these data may not always be available for the specific pixel; more often these data are available for large areas (average values in the municipality, district, etc.);
- additional data may not be available for free.

In all examined studies little attention is paid to the data preprocessing phase. Preprocessing methods, which help to prepare input data set for time series forecasting tasks so that forecasting accuracy can be increased, such as feature selection and feature extraction, are not used.

Box-Jenkins ARIMA model as well as linear regression analysis used to forecast NDVI time series in several studies are linear forecasting methods and are not robust to noise, while NDVI time series are nonlinear and noisy [37]. Moreover, the identification of a suitable ARIMA model is a time-consuming and resource-consuming procedure.

1.2. Analysis of Time Series Forecasting Studies With Signal Decomposition

Analysis of many time series forecasting studies, where decomposition methods are used, is shown in Table 1.2. The columns contain three decomposition methods: wavelet decomposition [33], [84], empirical mode decomposition [33], [34], [51], [64], [95] and variational mode decomposition [34], [51], [64], [84], [95], but the rows contain the relevant forecasting methods used in different studies. Besides, two of those time series decomposition methods are used in each study, providing an option of comparing the forecasting accuracy.

First, the forecasting method is used together with the first decomposition method, and then with the other decomposition method. For the decomposition method, which is used to achieve the highest forecasting accuracy, in the corresponding field "+" is applied , but for other decomposition method used in the research "–" is applied , as shown in Table 1.2. The third decomposition method is not used in these studies.

Table 1.2

Comparison of Decomposition Methods Used in Time Series Forecasting

Decomposition method / Forecasting method	Wavelet decomposition	Empirical mode decomposition	Variational mode decomposition
Feedforward neural network [33]	–	+	
Least squares support vector machine, extreme learning machine [84]	–		+
Weighted regularized extreme learning machine [34]		–	+
Spiking neural network [95]		–	+
Generalized regression neural network [51]		–	+
Multi-kernel regularized pseudo inverse neural network [64]		–	+

Therefore, the field where the third decomposition method and corresponding forecasting method intersect is coloured grey. It seems that most often the highest forecasting accuracy for time series is achieved using VMD method.

Some of the authors in their studies [33], [95] use decomposition method to entire input time series in the forecasting task and only then a training and test data set are created and training of forecasting model is performed. However, the results that usually show a high forecasting accuracy do not show the real situation for use of this approach, because only historical observations are forecasted. In this case, a test data set is made from a part of historical observations to which decomposition was previously applied. But in real time, there are new values of time series, and their forecasting in these scientific articles [33], [95] is not discussed. Therefore, it is impossible to determine whether these trained forecasting models can forecast with equal accuracy new values of time series that will come to the time series behind the last element of historical observations.

Other authors divide input time series into smaller fragments according to the length of the window and apply the decomposition to each fragment separately [30], [34]. This means that decomposition is applied to each data entry from the reconstructed phase space. In this way, the signal decomposition methods used for frequency analysis can also be used to forecast new values of time series (to forecast the values that come after historical data).

However, the main drawback of all studies of signal decomposition methods is that they are not intended for decomposition of time series within the framework of forecasting tasks. The first problem, if the VMD method in the input receives all historical time series observations, is that calculating sub-signal values at some point of time, decomposition methods use all values of input time series, including future observation values by time. Accordingly, if the decomposition method uses all historical observations of input time series, then each time, when a new observation arrives, the values of the sub-signals must be recalculated by using decomposition method, which is applied on all historical observations that also include latest observation, and all previously calculated values of sub-signals change in all time steps. The second problem is that studies [33], [95] that use decomposition method for entire input time series do not examine if it is possible to forecast a new value of time series with an equivalent accuracy compared to what was obtained from historical NDVI observation forecasting.

1.3. The Formal Statement of Task

To determine the value of process $y = \{y(t), t = 1, 2, \ldots, N\}$ at time $t = N + 1$, where N is the number of historical observations, it is necessary to establish a functional relationship between $y = \{y(t)\}$ values of historical observation and future values. Besides, it must be noted that for this relationship the influence of historical values of the derived time series $u_1 = \{u_1(t)\}$, $u_2 = \{u_2(t)\}$, …, $u_K = \{u_K(t)\}$ on the input time series should be taken into account, as described in Equation (1.2):

$$\begin{aligned} y(t+1) = {} & f(y(t), y(t-1), \ldots, y(t-m+1), u_1(t), u_1(t-1), \ldots, \\ & u_1(t-m_1+1), \ldots, u_K(t), u_K(t-1), \ldots, u_K(t-m_K+1)) + \varepsilon(t), \end{aligned} \tag{1.2}$$

where y – input time series;

K – number of derived time series;

u_1 – the first derived time series;

u_K – K-th derived time series;

m – length of the window, used for the input time series;

m_1 – length of the window, used for the first derived time series;

m_K – length of the window, used for the K-th derived time series;

t – time;

ε – white noise.

Functional relationship (1.2) is a forecasting model. It is necessary to obtain a forecasting model for which root mean square error (RMSE) value between true and forecasted values of time series $y = \{y(t)\}$ is minimal.

1.4. Conceptual Description of a Forecasting System

Based on the formal statement of the task, the structure of analysed data, as well as literature review and analysis, and a conceptual scheme of forecasting system is proposed, which includes methods and approaches of:

- signal processing (time series decomposition);
- data preprocessing (phase space reconstruction, creating data sets, feature selection and extraction);
- and machine learning.

Based on the literature review and analysis, the **requirements** for forecasting system are developed.

1. The forecasting system at the input receives only historical data of NDVI time series.
2. An approach should be developed that allows to approximate sub-signal values in each time step ensuring the following:
   - the values of sub-signal are available for all time steps where historical observations of NDVI time series are available;
   - in each time step, the values of sub-signals are obtained without using the variational mode decomposition method;
   - when calculating the values of sub-signals in each new time step, the values of sub-signals in previous time steps do not change.
3. To improve the forecasting accuracy of the forecasting model, it must be combined:
   - with the data preprocessing methods (feature selection and feature extraction methods);
   - with the proposed sub-signal approximation approach.

In developing a forecasting system based on the requirements there are several **limitations**:

1. The NDVI time series, each obtained from one pixel, are analysed and processed.
2. Only short-term forecasting is performed – one-step ahead (for MODIS Terra NDVI images used in Thesis, it is seven days).
3. The additional data are not used (such as data of air temperature, data of rainfall, other vegetation indices, and data of land surface categories).

It is proposed to use the decomposition for data preprocessing to obtain a certain number of sub-signals. Phase space reconstruction can be used to obtain an initial data set from the scalar time series, which is divided into three parts. Feature selection can be used to determine the informativeness of the features and to exclude non-informative features from all sets that may reduce the accuracy of the forecasting. It is proposed to use the feature extraction in the reduced feature sets to transform these sets according to certain criteria. Then these data sets are passed into the input to forecasting method, which by learning on these data sets finds the functional relationship (see Equation (1.2)) thus obtaining forecasting model that can be used to forecast new values of the NDVI time series.

2. OVERVIEW OF METHODS USED IN THE DEVELOPMENT OF THE FORECASTING SYSTEM

The chapter gives an insight into the methods of data preprocessing and forecasting used in the development of the forecasting system. One of the modern trends in the development of forecasting systems is a combination of different methods and approaches; usually, different data preprocessing methods are combined with one or more time series forecasting methods [35], [80], [81], [85], [110]. That allows to compensate weaknesses of separate methods or approaches and increase forecasting accuracy.

According to literature analysis, it is proposed to use a variational mode decomposition (VMD) method [118] as time series decomposition method. To divide the scalar NDVI time series and obtained sub-signals into fragments according to window length and obtain the initial data set, the phase space reconstruction can be used. For phase space reconstruction, it is proposed to use the popular time delay method [48]. The initial data set is divided into training, validation and test data sets, where each of these sets consists of attribute (or feature) set and forecasting parameter.

Each attribute in the data sets is a lagged value of NDVI time series and sub-signals. To reduce training, validation and test attribute sets by selecting only informative attributes, feature selection methods [26], [77] can be used. The popular feature selection method, which can be used in time series forecasting tasks, is a stepwise regression analysis [99], which is proposed to use in the developed system. It is important that the features do not linearly correlate with each other and do not complicate the forecasting model training. Principal component analysis (PCA) is a method that is often used for obtaining a linear uncorrelated feature set [79]. The principal component method is proposed to use in the developed system as a feature extraction method. As a forecasting method in the system, it is proposed to use the layer recurrent neural network (LRNN), due to its ability to forecast nonlinear, nonstationary and noisy time series with good accuracy [71].

