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Stochastic modeling for transport logistics

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Abstract

This paper deals with a logistic system consisting of a wholesale store, a retail store and automobiles that are taking part in goods delivery from a wholesale store to a retail store. Assuming random and coming at random time moments demands, we construct a stochastic model for this transport logistic scheme and derive Gaussian approximation for transport and stock level of goods dynamics.

Keywords: Logistic system; Transport logistics scheme; Gaussian approximation; Diffusion approximation; Markov process

1. Introduction

In the last years, research interest in transport logistics has increasingly focused on dynamical theory approach (see [1,5,6,7,8] and references there) for quantitative and qualitative analysis of system behavior. Even for most simple logistic dynamical system consisting of a wholesale store of capacity A, a retail store of capacity R and automobiles which are taking part in goods delivery from a wholesale store to a retail store the author of paper [1] by means of imitation modeling succeeded in finding such a complex mode of the operation as limit cycles and other irregular attractors. But in reality any transport logistics model is dependent at random demand and operates at random environment. Besides, a time moment for restocking of goods also is a random value. This means that for quantitative analysis for goods growth we have to calculate not only given by deterministic dynamical system stock level of goods bet also to estimate possible random deviations on these idealized representations. To do this in our

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paper we consider some complicate proposed in [1] deterministic model assuming that the demand be random and coming at random time moments. The expressed in paper [1] mathematical model for dynamical analysis of the above transport logistics scheme is system of three-dimensional ordinary differential equations.

$$\begin{cases} \dfrac{d}{dt}x(t) = kz(t)(R - y(t)), \\ \dfrac{d}{dt}y(t) = aR^{-1}x(t)(R - y(t)) - by(t), \\ \dfrac{d}{dt}z(t) = -hR^{-1}x(t)(R - y(t)) + cA^{-1}(A - z(t)). \end{cases} \tag{1}$$

with right part, that dependent on the number of involving in goods delivery transport $x(t)$ and stock levels of goods $y(t)$ and $z(t)$ in the corresponding stores. This model constructed under assumption that for any $t \geq 0$:

- The increments $\Delta x(t) := x(t + \Delta) - x(t)$ of involving in goods delivery number of trucks are proportional to Δ multiplied by to stock $z(t)$ at wholesale store and a number of vacancies $\mathrm{R} - y(t)$ at retail store
- The increments $\Delta y(t) := y(t + \Delta) - y(t)$ of stock levels of goods are proportional to Δ multiplied by involving in goods delivery number of trucks $x(t)$, a number of vacancies $\mathrm{R} - y(t)$ at retail store, after deduction of ordering for goods $by(t)\Delta$
- The increments $\Delta z(t) := z(t + \Delta) - z(t)$ of stock levels of goods are proportional to Δ multiplied by $x(t)$, a number of vacancies $\mathrm{A} - z(t)$ at wholesale store, after deduction of goods transportable from wholesale store to retail store.

To take into account random properties of demand for goods we have to model a demand at the time interval $[t, t + \Delta)$ as a random variable that can arrive or not with dependent on interval length probability. That is why we propose for dynamical analysis of the above logistic transportation scheme a stochastic model given by following finite-difference approximation:

$$\begin{aligned} x_\varepsilon(t_{k+1}) &= x_\varepsilon(t_k) + kz_\varepsilon(t_k)(R - y_\varepsilon(t_k))\Delta, \\ y_\varepsilon(t_{k+1}) &= y_\varepsilon(t_k) + aR^{-1}x_\varepsilon(t_k)(R - y_\varepsilon(t_k))\Delta - \Delta_y(t_k), \\ z_\varepsilon(t_{k+1}) &= z_\varepsilon(t_k) - hR^{-1}x_\varepsilon(t_k)(R - y_\varepsilon(t_k))\Delta + cA^{-1}(A - z_\varepsilon(t_k)), \end{aligned} \tag{2}$$

where $t_k = k\Delta, k \in N$, ε is a small positive parameter, and $\Delta_y(t_k)$ is a random sequence defined by dependent on two identically independent distributed (i.i.d.) independent uniform R(0,1) distributed series $\{\xi_k, k \in N\}$ and exponentially distributed with parameter $\varepsilon^{-1}\Delta$ series $\{\eta_k, k \in N\}$ as follows:

$$\begin{aligned} &P(\Delta_y(t_k) = 0 / \Delta < \eta_k) = 1 - e^{-\varepsilon^{-1}\Delta}, \\ &P(\Delta_y(t_k) = \varepsilon b(u) y_\varepsilon(t_k) / \Delta \geq \eta_k, \xi_k = u) = e^{-\varepsilon^{-1}\Delta}, \\ &k \in N, u \in [0,1]. \end{aligned} \tag{3}$$

