

INFORMATION TECHNOLOGY AND
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INFORMĀCIJAS TEHNOLOĢIJA UN
VADĪBAS ZINĀTNESTATISTICAL DESCRIPTION OF A DISTRIBUTION OF POPULATION DENSITY
OVER THE LATVIAN TERRITORY

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1. Introduction

A number of vital practical problems require the description of a distribution of population over the considered territory. In this connection we have set the task to describe in an analytical form the available data on the population of Latvia.

In Tables 1 and 2 the data about the population and the square of Latvia on cities and areas is represented, as well as their coordinates. Data was offered to us by Central Statistical Bureau of Latvia. In Figure 1 the map of population density of Latvia is shown. Information source is the Latvian Association of Local and Regional Governments. The different colours correspond with different allocation in the given region.

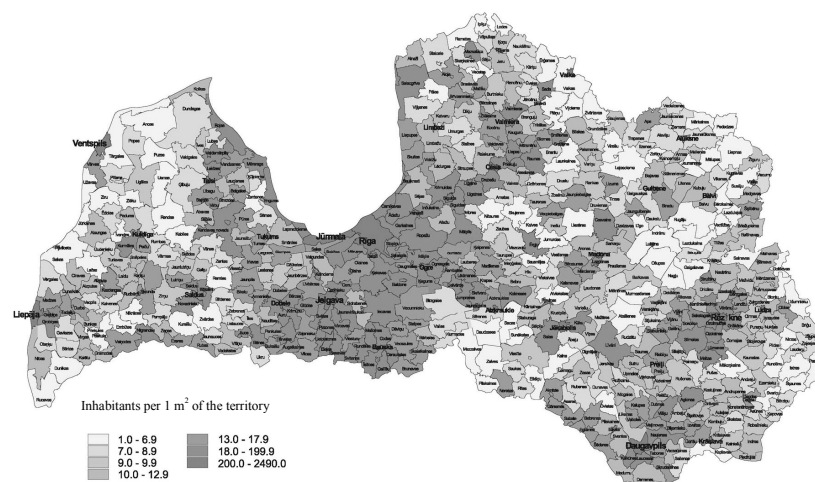


Figure 1. Population density in cities and areas

Table 1

Characteristics of cities

<i>City</i>	<i>Index</i>	<i>Coordinate</i> <i>x</i>	<i>Coordinate</i> <i>y</i>	<i>Population</i> <i>(Ĥ)</i>	<i>Square, km²</i> <i>(P)</i>
Rīga	j1	190	150	722 485	307
Daugavpils	j2	340	30	108 091	73
Jelgava	j3	170	110	66 051	60
Jūrmala	j4	150	180	55 408	100
Liepāja	j5	10	100	85 477	60
Rēzekne	j6	390	100	36 345	18
Ventspils	j7	50	200	43 544	55

Table 2

Characteristics of areas

<i>Area</i>	<i>Index</i>	<i>Coordinate</i> <i>x</i>	<i>Coordinate</i> <i>y</i>	<i>Population</i> <i>(Ĥ)</i>	<i>Square, km²</i> <i>(P)</i>
Aizkraukles rajons	i1	270	110	40 116	2 567
Alūksnes rajons	i2	360	210	24 483	2 245
Balvu rajons	i3	390	170	27 245	2 381
Bauskas rajons	i4	210	100	50 988	1 881
Cēsu rajons	i5	270	180	56 614	2 973
Daugavpils rajons	i6	340	40	39 496	2 526
Dobeles rajons	i7	130	100	37 980	1 632
Gulbenes rajons	i8	340	180	26 281	1 876
Jelgavas rajons	i9	170	110	36 941	1 605
Jēkabpils rajons	i10	300	90	52 593	2 997
Krāslavas rajons	i11	400	50	33 313	2 288
Kuldīgas rajons	i12	60	150	35 822	2 500
Liepājas rajons	i13	30	110	43 849	3 593
Limbažu rajons	i14	230	230	37 798	2 602
Ludzas rajons	i15	420	110	31 305	2 412
Madonas rajons	i16	330	140	42 918	3 349
Ogres rajons	i17	240	130	64 060	1 843
Preiļu rajons	i18	340	80	38 317	2 042
Rēzeknes rajons	i19	380	100	40 442	2 809
Rīgas rajons	i20	210	150	161 119	3 132
Saldus rajons	i21	50	110	36 735	2 182
Talsu rajons	i22	100	200	46 680	2 748
Tukuma rajons	i23	130	150	54 813	2 457
Valkas rajons	i24	300	220	31 723	2 441
Valmieras rajons	i25	260	240	58 328	2 373
Ventspils rajons	i26	60	190	13 945	2 462

Our problem consists in working out the analytical dependences, allowing one to calculate corresponding population density for the specified coordinate of the territory of Latvia. On this basis the following problem is solved: the weighted average centre of Latvia is

judging by a condition that the square of the circle and area coincide, and their joint part is maximal.

For the considered case, function $\xi_i(x, y)$ is described as follows:

$$\xi_i(x,y)= \begin{cases} 1, & \text{if } (y_i - r_i < y < y_i + r_i) \wedge [x_i - \sqrt{(r_i)^2 - (y - y_i)^2} < x < x_i + \sqrt{(r_i)^2 - (y - y_i)^2}] \\ 0, & \text{otherwise.} \end{cases}$$

We assume that population of area is distributed uniformly over its territory.