2.1. Variational Mode Decomposition

In variational mode decomposition model, it is assumed that the real signal f consists of several sub-signals u_k, where $k = 1, \ldots, K$ is sub-signal number and K is the number of sub-signals. A sub-signal or intrinsic mode function is an amplitude and frequency modulated (AM-FM) signal [118] and can be described with Expression (2.1):

$$u_k(t) = A_k(t)\cos\left(\phi_k(t)\right), \tag{2.1}$$

where $A_k(t)$ – k-th sub-signal amplitude;

$\phi_k(t)$ – k-th sub-signal phase;

t – time.

For any sub-signal, frequencies ω change in a small range. Each of K sub-signals must be centred around the centre frequency ω_k, which is calculated during decomposition. To calculate sub-signals, it is necessary to minimize the sum of K sub-signal frequency bandwidth, provided that the sum of all K sub-signals is equal to the original signal [118]. The unknowns are K sub-signal centre frequencies and K intrinsic mode functions that are centred on these frequencies. Since some of the unknowns are functions, variational calculations are used in VMD method.

To obtain the correct results in both signal endpoints, using the algorithm of variational mode decomposition, the original signal is expanded, using "mirroring" [118]. The expanded signal is twice as long as the original signal, and its length is T.

2.2. Phase Space Reconstruction With Time Delay Method

The phase space is an abstract space, which describes a set of possible states of the system, where each possible state corresponds to the point in phase space [11]. For a chaotic time series, a phase space reconstruction with a time delay method according to Taken's theorem [48] is used. Using Taken's theorem, a chaotic and dynamic system can be described with a set of delayed vectors. Using the phase space reconstruction method with a time delay, it is necessary to find two parameters – the suitable values of the embedding dimension m and time delay τ. The accuracy of the time series prediction depends on these parameters. In the formal statement, when searching for a functional relationship (1.2), as the window length m, before the data preprocessing is finished, the reconstructed phase space dimension is used.

2.3. Stepwise Regression Analysis

Stepwise regression analysis is a systematic method for sequent feature selection, where features (or attributes) are added to the multi-linear model and removed from it as a result of automatic procedures, based on the statistical significance of the attribute in the regression analysis [99].

The original stepwise regression model does not include any attribute. Then in each step F-statistic p-value is calculated to test models with certain attributes [111]. The attribute with the smallest p-value is added to the model, if it does not exceed the specified addition threshold and if the null hypothesis of if this attribute having a zero coefficient is rejected. Out of attributes that are already in the model, the attribute with the highest p-value is removed from the model, if p-value of attribute exceeds the specific removal threshold, and if the null hypothesis that this attribute has a zero coefficient is not rejected. However, the stepwise regression analysis is not globally optimal [16].

2.4. Principal Component Analysis

Principal component analysis (PCA) is a statistical feature extraction method, which is using the orthogonal transformation to transform a potentially correlating data (attributes $x_1, x_2, \ldots, x_m$ in this work) set to linearly uncorrelated data set $\hat{x}_1, \hat{x}_2,, \ldots, \hat{x}_p$, where m is the dimensionality of original data set but p is the number of principal components [79].

The first step in the PCA algorithm is to normalize data so that the mean value of each attribute would be zero. Then, based on the normalized data, principal components are calculated. Based on the attributes, a covariance matrix C of a sample set is calculated for which an eigenvector set M is obtained by performing the eigenvalue decomposition. Principal components $\hat{x}$ are chosen in such a way that the first principal component $\hat{x}_1$ contains the most variance of the original data, the second contains the second largest variance of the original data; besides, it linearly does not correlate with $\hat{x}_1$, and so on.

2.5. Artificial Neural Networks

Artificial neural networks (ANN) is a form of artificial intelligence, which tries to imitate the function of biological neurons that work in the human brain [85]. One of the most common artificial neural network models is a multi-layer perceptron, which includes an input layer, output layer and one or more hidden layers [85].

In the case of the prediction task addressed in this Thesis, the output layer contains one neuron that gives the forecasted value of the time series. Each neuron in every layer receives the weighted inputs from the previous layer, and these weighted inputs are summed up using the combination function; this result of summing up is fed as an argument to activation function, such as hyperbolic tangent, logistic or linear function [46], where this function value is the output of the neuron, and it is fed to the next layer [85].

The aim of the neural network training is to reduce the global error, which is calculated using a specific goal function. One of the most popular and effective non-linear optimization methods is the Levenberg-Marquardt backpropagation algorithm with a Bayesian regularization [85]. Bayesian regularization minimizes a linear combination of squared errors and weights, leading to a smaller weights for neural network model; it allows obtaining smoother forecasts and reduces overfitting capability for forecasting data [23]. As goal function, using Bayesian regularization, the regularized mean square error MSE_{reg} [23] is used.

Layer recurrent neural network (LRNN) is one of the dynamic, recurrent neural network forms, which is created by adding the feedback connections from the hidden layer to the context layer in multi-layer perceptron [18], [71]. The context layer stores the values of the hidden layer with a time delay, thus providing useful information about the previous input vector; besides, it determines the main quality – sequence memorizing [19].

3. DEVELOPING THE SUB-SIGNAL APPROXIMATION APPROACH

The chapter is devoted to the development of modification of variational mode decomposition (VMD) method and development of approximation approach to the sub-signal, obtained from modified VMD method, which allows to approximate value of sub-signal in each time step.

3.1. The Usage of Original VMD Method in NDVI Time Series Forecasting

Two experiments are performed to evaluate the applicability of the original variational mode decomposition (VMD) method in the NDVI time series forecasting task. The experiment uses 100 NDVI time series that are selected by generating random numbers corresponding to pixel rows and columns numbers in MODIS NDVI images. Each of the two experiments is repeated 100 times.

The accuracy of NDVI time series forecasting is first tested using the original VMD method to all historical observations of NDVI time series as it is done in the studies [33], [95]. In the experiment, the NDVI time series is fed into the input of the VMD method, obtaining a certain number of sub-signals. Data sets consisting of sets of attributes and a forecasting parameter are generated from the NDVI time series and all sub-signals. The data sets are preprocessed by selecting informative, linearly uncorrelated features. The preprocessed data sets are fed into an input to a layer recurrent neural network obtaining a forecasting model. The values of loss functions obtained in the experiment: root mean square error RMSE, directional symmetry DS, and adjusted coefficient of determination are given in Table 3.1.

In the next experiment, NDVI time series forecasting accuracy is tested using the original VMD method for each NDVI time series fragment with a certain window length as it is done in studies [30], [95]. The NDVI time series is divided into fragments with a certain window length, resulting in data sets consisting of attribute sets and a forecasting parameter. Attribute sets are preprocessed and the most informative, uncorrelated attribute sets are obtained. The preprocessed data sets are fed into an input to a layer recurrent neural network obtaining a forecasting model. The experiment is repeated for all 100 time series. The mean values of loss functions for all 100 time series obtained in this experiment are given in Table 3.1.

Table 3.1

The Mean Values of Loss Functions in Experiments With Original VMD Method

	Experiment No. 1			**Experiment No. 2**		
Data set	***RMSE***	***DS***	R^2	***RMSE***	***DS***	R^2
Training data set	0.000027	99.94 %	1.00	0.0013	97.53 %	0.99
Validation data set	0.000040	99.99 %	1.00	0.0016	97.45 %	0.99
Test data set	0.046000	85.59 %	0.96	0.0011	97.69 %	0.99

In the first experiment obtained values of forecast loss functions, which are shown in Table 3.1.1, present a high forecast accuracy on training and validation data set. However, obviously mean value of all three loss functions on test data set present unsatisfactory forecast accuracy, and this forecasting model is not practically suitable for NDVI time series forecasting. This is because the variational mode decomposition, calculating sub-signal values in every time step *t*, uses all input signal values. In other words, each sub-signal value in every time step *t* contains information about time series values in previous and future time steps. To make the signal data available before the first and after the last NDVI time series observation, the authors of original VMD method use signal "mirroring" extending time series with values from historical observations [118]. In the second experiment the mean values of loss functions, as can be seen in Table 3.1, show stable forecast accuracy on all three data sets.

3.2. Modification of Variational Mode Decomposition Method

The original VMD method uses a "mirroring" algorithm [118]. If the signal input length is *N*, then the signal midpoint is calculated: an integer from $N / 2$. From the input signal all elements from the first to $N / 2$ are taken and in reverse order are placed in front of the input signal. Then all elements from $N / (2 + 1)$ to *N* are taken and in reverse mode are placed at the end of the input signal. Thereby variational mode decomposition method works with a signal whose length is *T*. However, from the perspective of time series forecasting, this "mirroring" approach causes certain problems – extended parts of time series give investment in sub-signal computing process, and hence affect the sub-signal values in time steps from $t = 1$ to $t = N$. It leads to a problem that they can forecast with high accuracy only historical observations of time series when developing forecasting models.