This means that there are random time moments $\{\tau_k, k \in N\}$ when the trajectory for stock levels of goods $y_\varepsilon(t)$ has small jumps $\varepsilon b(\xi_k) y_\varepsilon(t_k)$ bet these jumps occur very close: $\forall k \in \mathrm{N} : \mathrm{E}\{\tau_{k+1} - \tau_k\} = \varepsilon$. The sample trajectories for equations (2) - (3) with parameters $\varepsilon = 0.01, \Delta = 0.001, k = 1, h = 1, R = 10, \mathrm{c} = 1,$ and trajectories for solutions of equitation (1) with initial conditions $x_\varepsilon(0) = x(0) = 2, y_\varepsilon(0) = y(0) = 2, z_\varepsilon(0) = z(0) = 2$ are shown at the Fig. 1. As we can see most dependent on random demand are dynamics for stock levels of goods $y_\varepsilon(t)$. At the next sections applying the stochastic averaging method [3] we derive approximate solution for (2) - (3) as a three dimensional Gaussian process and discuss a behaviour of mean value and variances for stock levels of goods $\{z_\varepsilon(t), y_\varepsilon(t), z_\varepsilon(t)\}$.

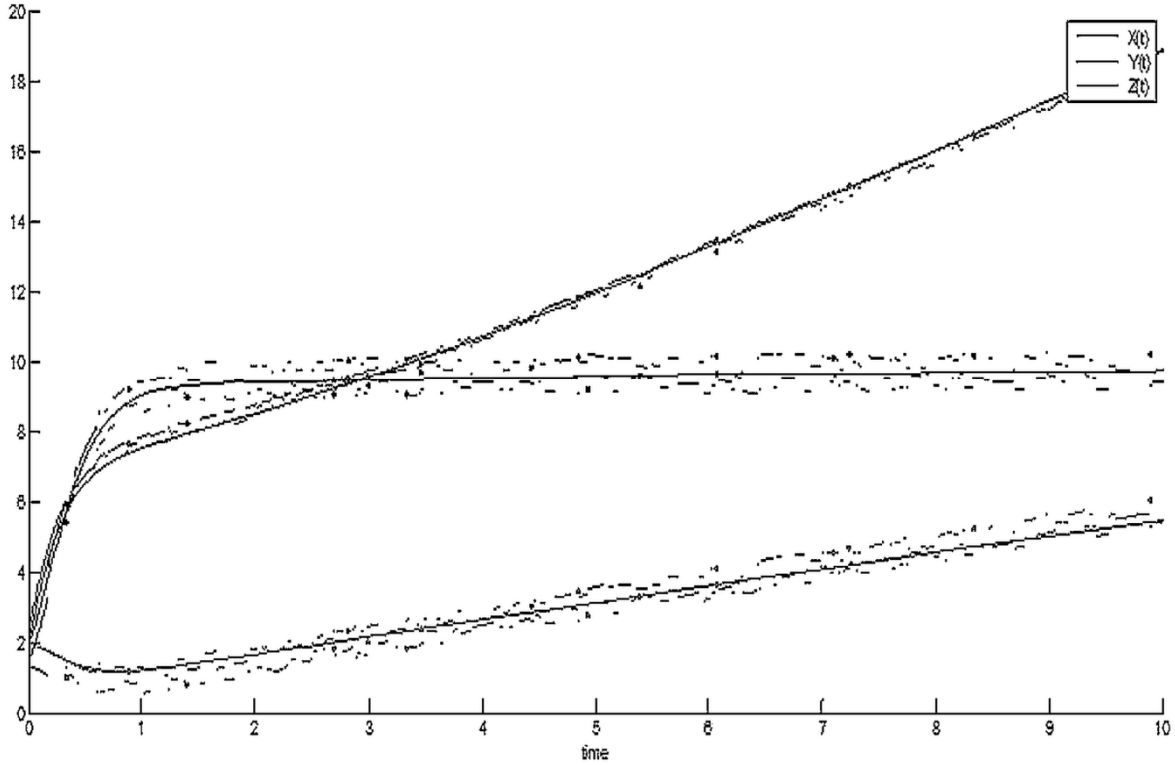

Fig. 1.Sample trajectory for (1) (unbroken line) and corresponding solutions of (2) - (3) (broken lines).

