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Discrete Models in Research of Wave Processes in Rod Structures of Radio-Electronic Means

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***Abstract* – The article shows the relevance of the application of discrete models of rod structures of radio-electronic means (REM) for the study of their behaviour under transient loading. A discrete model of the propagation of harmonic waves in the rod and the study of standing waves are proposed. Computational experiments using the proposed model are conducted. The results show that the model accurately reflects qualitative dynamics of the physical processes in the elastic rod while the waves of elastic deformations are passing through. The proposed models are used for software implementations of systems of mechanical simulation of the behaviour of rod structures.**

***Keywords* – Discrete model, displacement wave, elastic rod, standing wave, resonance.**

I. Introduction

In [1], the need to study the dynamics of rod structures of radio-electronic means (REM) mounted on mobile carriers is noted. The problems of using mathematical modelling techniques for research of physical processes in rod elements under the influence of a short-term shock pulse are considered. The losses of energy in the body of the rod are not taken into account, and it is considered that the pulse keeps its shape and amplitude unchanged for all the movement along the rod [2], [3].

In the present article, we consider the case when not a single pulse, but a periodic motion is set at one end of the rod (or both), which will be distributed along the rod with a finite velocity. Therefore, all other points of the rod will gradually make the periodic motion at the same frequency, and as a result of the energy losses, oscillation amplitudes of individual points of the rod will gradually decrease at the distance from the point, which is driven to oscillate. Such oscillatory motions propagating along the rod, gradually fading, belong to the class of wave motions or waves [2].

Article [1] shows the developed discrete model for the study of the propagation of a single displacement pulse in the elastic rod under the edgewise impact. The energy loss is not taken into account. In [2], [4] the model of an “infinitely long” non-fading rod is used for the study of wave propagation in the rod. The real rod elements of REM structures have finite length, and one must take into account all the factors that characterise the dynamics of wave processes during the propagation of elastic waves in the rod. The appropriate model adequate to these processes is needed.

II. Development of the Discrete Propagation Model for the Elastic Wave of Displacement in the Rod

Let us consider the equation of the rod motion under the forced oscillations. Let us suppose that the left end of the length l is fixed, while the right end is free and oscillates according to the law $u = U_0 \sin \omega t$ in the direction of the rod length (Fig. 1).

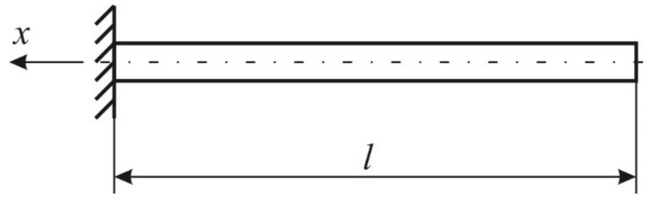

Fig. 1. Geometrical model for the rod.

These fluctuations (as well as a separate longitudinal pulse) will be transmitted along the rod from one layer to another: the elastic longitudinal wave will run along the rod. Each point of the rod, being at a distance x_i from the beginning, will make the same harmonic motion, as the starting point, but in this movement, it will fall to the time required for the propagation of the wave at a distance x_i. This time is equal to x_i / v, where v is the speed of propagation of the wave along the rod [5]. Such a harmonic motion of individual points (cross-sections) of the rod, extending along the rod with a certain speed, is called a harmonic traveling wave.

Longitudinal vibrations arising in the rod are described by the wave equation [6]–[8]:

$$E = \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}, \qquad (1)$$

where $u(x, t)$ is the offset of the current cross-section of the rod along the axis x; E is the Young’s modulus; ρ is the material density.