To solve the problem caused by "mirroring", one of the options is not to use the extension of time series but twice cut the number of original signal observations received by the VMD method at the input. If all the original time series (that is used as extended time series) is with the number of observations *T*, then the middle part of time series contains N observations, and the obtained sub-signal is corresponding to those N observations. Now, the real observations of the original time series will be replacing the extensions, and sub-signal values will contain information about the true value of input time series in the next time steps. The division of time series in the "extended" and "original" parts is shown in Fig. 3.1. Accordingly, the extension at the beginning of the time series will include observations from the first observation to the point obtained by the formula (3.1):

$$A = \frac{T}{4} + 1. \tag{3.1}$$

The extension at the end of the time series corresponds to the observations of the original time series from point *C* to the last observation. The endpoint of the new "original" NDVI time series *C* is obtained by calculating the formula (3.2):

$$C = 3\frac{T}{4} + 1. \tag{3.2}$$

The calculated proportionality of “original” (fragment AC) and “extended” time series correspond to the proportionality of original and extended time series obtained using the original VMD method. In other words, the length of original times series is half of the length of extended time series; besides, the extended values are located at both endpoints of the original time series.

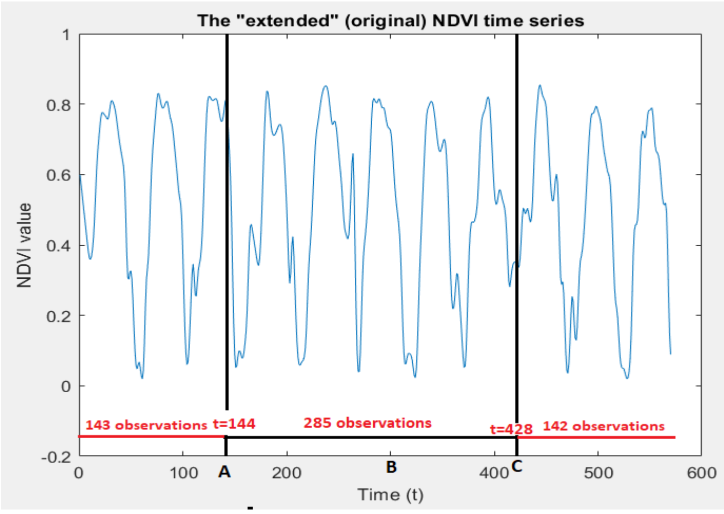

Fig. 3.1. Division of NDVI time series to the “extended” and “original” part.

However, this modification is not yet applicable for forecasting new values of the NDVI time series, because the obtained sub-signal values are available only for the middle part of the input time series, as shown in Fig. 3.1. Accordingly, the time series value that can be forecasted one time-step ahead will be located behind these middle part observations – in time step $t = N + 1$, while in the NDVI forecasting task the value has to be forecasted in time step $t = T + 1$.

An experiment is being conducted to test the accuracy of forecasting using the modified VMD method for all historical observations of the NDVI time series. In the experiment 100 NDVI time series is used where each time series originally contains 814 observations. Each time series is taken, and several iterations are performed, where first the first 70 % observations are selected from time step $t = 1$ to $t = 570$, which together compose 570 observations ($T = 570$). The selected 570 observations are fed to modified VMD method input and then sub-signals are obtained in time steps from $t = 143$ to $t = 428$, and together each sub-signal contains 285 observations ($N = 285$). From NDVI time series and obtained sub-signals reconstructed phase spaces are combined into a single data set, which is divided into training and validation data sets. On these sets the selection of informativeness features is performed using stepwise regression analysis, and obtaining of linearly uncorrelated features using principal component analysis. The preprocessed training and validation data are fed to an input of the layer recurrent neural network, which is learning on data.

When the forecasting model is obtained, it is used to forecast the value of NDVI time series in time step $t = N + 1$. When the forecast is obtained, the first iteration of the experiment is completed and the next 70 % observations are taken, and the process is repeated. The observations of NDVI time series from $t = N + 1$ to $t = T$ form a test data set. Mean values of the loss functions from all 100 NDVI time series overall data sets obtained in the third experiment are shown in Table 3.2.

Table 3.2

The Obtained Mean Values of Loss Functions in the Experiment With Modified VMD Method

Data set	*RMSE*	*DS*	R^2
Training data set	0.000058	99.88 %	1.00
Validation data set	0.000076	100 %	1.00
Test data set	0.000074	100 %	0.99

RMSE values obtained depending on the balancing parameter of the data-fidelity constraint α at different sub-signal number K are shown in Fig. 3.2. Analysing Fig. 3.2, it can be concluded that, firstly, RMSE values have a tendency to grow by increasing the accuracy of the sub-signal sum values regulating parameter α. Secondly, the lowest RMSE values are achieved using only one sub-signal. While analysing Table 3.2, it can be concluded that forecasting new values of time series (one value forward in the selection of observations) now shows high accuracy.

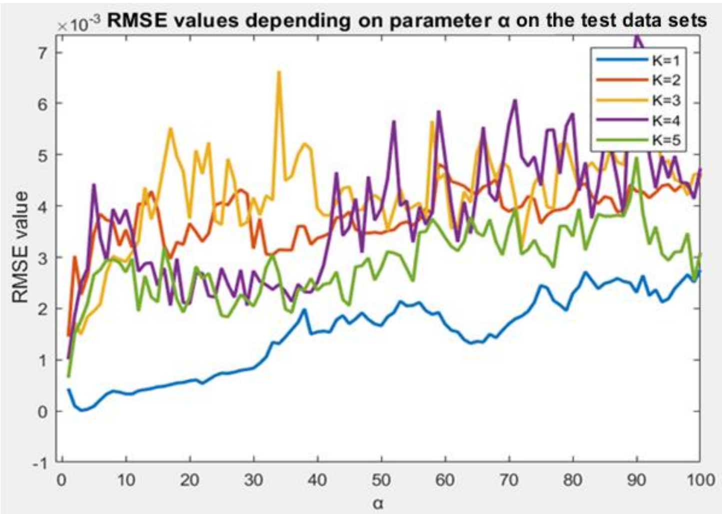

Fig. 3.2. RMSE values depending on parameter α.

However, due to the reduction of the original time series forecasting is only possible in the middle part of the original NDVI time series. It is also necessary to perform an experiment to evaluate how the RMSE values change depending on how many digits of decimal places the sub-signal is using. The description of experiment matches the description of the third experiment, where the modified VMD method for all historical observations of the NDVI time series was used, except that a certain number of the last digits is successively discarded for sub-signal. In the fourth experiment, 16 mean RMSE values (see Fig. 3.3) are obtained using the sub-signal with the different number of decimal places.

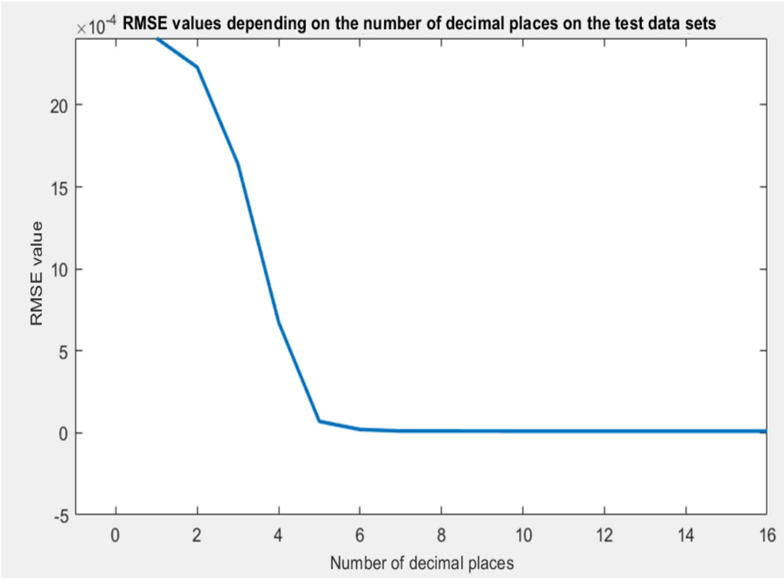

Fig. 3.3. RMSE values depending on decimal places of sub-signal.

Analysing Fig. 3.3 it can be concluded that RMSE value falls sharply from one to five decimal places. The changes of RMSE values from six to sixteen digits of the decimal places are small. The mean RMSE values from one to three digits of the decimal places vary between 0.00164 and 0.0024. The mean RMSE value with four digits of the decimal places is 0.00067. The mean RMSE values starting from five to sixteen digits of the decimal places vary between 0.00049 and 0.000074.

In the second experiment the original VMD method is used for each fragment of NDVI time series, and on the test data set obtained mean RMSE value is 0.0011. Therefore, in order to obtain highest forecasting accuracy of NDVI time series, for approximated sub-signal at least four correct digits of decimal places should be obtained compared with an original sub-signal given by the modified VMD method.