2. Diffusion approximation procedure

The defined in previous section stochastic dynamical system in more general form has been analysed in our previous paper [2]. The corresponding to finite-difference equation (2) - (3) random process possess Markov property and may be analysed through intermediary of generator [3]:

$$\mathsf{L}(\varepsilon)v(x,y,z) := \lim_{\Delta\to 0}\frac{1}{\Delta}\mathrm{E}\{v(x_\varepsilon(t_k+\Delta), y_\varepsilon(t_k+\Delta), z_\varepsilon(t_k+\Delta)) - v(x,y,z)\big|_{x_\varepsilon(t_k)=x, y_\varepsilon(t_k)=y, z_\varepsilon(t_k)=z}\} =$$

$$= \left\{ kz(\mathrm{R}-y)\frac{\partial}{\partial x} + a\,\mathrm{R}^{-1}x(\mathrm{R}-y)\frac{\partial}{\partial y} + [c\mathrm{A}^{-1}(\mathrm{A}-z) - h\,\mathrm{R}^{-1}x(\mathrm{R}-y)]\frac{\partial}{\partial z}\right\} v(x,y,z) + \tag{4}$$

$$+\frac{1}{\varepsilon}\int_0^1 [v(x, y+\varepsilon b(u)y, z) - v(x,y,z)]du$$

where $v(x,y,z)$ is an arbitrary sufficiently smooth bounded function . Now we have to derive a limit $\lim_{\varepsilon\to 0}\mathsf{L}(\varepsilon)v(x,y,z) := \mathsf{L}\,v(x,y,z)$, where

$$\mathsf{L} = kz(\mathrm{R}-y)\frac{\partial}{\partial x} + \left[a\,\mathrm{R}^{-1}x(\mathrm{R}-y) - by\right]\frac{\partial}{\partial y} + [c\mathrm{A}^{-1}(\mathrm{A}-z) - h\,\mathrm{R}^{-1}x(\mathrm{R}-y)]\frac{\partial}{\partial z} \tag{5}$$

and $b = \int_0^1 b(u)du$. The operator (5) can be interpreted as an infinitesimal operator [9] for defined by system (1) continuous semigroup.

Therefore [2] for sufficiently small $\varepsilon > 0$ sample trajectories of defined by finite-difference equation (2) - (3) random dynamical system we can approximate by corresponding solutions of equation (1), that is, if $x_\varepsilon(0) = x(0), y_\varepsilon(0) = y(0), z_\varepsilon(0) = z(0),$ then for any $T > 0$:

$$\mathrm{P}\left\{\lim_{\varepsilon\to 0}\sup_{0\le t\le T}[|\,x_\varepsilon(t) - x(t)\,| + |\,y_\varepsilon(t) - y(t)\,| + |\,z_\varepsilon(t) - z(t)\,|] = 0\right\} = 1. \tag{6}$$

As it has been proven in [2] the deviations of solutions (2) - (3) on corresponding solutions of (1) have an order $\sqrt{\varepsilon}$ and we may analyse these deviations applying diffusion approximation procedure to no homogeneous three dimensional Markov process:

$$X_\varepsilon(t) = \frac{x_\varepsilon(t) - x(t)}{\sqrt{\varepsilon}}, Y_\varepsilon(t) = \frac{y_\varepsilon(t) - y(t)}{\sqrt{\varepsilon}}, Z_\varepsilon(t) = \frac{z_\varepsilon(t) - z(t)}{\sqrt{\varepsilon}} \tag{7}$$

with zero initial conditions. The same as before we should derive a generator for (7):

$$\Lambda(\varepsilon)v(X,Y,Z) := \lim_{\Delta\to 0}\frac{1}{\Delta}\mathrm{E}\{v(X_\varepsilon(t_k+\Delta), Y_\varepsilon(t_k+\Delta), Z_\varepsilon(t_k+\Delta)) - v(X,Y,Z)\big|_{X_\varepsilon(t_k)=X, Y_\varepsilon(t_k)=Y, Z_\varepsilon(t_k)=Z}\}$$

and pass to limit $\lim_{\varepsilon\to 0}\Lambda(\varepsilon)v(X,Y,Z) := \Lambda v(\mathrm{X},Y,Z)$, where

$$\Lambda v(\mathrm{X},Y,Z)=\Big\{[-kz(t)Y+(\mathrm{R}-y(t))Z]\frac{\partial}{\partial X}+$$
$$+[-a\,\mathrm{R}^{-1}x(t)Y+a\,\mathrm{R}^{-1}X(\mathrm{R}-y(t))]\frac{\partial}{\partial Y}+\frac{1}{2}\beta^2y^2(t)\frac{d^2}{dY^2}+ \tag{8}$$
$$+[-c\mathrm{A}^{-1}Z+h\,\mathrm{R}^{-1}x(t)Y-X(\mathrm{R}-y(t))]\frac{\partial}{\partial Z}\Big\}v(\mathrm{X},Y,Z)$$