Since during the propagation of the traveling wave the oscillation energy is gradually dissipated as a result of internal friction [7], [9]–[11], we will take into account the loss of energy in the form of a dissipative force proportional to the rate of deformation in (1), and add an external force $F(x, t)$ to the right side, exciting vibrations in the end section of the rod

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(Fig. 1). Then the equation of the forced longitudinal oscillations of the rod can be written as follows:

$$E\frac{\partial^2 u}{\partial x^2}+\eta\frac{\partial}{\partial t}\left(E\frac{\partial^2 u}{\partial x^2}\right)-\rho\frac{\partial^2 u}{\partial t^2}=F(x,t), \qquad (2)$$

where η is the material viscosity coefficient.

In accordance with the method of finite differences, we construct a geometric discrete model for the rod consisting of N nodes, connected by elastic couplings (Fig. 2).

Fig. 2. Discrete model for the rod.

Replace the first time derivative in the left part (2) by its difference analogue and put it in (2) – $L(u)=\frac{\partial^2 u}{\partial x^2}$. Then equation (2) can be written as:

$$EL(u)_t+\frac{\eta}{\tau}\left[EL(u)_t-EL(u)_{t-\tau}\right]=\rho\frac{\partial^2 u}{\partial t^2}, \qquad (3)$$

where τ is a time sampling rate, and force $F(x, t)$ is taken into account in the initial conditions.

Expanding the brackets and grouping similar terms in (3), we obtain:

$$\left[\frac{\left(1+\frac{\eta}{\tau}\right)E}{\rho}L(u)_t-\frac{\frac{\eta}{\tau}}{\rho}L(u)_{t-\tau}\right]=\frac{\partial^2 u}{\partial t^2}. \qquad (4)$$

Given that the second derivative of displacement with respect to time is the acceleration a of the node, let us write (4) in the form of $a=\frac{\partial^2 u}{\partial t^2}$, and by replacing the second derivative with respect to time by the difference analogue, we get:

$$\tau^2 a=u_x(t+\tau)-2u_x(t)+u_x(t-\tau). \qquad (5)$$

Let us transform (5) in the form of the explicit difference equation:

$$\tau^2 a+2u_x(t)-u_x(t-\tau)=+u_x(t+\tau), \qquad (6)$$

which, being complemented by the boundary and initial conditions, forms a clear difference scheme, giving the calculated rod model in conjunction with the geometric model, which is simply realised by a computer.

The degree of information reliability obtained in discrete models must be confirmed by the research materials of models after their design and construction of algorithms [12]–[15].

The article presents the research results of the rod structure model, conducted with the help of computational experiments. Numerical solutions obtained by the developed models for a number of tasks are compared with the known analytical solutions of these problems, and the degree of the model accuracy is assessed.

III. Computational Experiments

In the computational experiments, the process of passing of the displacement harmonic traveling wave along the rod has been investigated, and the distribution of the displacement amplitudes by length at different frequencies of normal vibrations and under different conditions of fixing the ends of the rod (a cantilever rod, a rod with fixed ends) has been defined.

We have investigated the formation of standing waves in the rod (the link of this fact with the resonance phenomenon is described in [2]); the conditions, in which the amplitude of the standing wave in the antinode of a given displacement amplitude of the rod ends will be maximum, are described.

Task 1. We have considered the case when the ends of the rod are under different conditions: the left end of the rod of l = 1500 mm length is fixed rigidly, the right end is free and performs the harmonic motion according to the law $u = U_0 \sin\omega t$ in the direction of the rod length (Fig. 3).

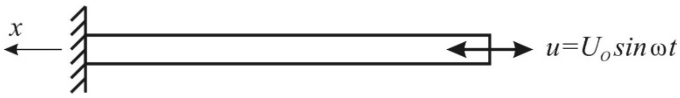


Fig. 3. The experimental scheme.

The problem considered in [15] shows that in this case such displacement amplitude distribution is only possible, when the antinode is formed at the free end, and the node is formed at the fixed end. This condition is satisfied only when the rod length fits the odd number of the wave quarters, i.e., the wave lengths corresponding to different harmonics satisfy the relation $\lambda_k = 4l/k$, where k = 1, 3, 5 … ($2n-1$) (n is any integer).