3.3. Approximated Calculation of Sub-Signal Values

In order to perform NDVI time series prediction using sub-signal values as attributes, and to look for a functional relation calculated by Equation (1.2), it is necessary to develop an approach that allows approximate calculation of sub-signal values at any time step *t*. For this purpose, the author proposes to use the solutions of linear equation systems (LES). The system of linear equations [55] is described by Equation (3.3):

$$Aw = b, \tag{3.3}$$

where A – the coefficient matrix ($m \times n$);

b – the vector of constant terms ($m \times 1$);

w – the vector of unknowns ($n \times 1$);

m – the number of equations;

n – the number of unknowns.

The kernel functions used in machine learning can be used to obtain a quadratic coefficient matrix. The kernel function provides the transformation of data from an input space to a multi-dimensional attribute or feature space [74]. If the data set used in approximation algorithm consists of sub-signal u, whose number of observations is N, then by submitting this sub-signal as a vector to the linear kernel function, $N \times N$ matrix is obtained by Equation (3.4):

$$K(u,u) = \begin{bmatrix} u(1)u(1) & u(1)u(2) & \dots & u(1)u(N) \\ u(2)u(1) & u(2)u(2) & \dots & u(2)u(N) \\ \dots & \dots & & \dots \\ u(N)u(1) & u(N)u(2) & \dots & u(N)u(N) \end{bmatrix}. \tag{3.4}$$

Solving the homogenous linear equations system with the singular value decomposition (SVD) method where matrix (3.4) is used as the coefficient matrix, null space x is obtained where the set of linearly independent solutions x describes Matrix (3.5):

$$x = \begin{bmatrix} x_1(1) & x_1(2) & x_1(3) & \dots & x_1(N) \\ x_2(1) & x_2(2) & x_2(3) & \dots & x_2(N) \\ \dots & \dots & \dots & & \dots \\ x_{N-1}(1) & x_{N-1}(2) & x_{N-1}(3) & \dots & x_{N-1}(N) \end{bmatrix}. \tag{3.5}$$

Matrix (3.5) consists of $N-1$ linearly independent solutions and N variables. Now it is possible to overwrite the linear kernel function (3.4) as a linear kernel function, which at the input receives a data set of two vectors: NDVI time series y with N observations and sub-signal u, as shown in Matrix (3.6):

$$K([yu],[yu]) = \begin{bmatrix} y(1)y(1)+u(1)u(1) & \dots & y(1)y(N)+u(1)u(N) \\ y(2)y(1)+u(2)u(1) & \dots & y(2)y(N)+u(2)u(N) \\ \dots & & \dots \\ y(N)y(1)+u(N)u(1) & \dots & y(N)y(N)+u(N)u(N) \end{bmatrix}. \tag{3.6}$$

The null space (3.6) has one solution less than the number of unknowns. It is necessary to obtain one more equation – to find the particular solution w of the first non-homogenous linear equations system, where Matrix (3.7) is used as coefficient matrix A_1:

$$A_1 = \begin{bmatrix} 1 & y(1)y(1)+u(1)u(1) & ... & y(1)y(N-3)+u(1)u(N-3) & y(1) \\ 1 & y(2)y(1)+u(2)u(1) & ... & y(2)y(N-3)+u(2)u(N-3) & y(2) \\ ... & & ... & ... & ... \\ 1 & y(N)y(1)+u(N)u(1) & ... & y(N)y(N-3)+u(N)u(N-3) & y(N) \end{bmatrix}. \tag{3.7}$$

In Matrix (3.7) with size $N \times (N-1)$, the first attribute is one's vector, which allows calculating the regression constant, but starting from the second to the penultimate attribute has the corresponding attributes from linear kernel function (3.6). For depending parameter b it is proposed to use the NDVI time series y. The solution is obtained using the non-linear least squares method. Function (3.8) is used as a non-linear function f:

$$f = w(1) + \frac{\sqrt{S^2+1}-1}{2} + S + yw(N)\,, \tag{3.8}$$

where S – the weighted sum.

While the element of the weighted sum $S(t)$, where $t = 1,...,N$ is calculated by (3.9):

$$S(t) = w(2) + \sum_{i=2}^{N-2} A_1(t,i)w(j),\ j = 3,...,N-1\,. \tag{3.9}$$

By adding the obtained solution vector w to the null space solutions, the coefficient matrix A_2 is obtained as shown in Equation (3.10):

$$A_2 = \begin{bmatrix} x_1(1) & x_1(2) & ... & x_1(N-2) & x_1(N-1) & x_1(N) \\ x_2(1) & x_2(2) & ... & x_2(N-2) & x_2(N-1) & x_2(N) \\ ... & ... & ... & ... & ... & ... \\ x_{N-1}(1) & x_{N-1}(2) & ... & x_{N-1}(N-2) & x_{N-1}(N-1) & x_{N-1}(N) \\ w(3) & w(4) & ... & 0 & 0 & 0 \end{bmatrix}. \tag{3.10}$$

The vector of constant terms b_t used in the second non-homogenous LES is also composed of N elements. The elements of vector $b_t(i)$, where $i = 1, \ldots, N-1$ in the time step t are obtained using the equation system (3.11):

$$b_t(i) = \begin{bmatrix} y(t)y(1)x_1(1) & + & y(t)y(2)x_1(2) & + & ... & + & y(t)y(N)x_1(N) \\ y(t)y(1)x_2(1) & + & y(t)y(2)x_2(2) & + & ... & + & y(t)y(N)x_2(N) \\ ... & ... & ... & & ... & ... & ... \quad ... \\ y(t)y(1)x_{N-1}(1) & + & y(t)y(2)x_{N-1}(2) & + & ... & + & y(t)y(N)x_{N-1}(N) \end{bmatrix} \tag{3.11}$$

.

Using the particular solution w and the vector of constant terms b, an unknown weighted sum $\hat{S}$ can be calculated (from Equation (3.7). The vector element $b_t(i)$, where $i = N$ then is calculated by Equation (3.12):

$$b_t(i) = \hat{S}(t) - w(2)\,. \tag{3.12}$$

Thus, by solving the second non-homogenous linear equation system using the least squares method in every time step t, where the coefficient matrix A_2 is obtained by Equation (3.10) and the vector of constant terms b_t after equation system (3.11)) and Equation (3.12), the particular solution w_t is obtained. To obtain sub-signal u approximated value in the time step t, first Equation (3.13) is used, which allows obtaining N different $\hat{u}_i(t)$ values, where $i = 1, \ldots, N$:

$$\hat{u}_i(t) = \frac{\left(w_t(i) - y(t)y(i)\right)}{u(i)} . \qquad (3.13)$$

And by all i the average value is calculated by Equation (3.14):

$$\hat{u}(t) = \frac{\sum_{i=1}^{N} \hat{u}_i(t)}{N} . \qquad (3.14)$$

The result obtained by Equation (3.14) is an approximated value of sub-signal u in time step t. Depending on the condition number of the coefficient matrix (see matrix (3.10)) used in the second non-homogeneous system of linear equations, the resulting approximated values may have different degrees of precision. Using the condition number of the matrix of coefficients (3.10), the approximate number of digits can be calculated (Expression (3.15)), by which the accuracy decreases, having obtained a solution to the system of linear equations:

$$M = \log_{10}(\mathrm{cond}(A_2)) . \qquad (3.15)$$

The second factor that affects the accuracy of the approximated sub-signal is the error values of the solutions of the linear equation systems. This approximation approach uses quadruple precision, which uses 34 decimal places.

4. DEVELOPMENT OF FORECASTING SYSTEM NDVI FS

The chapter is devoted to the development of the NDVI time series forecasting system (NDVI FS). The forecasting system NDVI FS consists of a user interface, data preprocessing module, machine learning module, and a data store. The user interface is responsible for the input of the pixel coordinates, as well as an option of sub-signal approximation approach (use or not) obtained from modified VMD method.

When getting started, the user enters into the system three required element values via the appropriate interface:

- The *X* coordinate or geographic longitude in the MODIS NDVI images according to the coordinate system of these images.
- The *Y* coordinate or geographic latitude in MODIS NDVI images according to the coordinate system of these images.
- The value of the optional element (1: use VMD sub-signal approximation approach, 0: not use).

In the data preprocessing module, the data sets are created and preprocessed. In the machine learning module, the forecasting model is trained and the NDVI time series forecasting is performed. If the forecasting model and data preprocessing parameters for user-selected (according to the input pixel coordinates) time series are not available in the data store, then all available NDVI historical values are selected for the given pixel and saved in the comma-separated value (CSV) file. This time series is fed to data preprocessing module.

4.1. Data Preprocessing Module

In the data preprocessing module in the block "DP1", a decomposition of NDVI time series and obtained sub-signal approximation is performed, if the user has chosen it (see Fig. 4.1).