and $\beta^2=\int_0^1 b^2(u)du$. This operator can be interpreted [3] as a generator for no homogeneous Markov process $\{X(t),Y(t),Z(t),t\ge 0\}$ which satisfies to systems of two ordinary equations and one stochastic Ito equation:

$$\frac{d}{dt}X(t)=-kz(t)Y(t)+(\mathrm{R}-y(t))Z(t), \tag{9}$$
$$dY(t)=[-a\,\mathrm{R}^{-1}x(t)Y(t)+a\mathrm{R}^{-1}X(t)(\mathrm{R}-y(t))]dt+\beta y(t)dw(t) \tag{10}$$
$$\frac{d}{dt}Z(t)=-c\mathrm{A}^{-1}Z(t)+h\mathrm{R}^{-1}x(t)Y(t)-X(t)(\mathrm{R}-y(t)), \tag{11}$$

with initial conditions $\{X(0)=0,Y(0)=0,Z(0)=0\}$. As it has been proved in [2] finite dimensional distributions of the defined by equations (2) - (3) Markov process $\{x_\varepsilon(t),y_\varepsilon(t),z_\varepsilon(t)\}$ may be approximated by corresponding finite dimensional distributions of the process

$$\bar{x}_\varepsilon(t)=x(t)+\sqrt{\varepsilon}X(t),\ \bar{y}_\varepsilon(t)=y(t)+\sqrt{\varepsilon}Y(t),\ \bar{z}_\varepsilon(t)=z(t)+\sqrt{\varepsilon}Z(t). \tag{12}$$

Unfortunately, we cannot analyse variance separately approximation for stock levels of goods given by equation (10). We have to derive and solve the system of differential equations for all elements of a covariance matrix for the three dimensional Gaussian random vector $\{X(t),Y(t),Z(t)\}$:

$$\frac{d}{dt}q_{XX}(t)=-2kz(t)q_{XY}(t)+2(\mathrm{R}-y(t))q_{XZ}(t),$$
$$\frac{d}{dt}q_{XY}(t)=-kz(t)q_{YY}(t)+(\mathrm{R}-y(t))q_{YZ}(t)-a\,\mathrm{R}^{-1}x(t)q_{XY}(t)+a\,\mathrm{R}^{-1}q_{XX}(t)(\mathrm{R}-y(t)),$$
$$\frac{d}{dt}q_{XZ}(t)=-kz(t)q_{YZ}(t)+(\mathrm{R}-y(t))q_{ZZ}(t)-c\mathrm{A}^{-1}q_{XZ}(t)+h\,\mathrm{R}^{-1}x(t)q_{XY}(t)-q_{XX}(t)(\mathrm{R}-y(t)),$$
$$\frac{d}{dt}q_{YY}(t)=-2a\,\mathrm{R}^{-1}x(t)q_{YY}(t)+2a\,\mathrm{R}^{-1}q_{XY}(t)(\mathrm{R}-y(t))+\beta^2y^2(t), \tag{13}$$
$$\frac{d}{dt}q_{YZ}(t)=-(a\,\mathrm{R}^{-1}x(t)+c\mathrm{A}^{-1})q_{YZ}(t)+a\,\mathrm{R}^{-1}q_{XZ}(t)(\mathrm{R}-y(t))+h\,\mathrm{R}^{-1}x(t)q_{YY}(t)-q_{XY}(t)(\mathrm{R}-y(t)),$$
$$\frac{d}{dt}q_{ZZ}(t)=-2c\mathrm{A}^{-1}q_{ZZ}(t)+2h\,\mathrm{R}^{-1}x(t)q_{YZ}(t)-2q_{XZ}(t)(\mathrm{R}-y(t)),$$

with zero initial conditions. Applying the Runge-Kutta method for solution of equations (13) we can calculate approximation for covariance matrix for stock levels of goods for $A = 100, b(u) = 2bu$, $\varepsilon = 0.01, h = 1, R = 10, \mathrm{c} = 1, a = 5$ with the same parameters in (1)-(2)-(3) and initial values as for the Fig.1.

3. Conclusion

The proposed stochastic model for transport logistics in a form of nonlinear difference-differential equations with stochastic Poisson type increments in tandem with stochastic approximation procedure permits to take into account random character of demand for freight services and to supplement the classical deterministic analysis with Gaussian approximation for possible random deviations.

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