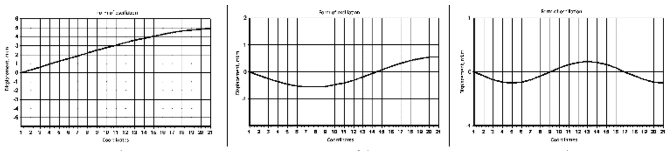
In modelling, the program evaluates 1, 3 and 5 natural frequencies of longitudinal vibrations of a cantilever rod by the analytical formula [16]:

$$\omega_i=\frac{(2n-1)\pi}{2l}\sqrt{\frac{E}{\rho}}, \qquad (7)$$

where $I = n$.

Further, fluctuations are defined in series at each frequency at the free end of the rod. The solving goes on up to the standing wave formation in the rod. As a result, graphics for longitudinal vibrations of a cantilever rod are built (Fig. 4) for a single point in time.

The graphs in Fig. 4 show that a standing wave of displacements is set at each oscillation frequency of the external action in the rod, and the displacement amplitude at the free end is equal to the amplitude of the external action, and a displacement node is mounted at the fixed end. In general, the distribution of the displacement amplitudes along the length of the rod, and the relative position of nodes and antinodes correspond to the first (Fig. 4a), the third (Fig. 4b) and the fifth (Fig. 4c) harmonica, which fully corresponds to the picture of the analytic solution given in [17].



a) b) c)

Fig. 4. Forms of longitudinal vibrations of the rod.

Task 2. The nature of the normal oscillations of the rod is also dependent on its edge conditions [15], [18]. Task 1 considers the case when the conditions at the ends of the rod are different.

Here, we consider the case when both ends of the rod are in the same conditions, namely, both are fixed (Fig. 5).

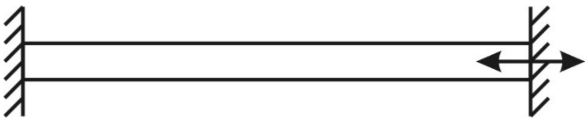


Fig. 5. Scheme for the rod attachment.

The force acts at the right edge of the rod through the fixation, transmitting to it the harmonic motion with a predetermined amplitude, frequency and phase.

The rod length is l = 1500 mm. The second natural frequency of the longitudinal oscillations (according to formula [3]) is f_2 = 3459 Hz (taken as an example). The right edge of the rod is driven at this frequency through the fastening; the harmonic wave will propagate along the rod with the same speed as in the rod with free ends. Distribution of displacement amplitudes along the rod is given by the expression $\sin\frac{n\pi x}{l}$, where n = 1, 2, 3…. This expression allows us to establish which distribution function corresponds to a harmonic [19].

There is $n\frac{\lambda_n}{2}$ of the wavelength for such an attachment to fit the length of the rod, where λ_n is the wavelength corresponding to a given value n. In [2]–[6] it is shown that $n = k$, where k is a wave number, and if $n = k = 2$ (the second natural frequency), the wavelength is $\lambda_2 = l$, and $\omega_2 = 2\pi\upsilon/l$ is the angular frequency, i.e., it corresponds to the second harmonic. This means that k of half-waves of the k-th harmonic is placed over the length of the rod.

The solution is obtained by the model, and the displacement amplitude distribution graph is plotted, which corresponds to the second harmonic of vibrations (Fig. 6): two half-waves are placed along the length of the rod.

There is a displacement node at the left fixed end, and at the right end, where the disturbance is given, the node of the formed wave is shifted inside (towards the left edge) by the grid length of only 0.0625, i.e., close to the right end of the rod, and when it falls in the right wing node of the model under fluctuations, the second harmonic form is obtained (Fig. 6). Since there are displacement nodes at both ends of the model, the amplitude at the wave antinode is as large as possible, namely, it is 10 µm in comparison with 0.3 µm amplitude of external disturbance (Fig. 6).