- The phase spaces are reconstructed for both time series using time delay method (or only for the NDVI time series if the user has not selected the approximation of the sub-signal).
- In the data set creation block, phase spaces are merged (if there are two phase spaces) and then are divided into training (70 % of records), validation (15 % of records), and test (70 % of data records) data sets. Each data set consists of the attribute (feature) set and forecasting parameter.
- In block "DP3", informative features are selected from all sets using stepwise regression analysis, and linearly uncorrelated feature sets are obtained using principal component analysis.

Now the obtained linearly uncorrelated training, validation and test attribute (feature) sets with a forecasting parameter of training, validation, and test data sets are the output of data preprocessing module.

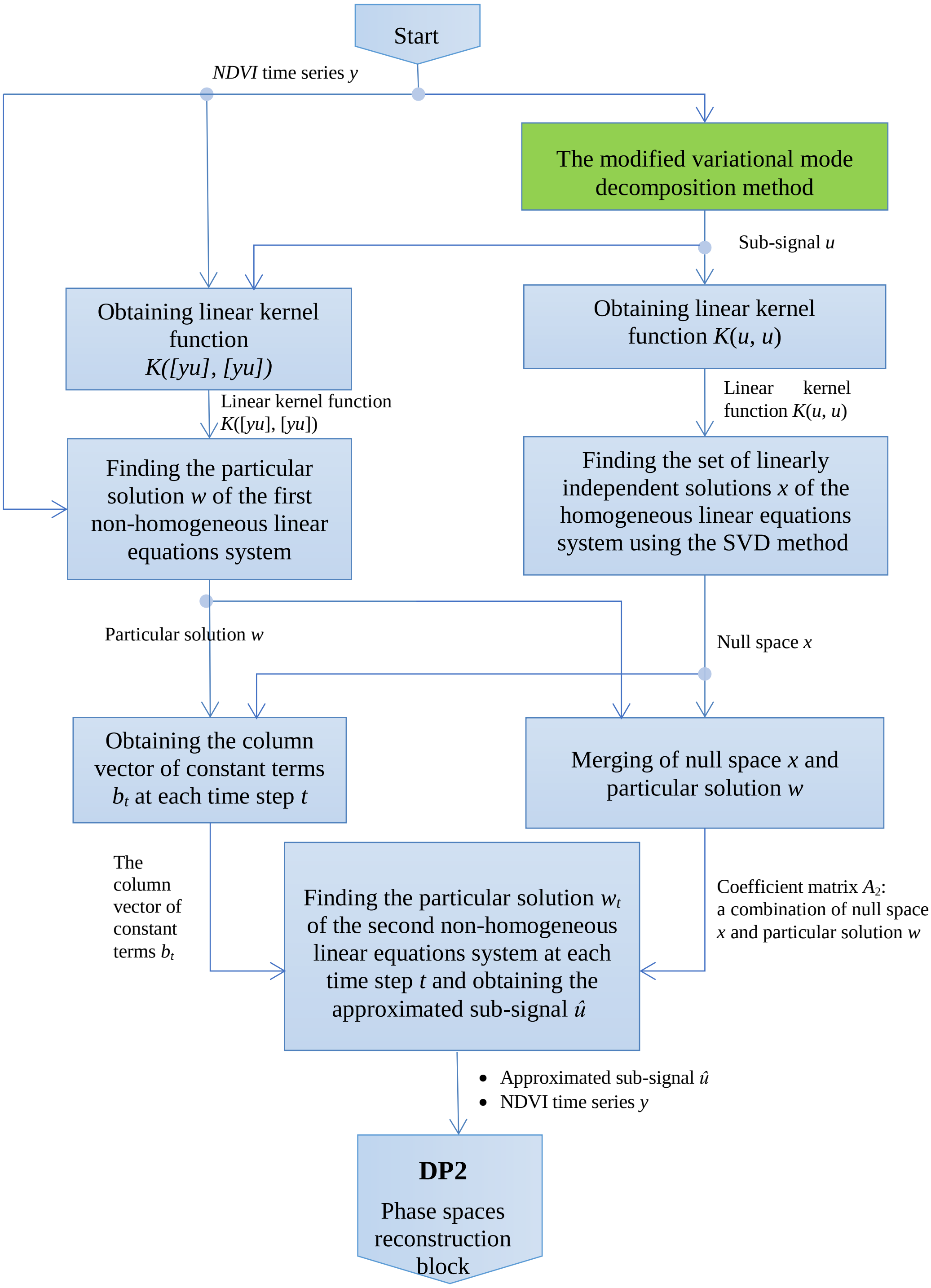

Fig. 4.1. Sub-signal approximation block "DP1".

All the necessary data preprocessing parameters: coefficient matrix A_2; particular solution w; indices of informativeness features; the mean values of features and set of eigenvectors M are saved in the data store with a unique identifier linking them with the particular NDVI time series.

4.2. Machine Learning Module

The machine learning module consists of two components: a training block and forecasting block. In the input, the module receives a preprocessed training, validation, and test data set from data preprocessing module. The output of the module is a forecast that is provided to the user.

- In the training block the layer recurrent neural network performs training on the training data set, evaluating the RMSE on validation data set. As a most suitable LRNN forecasting model is chosen the one for which the smallest value of loss function RMSE is achieved.
- With the best-found forecasting model short-term forecasting is performed on training, validation and test data set forecasting parameter, and the values of loss functions RMSE, DS and R^2_{adj} on each of these sets are evaluated.
- When the most suitable LRNN forecasting model is found, then the parameters of this model (weights and bias) are saved in the data store linking them by the unique identifier with other data preprocessing parameters. The block output is the forecasting model.
- When the forecasting model is obtained, it can be used for forecasting the new values of NDVI time series by performing it in the forecasting block of NDVI time series (see Fig. 4.2). After a certain time period, which in case of MODIS NDVI images is seven days, the system receives a new available image, and the forecasting process can be repeated by forecasting the next value.

The practical implementation of the NDVI FS system in the form of an application is done using the high-level programming language MATLAB. The system is implemented using a set of interrelated MATLAB functions (both built-in and author-developed) and script for business logic, a Character-based User Interface (CUI), and a data store. The script is used to start the system and call up all the necessary functions in a specific order, starting with entering the user parameters. The data store is implemented in the form of data files stored on the user’s data carrier.

The system's business logic includes calculating the NDVI time seriesshort-term forecast using user input parameters, MODIS NDVI images in the data store, and certain algorithms. In the developed system NDVI FS, all data preprocessing, LRNN model training and time series forecasting are fully automated.

The development of the modified VMD method *vmd_modified* code is based on the original VMD method code created by the authors of this method [119]. The development of the sub-signal approximation approach, the search for suitable values for all required data preprocessing parameters, as well as the search for suitable values of the LRNN parameters are realized using the author’s MATLAB functions. The author is developing the sub-signal approximation approach code using the MATLAB built-in function for null space calculation (function *null*), nonlinear regression analysis with the least squares method (function *lsqcurvefit*), and linear regression analysis with the least squares method (function *lsqlin*).

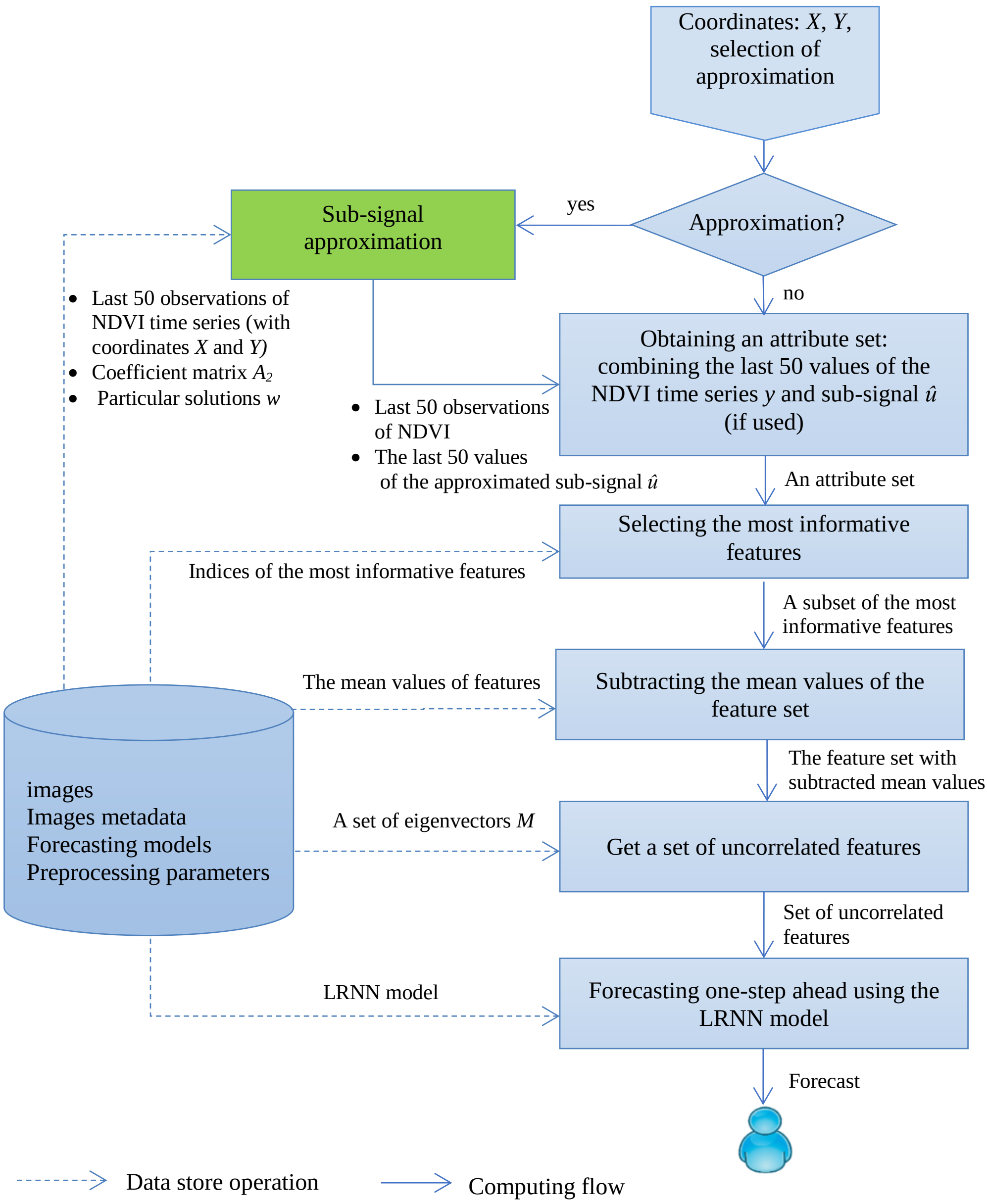

Fig. 4.2. Forecasting block.

Phase space reconstruction with time delay method is implemented with the help of the function *phasespace*, which is part of the Chaotic Systems Toolbox [54]. For stepwise regression analysis the built-in function *stepwisefit* is used. Principal component analysis (PCA) is performed using the MATLAB Toolbox for Dimensionality Reduction [59]. For the layer recurrent neural network (LRNN) model training the built-in function *layrecnet* is used.

The architecture, computing flow and data store operations of forecasting system NDVI FS are shown in

Fig. 4.3. The NDVI FS system receives images from the preparation platform of the MODIS images. The output of system NDVI FS is a short-term forecast of the NDVI time series selected by the user (e.g. a farmer).

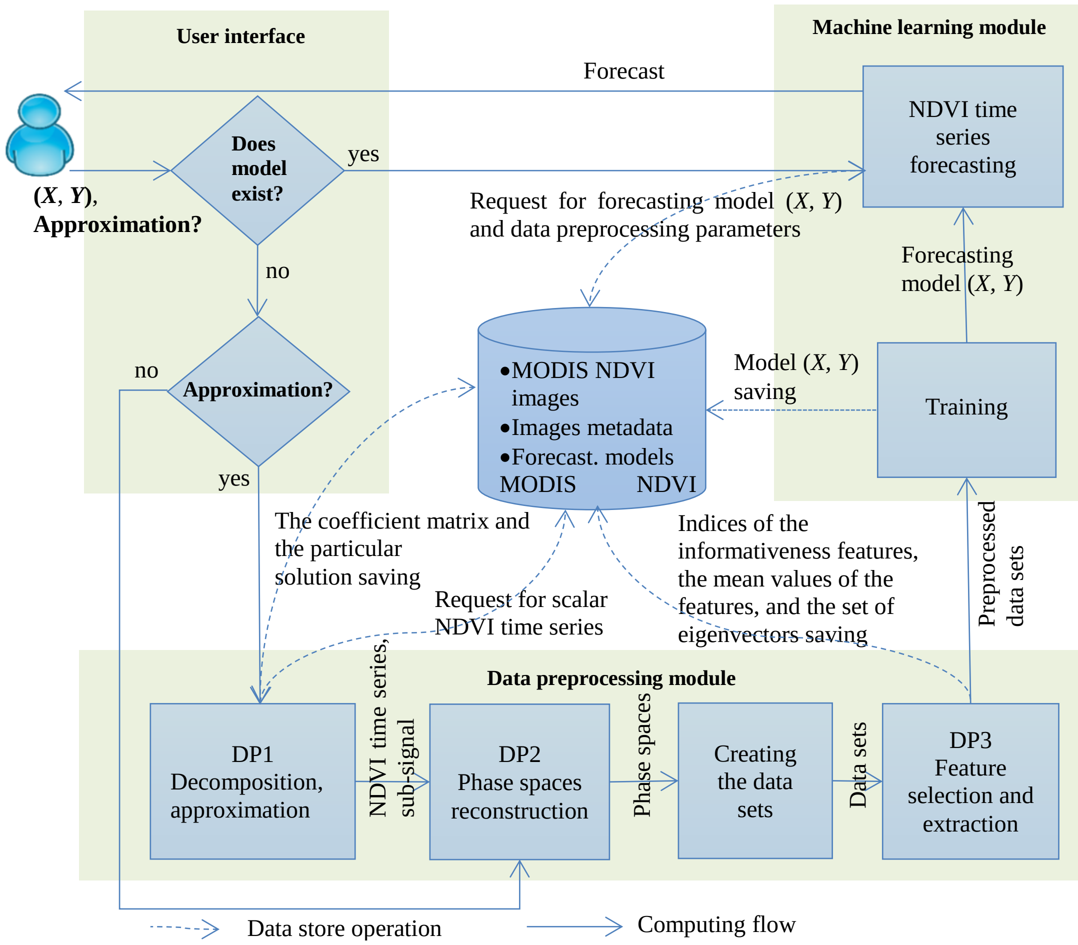

Fig. 4.3. NDVI time series forecasting system NDVI FS.

Thus, the developed forecasting system NDVI FS is the stage in precision agriculture that provides input data for the decision support system (see Fig. 4.4).

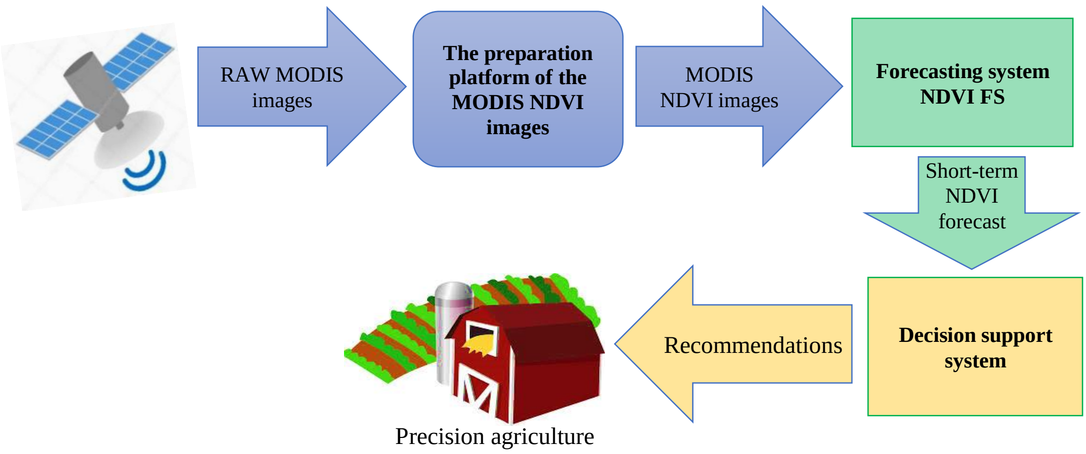

Fig. 4.4. The developed system NDVI FS in precision agriculture.

On the basis of the NDVI index forecast the decision support system makes certain decisions and issues recommendations to farmers.

5. ASSESSMENT OF ACCURACY OF THE DEVELOPED FORECASTING SYSTEM

The chapter is devoted to experiments with the developed forecasting system NDVI FS. The accuracy of the forecasting system is compared with the accuracy that is achieved using other forecasting methods. Each experiment is repeated 100 times each time taking a different NDVI time series.

5.1. NDVI Forecasting With the Classical Methods

There are three experiments using classical prediction methods. The aim of one experiment is to explore the simple moving average accuracy of forecasting task of NDVI time. In the experiment for the NDVI time series phase space with different values of dimension m is reconstructed. The phase space dimension m is searched in the interval [1; 50], but time delay $\tau = 1$. The appropriate dimension m, and accordingly simple moving average period is the value at which the minimum value of loss function RMSE is reached.