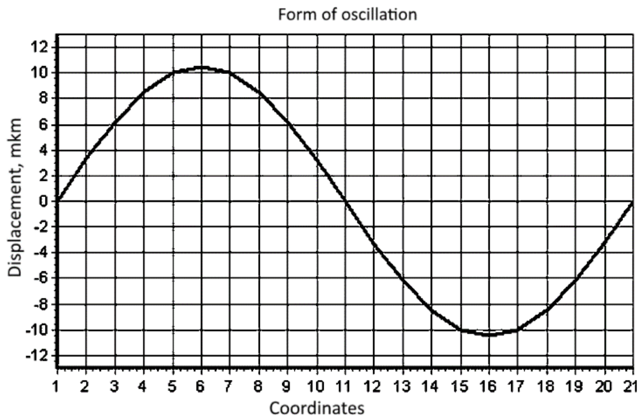


Fig. 6. The shape of the rod oscillations with fixed ends.

The modelling results of the study are fully confirmed by the analytical solution of this problem given in [2], [5].

Task 3. The formation of standing waves under longitudinal vibrations in the rod has been studied. We have used the model of the rod with rigidly fixed edges (Fig. 5). Sinusoidal oscillations have been given to the extreme left and right model nodes, either in phase or in antiphase, with the same amplitude.

Two harmonic traveling waves will extend towards each other under this impact on the rod, and the vibrations of each cross-section of the rod (or the model node) can be regarded as the composition of two traveling waves. By attaching conditions, each traveling wave forms the form node at the opposite end (Fig. 6), so the addition of two waves gives a standing wave. The amplitudes of the standing waves in the antinodes reach a maximum, when the conditions at the ends of the rod are the same, and the frequencies of the rod normal vibrations and the frequency of external force acting on the rod coincide.

According to analytical formula [3], the first four natural fluctuation frequencies of the rod fixed at the ends of l = 1500 mm length, are identified: f_1 = 1729 Hz, f_2 = 3459 Hz, f_3 = 5188 Hz, f_4 = 6198 Hz.

Fluctuations with these frequencies have been set consistently on both ends of the rod in phase and in antiphase. Figure 7 shows a picture of the standing wave formation for all cases of external action.