The aim of the next experiment is to explore the accuracy of continuous state space Markov chains in the forecasting task of NDVI time series. The author described an experiment using discrete time, discrete state space Markov chains for NDVI time series forecasting in a study [91], while the use of m-th order discrete time, continuous state space Markov chains for NDVI time series forecasting is described in the author's paper [93]. In the experiment, the continuous state space Markov chains with memory m are used. Since the m-th order Markov chain is used, last m states or last m observations of time series are used to forecast the next state. This combination of m states of Markov chain creates a vector that is formally identical to state vector in the reconstructed phase space.

To forecast the next value of time series, the Euclidean distance between the last delayed vector in phase space and all other delayed vectors is calculated. For the delayed vectors with a low Euclidean distance the next values of time series that follows this vector in time are taken and the forecast is obtained as the average value of all those next values.

The aim of the last experiment is to explore the accuracy of ARIMA methods in the forecasting task of NDVI time series. In the experiment, the first 70 % observations are selected for the NDVI time series on which model of $ARIMA(p, d, q)$ is trained, where p is the order of the autoregressive polynomial, d is differential operator order, and q is the order of moving average polynomial. A model with the different value of p, d and q is created and the forecast one time series unit forward is performed. Then the next 70 % of observations of NDVI time series are selected, starting from time step $t = 2$, and forecasting for the next value of time series is performed.

All values of time series corresponding to forecasted values of time series create a test data set. In the end, those values of p, d and q are selected where the value of the loss function RMSE between forecasted and observed values from the test data set is minimum.

5.2. Characteristics of the Forecasting System NDVI FS Experiments

Two experiments are performed to evaluate the accuracy of the developed forecasting system NDVI FS. The author described an experiment with Elman recurrent neural networks for NDVI time series forecasting in a study [92], an experiment with LRNN for NDVI time series forecasting in a study [87], and the use of a forecasting system NDVI FS that does not apply VMD based approximation approach is described in the author's studies [88], [94].

The aim of the first experiment is to explore the accuracy of forecasting system NDVI FS in forecasting task of NDVI time series without using the sub-signal approximation approach obtained from the modified VMD method. The experiment is repeated 100 times.

- The phase space with dimension $m = 50$ and time delay $\tau = 1$ is obtained from time series of normalized difference vegetation index and thus input data set is obtained. Input data set is divided into two parts: a set of attributes and forecasting parameter. Both the set of attributes and forecasting parameter are divided into three parts, providing training, validation and test data set.
- From each feature set informative features are selected using stepwise regression analysis, and the linearly uncorrelated feature sets are obtained using principal component analysis.
- Preprocessed training and validation data set are passed in layer recurrent neural network input for training and for obtaining forecasting model.

The aim of the next experiment is to explore the accuracy of the forecasting system NDVI FS of forecasting task of NDVI time series, using the sub-signal approximation approach obtained from the modified VMD method.

- Using the developed modification of the variational mode decomposition (VMD) method with the sub-signal value approximation approach obtained from this method, an appropriate sub-signal is obtained for normalized difference vegetation index. A phase space with $m = 50$ and time delay $\tau = 1$ is reconstructed for sub-signal as well as phase space is reconstructed with the same parameters for the NDVI time series.
- Phase spaces are combined to produce an input data set that is divided into two parts: an attribute set and a forecasting parameter. Both the attribute set and forecasting parameter are divided into three parts providing training validation and test data set.
- From each feature set informative features are selected using stepwise regression analysis, and linearly uncorrelated feature sets are obtained using the PCA.
- The preprocessed training and validation data set are passed in layer recurrent neural network input for training and for obtaining forecasting model.

In five experiments, the mean values of three loss functions: root mean square error RMSE, directional symmetry DS, and adjusted coefficient of determination R^2_{adj} on the test data sets are given in Table 5.1.

Table 5.1

The Mean Values of Loss Functions

Forecasting method or system	*RMSE*	*DS*	R^2_{adj}
The simple moving average	0.0442	93.40 %	0.94
A discrete time, continuous state space *m*-th order Markov chains	0.0214	83.76 %	0.90
Autoregressive integrated moving average	0.0108	93.85 %	0.96
Developed forecasting system NDVI FS without sub-signal approximation approach	0.0012	97.28 %	**0.99**
Developed forecasting system NDVI FS with sub-signal approximation approach	**0.0009**	**98.80 %**	**0.99**

Forecasting methods and systems in Table 5.1 are arranged by the decrease of the average RMSE values or the increase of forecasting accuracy. It is concluded that a higher forecasting accuracy is shown by the developed NDVI FS using the sub-signal approximation approach obtained from modified variational mode decomposition method ($RMSE$ = 0.0009, DS = 98.80 % and adjusted R^2_{adj} = 0.99).

5.3. Transfer of the Data Preprocessing Parameters and Forecasting Models

Two experiments are carried out with transferring of the data preprocessing parameters and models. The aim of one experiment is to explore the transfer of forecasting models to NDVI time series of neighbouring pixels by forecasting without the new training, and without using the sub-signal approximation approach.

- When on randomly chosen NDVI time series data preprocessing and training is performed using the developed NDVI FS without approximation approach, all the necessary parameters are saved, and a suitable forecasting model is obtained.
- Around this corresponding pixel of NDVI time series in satellite image, a grid is drawn that corresponds to radius $r = 5$. Thus, around the chosen corresponding pixel of NDVI time series a large grid of size 11×11 is drawn, where there are 120 pixels excluding the trained central pixel.
- For each of these neighbouring pixels 120 NDVI time series are obtained and each of these time series first is preprocessed and forecasted using from the training time series obtained data preprocessing parameters and forecasting model.
- Then, on each of these time series data preprocessing parameters and the forecasting model are individually obtained, then forecasts and RMSE values are calculated.

For each time series both RMSE values are compared. The experiment is repeated 100 times, each time forecasting 120 time series. In fifteen cases of the experiment obtained results are shown in Fig. 5.1.

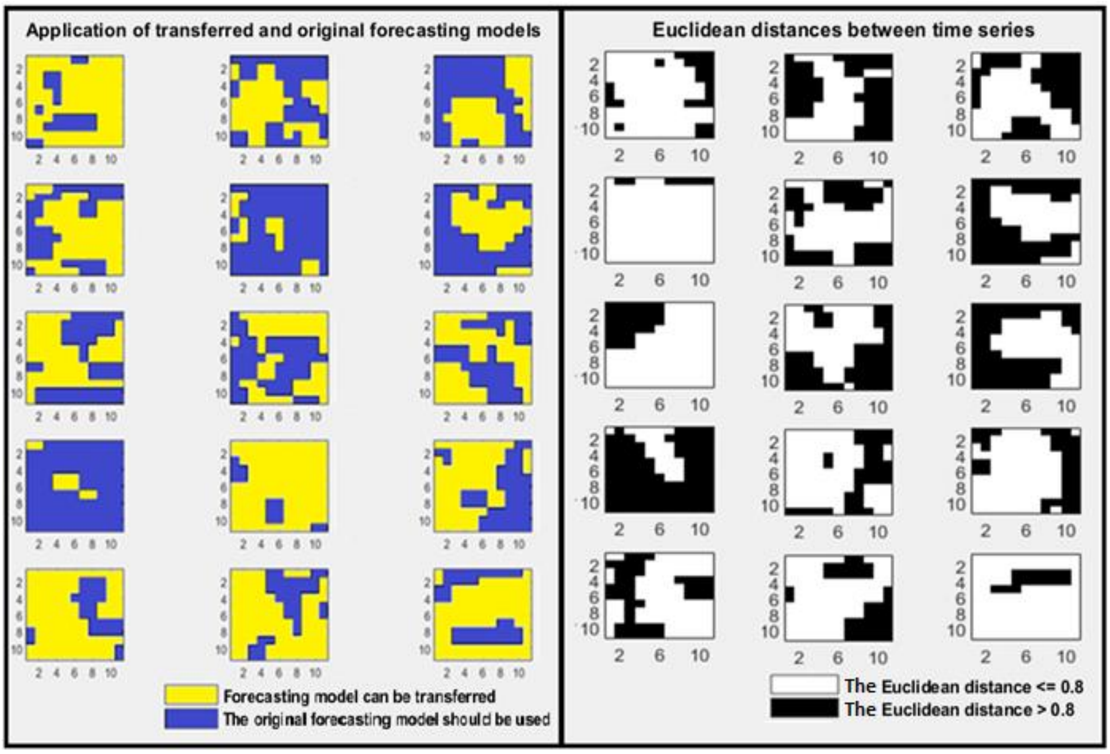

Fig. 5.1. Application of the forecasting model without approximation approach.

On the left side of the binary image (see Fig. 5.1), the pixels that are marked yellow (the value is one) are the ones on which time series can use transferred data preprocessing parameters and forecasting model. The pixels that are marked blue (the value is zero) are the ones on whose time series it is necessary to individually use the obtained data preprocessing parameters and original forecasting model. On the right-hand side of Fig. 5.1, a Euclidean distance with a threshold 0.8 is shown. Euclidean distances are calculated between the central pixel time series and all other time series. Analysing Fig. 5.1, it is concluded that by transferring preprocessing parameters and forecasting models and by performing the forecasting a similar RMSE value can be obtained, if between time series used in training the and time series where there are used transferred parameters and models the Euclidean distance is similar or lower than the value of threshold 0.8.

The aim of the next experiment is to explore the transfer of forecasting models to NDVI time series of neighbouring pixels by forecasting without the new training, and using the sub-signal approximation approach.

- The 11 × 11 pixel grid is obtained from the satellite image (121 pixels in total), where the central pixel is the pixel on which NDVI time series preprocessing and training are performed.
- Each of the 120 time series is preprocessed and predicted, first, using the transmitted parameters and models derived from the central pixel NDVI time series, and then using the individual parameters and models obtained on each time series, in both cases calculating the RMSE values between the observed and the predicted NDVI time series.

The experiment is repeated 100 times each time forecasting 120 time series. The results obtained from the first fifteen cases of the experiment are shown in Fig. 5.2.

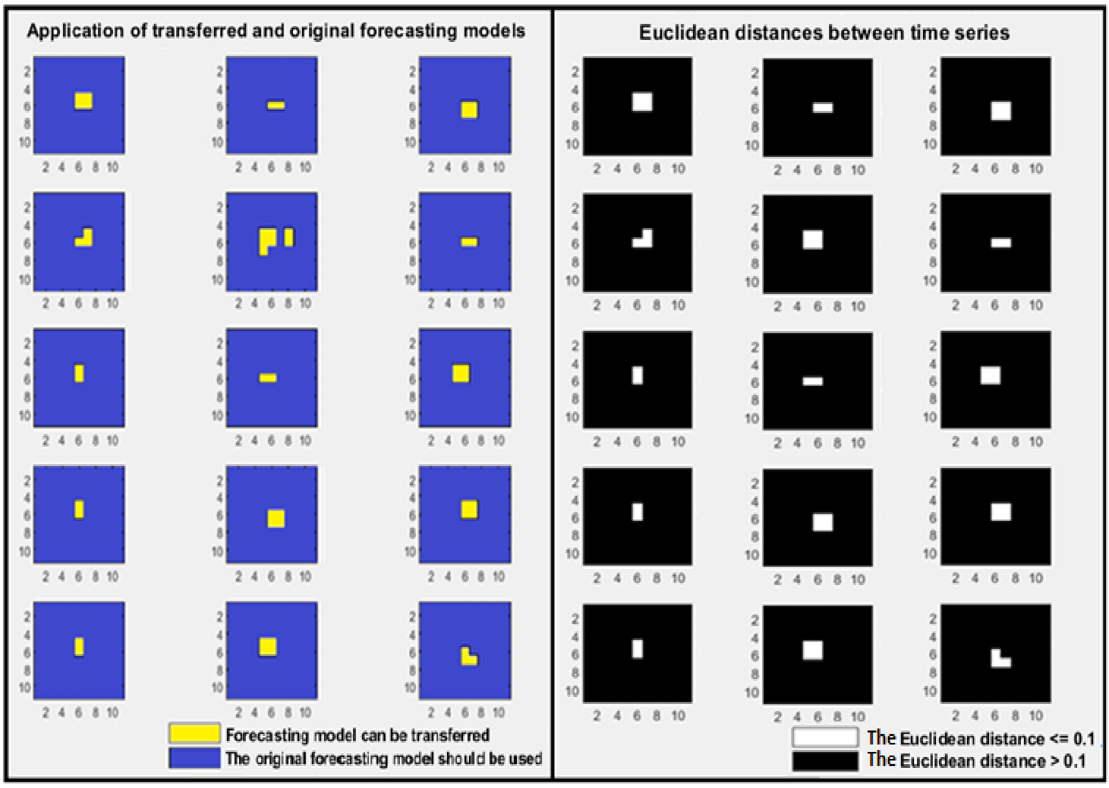

Fig. 5.2. Application of the forecasting model with approximation approach.

Also, on the left in Fig. 5.2. there is a binary image where in yellow are marked those pixels on which time series can transfer data preprocessing parameters and forecasting model, but in blue are marked those pixels on which time series it is not possible to perform transfer. On the right in Fig. there is the binary image of the Euclidean distance with a threshold 0.1 where Euclidean distances are calculated between time series from the central pixel and all other time series. Analysing Fig. , it is concluded that transferring of preprocessing parameters and forecasting model that are obtained from central pixel time series of normalized difference vegetation index can be used to forecast other NDVI time series with the similar values of RMSE if between the time series used in training and time series where transferred parameters and models are used, a Euclidean distance should be lower or equal than the value of threshold 0.1.

RESULTS AND CONCLUSIONS

Within the framework of the Doctoral Thesis, a forecasting system of NDVI time series NDVI FS has been developed, which provides forecasts of dynamics of the short-term vegetation changes that are extremely important in precision agriculture. The developed forecasting system is evaluated using the author's proposed approximation approach and without this approach to test the first hypothesis. To test the second hypothesis, model transfer experiments were performed, using parameters and models of one NDVI time series to forecast other NDVI time series. When the **tasks** set in the Doctoral Thesis were solved, the following main **results** were obtained.

1. Analysis of scientific literature on forecasting time series of normalized difference vegetation index was performed. Defined requirements and the choice of forecasting method for developing the system were reasoned.
2. Analysis of scientific literature on forecasting time series using a different signal decomposition method used in frequency analysis was performed. The choice of the decomposition method in forecasting task of NDVI time series and the need for approximation of the sub-signal was justified.
3. Approximation approach of sub-signal that is obtained usingmodified variational mode decomposition method is developed, which allows to approximate sub-signal values for all time steps for which historical observations of normalized difference vegetation index are available.
4. The forecasting system of time series of normalized difference vegetation index is developed, which is based on a set of specialized methods and approaches. It allows increasing the accuracy without using additional input data such as air temperature, rainfall, and land surface categories, as well as other vegetation indices. The whole in the system implemented data preprocessing process, LRNN model training and time series forecasting is automatized.
5. Evaluation of the developed system NDVI FS was performed, its accuracy both using and without using sub-signal approximation approach has been compared to a simple moving average, a discrete time, continuous state space *m*-th order Markov chains and an autoregressive integrated moving average accuracy in NDVI time series forecasting.
6. An approach is developed for transferring the trained forecasting model and appropriate data preprocessing parameters to the territory where forecasting models are not available. The forecasting accuracy in many cases is similar to the results provided by a trained forecasting model for a specific time series of normalized difference vegetation index.

The following **conclusions** were reached during the development of the Doctoral Thesis.

1. In studies on forecasting of time series of normalized difference vegetation index data preprocessing methods are not sufficiently used, which does not allow to achieve high forecasting accuracy.

2. The developed forecasting system performs a short-term forecast of the new value of the time series of normalized difference vegetation index using only the NDVI historical values.
3. The developed approach approximates the sub-signal of the variational mode decomposition at any time step where the historical values of the time series of normalized difference vegetation index are available; it allows using the approximated sub-signal to forecast the new values of the NDVI time series.
4. The accuracy of the approximated sub-signal depends on the condition number of the coefficient matrix of the second linear equations system and the error size of the particular solution of the first linear equations system.
5. The accuracy of forecasting that uses the system developed with the approximation approach proposed by the author is higher than without using this approach, which confirms the first hypothesis.
6. The forecasting accuracy of forecasting system NDVI FS used with or without the approximation approach proposed by the author is higher than the accuracy achieved by forecasting NDVI time series with a simple moving average, a discrete time, continuous state space m-th order Markov chains and an autoregressive integrated moving average.
7. Using forecasting system NDVI FS and training on NDVI time series the data preprocessing parameters and forecasting model can be obtained, which can be used to forecast other NDVI time series with similar accuracy compared to the accuracy that can be achieved by forecasting the time series with individually obtained data preprocessing parameters and forecasting model, if a Euclidean distance between both the time series is less or equal to the defined threshold, which confirms the second hypothesis.
8. The developed system NDVI FS without approximation approach can be used, if it is necessary to forecast values of the time series of normalized difference vegetation index for a large area in a relatively short period and where a slight decrease in forecasting accuracy is allowed.
9. The developed forecasting system NDVI FS with the approximation approach can be used when forecasts of the time series of NDVI with higher accuracy are needed, but for a relatively small area.

Further research is related to the improvement of the models and parameters transfer approach with a more precise definition of conditions where such a transfer can be performed as well as an improvement of the sub-signal approximation approach.

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