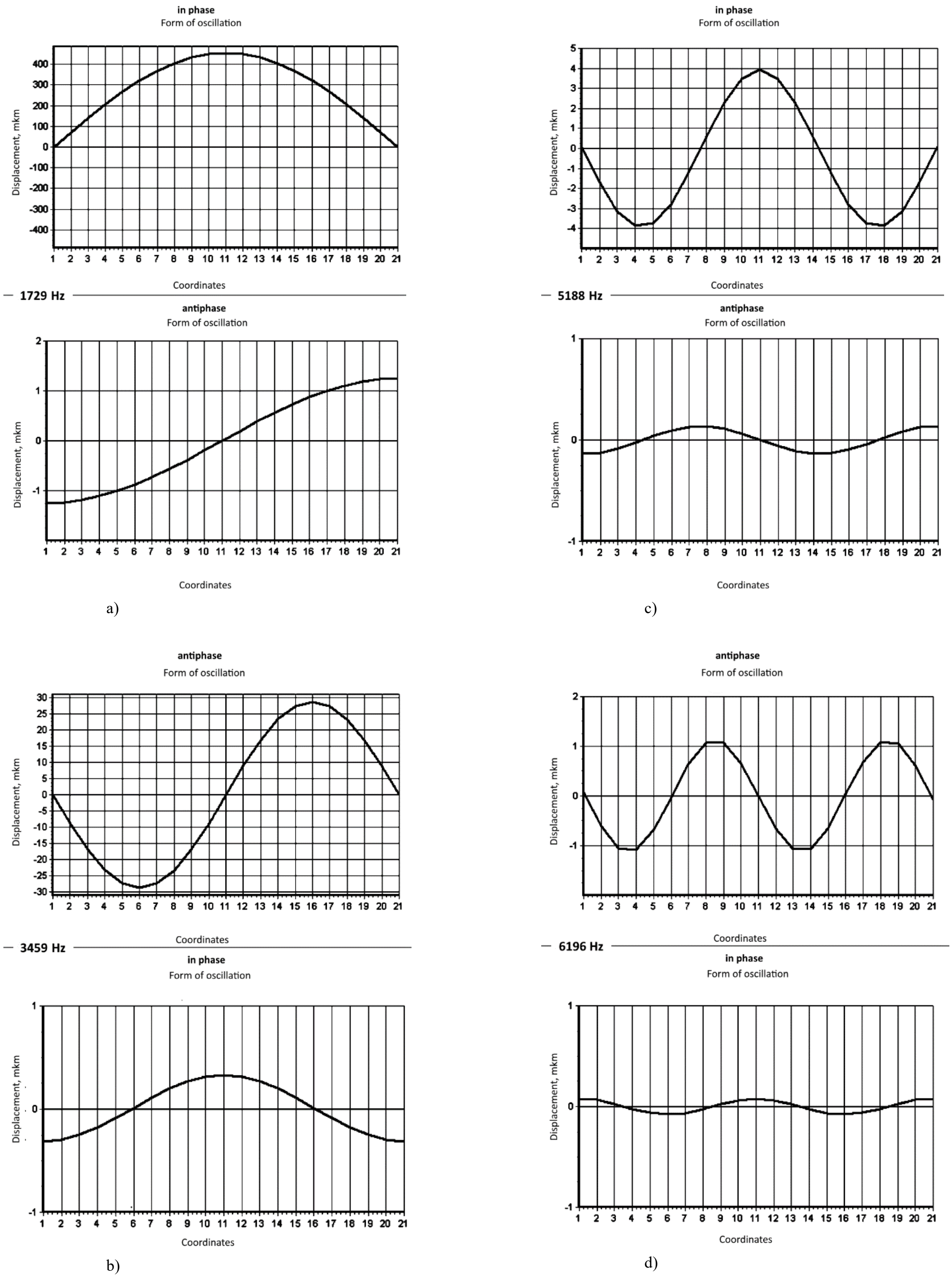


Fig. 7. Distribution of displacement amplitudes during the standing wave formation in the rod.

Fluctuations with these frequencies have been set consistently on both ends of the rod in phase and in antiphase. Figure 7 shows a picture of the standing wave formation for all cases of external action.

As seen from Fig. 7, a standing wave with large amplitude in the antinodes is formed in odd harmonics under the same conditions at the end of the rod: fluctuations in the phase (Fig.7a and Fig. 7c). During vibrations of the ends in antiphase (different conditions at the ends) odd harmonics are absent, and the amplitude of the formed waves does not exceed the amplitude of external force (Fig. 7c).

Even harmonics (f_2, f_4) form a standing wave of significant amplitude when the ends of the rod vibrate in antiphase (Fig. 7b, d), and if the ends vibrate in phase, then the amplitude of the resulting oscillation is not more than the external force amplitude (Fig. 7b and Fig. 7d).

The modelling results are consistent with the results of analytical studies and indicate under what conditions the forced oscillations can be obtained in the rod, which frequency and amplitude distribution will be close to the frequency and amplitude distribution of one of the normal vibrations of the rod. These normal oscillations are identical with the standing waves that can occur in a continuous system.

The emergence of the standing waves of significant amplitude in the rod under the action of a harmonic external force is the resonance phenomenon, the analysis of which in rod structures of REM is an urgent task.

IV. Conclusion

The conducted research of the suggested model of the rod structure has shown that the model reflects qualitatively correctly the dynamics of the physical processes occurring in the elastic rod under the forced oscillations. The proposed models are used for software implementation of systems of mechanical simulation of the behaviour of rod structures.

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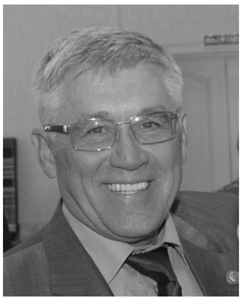

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