2.2. The population of big cities

Let j be a city index, $j = 1, 2, \dots, m$, where m is the number of considered big cities ($m = 7$). Let's denote:

SP_j is the territory of the j -th city;

$\psi_j(x, y)$ is an indicator function of the j -th city:

$$\psi_j(x,y)= \begin{cases} 1, & \text{if } (x,y) \in SP_j, \\ 0, & \text{otherwise;} \end{cases}$$

For the considered case the function $\psi_j(x, y)$ is described by a formula:

$$\psi_j(x,y)= \begin{cases} 1, & \text{if } (y_j - r_j < y < y_j + r_j) \wedge [x_j - \sqrt{(r_j)^2 - (y - y_j)^2} < x < x_j + \sqrt{(r_j)^2 - (y - y_j)^2}] \\ 0, & \text{otherwise;} \end{cases}$$

H_j is the population density for the j -th city.

The population densities of the cities calculated by formula $H_j = \hat{H}/P$, are presented in Table 3.

The density of distribution of the population of the j -th city over the territory will be described by two-dimensional normal density:

$$f^{(p)}(x,y) = \frac{H_j}{2\pi\sigma_j^2} \exp\left\{-\frac{1}{2\sigma_j^2} [(x - \mu_{1,j})^2 + (y - \mu_{2,j})^2]\right\}, \quad (2)$$

where $(\mu_{1,j}, \mu_{2,j})$ is coordinates of the centre of the j -th city, σ_j is the standard deviation.

The parameter σ_j is calculated by the formula:

$$\sigma_j = \rho r_j, \quad (3)$$

where $\rho > 0,5$ is a coefficient of city's attraction (in our case let $\rho = 0,7$),

r_j is a radius of the j -th city, calculated by formula

$$r_j = \sqrt{P_j/\pi}, \quad (4)$$

where P_j is square of the j -th city (see Table 1).

The main characteristics of the areas and the cities are shown in Table 3.

Definitive expression for population density of Latvia at point (x, y) will be

$$f(x, y) = f^{(r)}(x, y) + \sum_{j=1}^m \psi_j(x, y) f^{(p)}(x, y). \quad (5)$$

Table 3

Population density, radius of city/area, and standard deviation for the cities

City	Density	Radius	σ_j
Rīga	2353,37	9,89	6,92
Daugavpils	1480,70	4,82	3,37
Jelgava	1100,85	4,37	3,06
Jūrmala	554,08	5,64	3,95
Liepāja	1424,62	4,37	3,06
Rēzekne	2019,17	2,39	1,68
Ventspils	791,71	4,18	2,93
Aizkraukles rajons	15,63	28,59	-
Alūksnes rajons	10,91	26,73	-
Balvu rajons	11,44	27,53	-
Bauskas rajons	27,11	24,47	-
Cēsu rajons	19,04	30,76	-
Daugavpils rajons	15,64	28,36	-
Dobeles rajons	23,27	22,79	-
Gulbenes rajons	14,01	24,44	-
Jelgavas rajons	23,02	22,60	-
Jēkabpils rajons	17,55	30,89	-
Krāslavas rajons	14,56	26,99	-
Kuldīgas rajons	14,33	28,21	-
Liepājas rajons	12,20	33,82	-
Limbažu rajons	14,53	28,78	-
Ludzas rajons	12,98	27,71	-
Madonas rajons	12,82	32,65	-
Ogres rajons	34,76	24,22	-
Preiļu rajons	18,76	25,50	-
Rēzeknes rajons	14,40	29,90	-
Rīgas rajons	51,44	31,57	-
Saldus rajons	16,84	26,35	-
Talsu rajons	16,99	29,58	-
Tukuma rajons	22,31	27,97	-
Valkas rajons	13,00	27,88	-
Valmieras rajons	24,58	27,48	-
Ventspils rajons	5,66	27,99	-

3. Weighted average coordinates of Latvia

The weighted average coordinate of Latvia is a point (u, v) for which the following condition is satisfied: if all inhabitants of Latvia have to gather at one point, then (u, v) is a point, for which the total distance passed by all, will be minimum.

For simplification, we will consider all population of the j -th city concentrated in coordinate of its centre $(\mu_{1,j}, \mu_{2,j})$. This is a reasonable assumption, because the squares of cities are essentially less than squares of areas.

Now mathematically the problem is formed in this way:

to find a point (u, v) which minimizes objective function

$$g(u, v) = \sum_{i=1}^k h_i \int_{y_i-r_i}^{y_i+r_i} \int_{x_i-\sqrt{r_i^2-(\omega-y_i)^2}}^{x_i+\sqrt{r_i^2-(\omega-y_i)^2}} \sqrt{(z-u)^2 + (\omega-v)^2} dz d\omega + \sum_{j=1}^m \sqrt{(\mu_{1,j}-u)^2 + (\mu_{2,j}-v)^2} \hat{H}_j. \quad (6)$$

It is possible to find the minimum point (u, v) by a gradient method. Gradient of function (6) is given by expression:

$$\nabla g(u, v) = \begin{pmatrix} \frac{\partial}{\partial u} g(u, v) \\ \frac{\partial}{\partial v} g(u, v) \end{pmatrix} = \begin{pmatrix} -\sum_{i=1}^k h_i \int_{y_i-r_i}^{y_i+r_i} \int_{x_i-\sqrt{r_i^2-(\omega-y_i)^2}}^{x_i+\sqrt{r_i^2-(\omega-y_i)^2}} \left[(z-u)^2 + (\omega-v)^2 \right]^{\frac{1}{2}} (z-u) dz d\omega - \sum_{j=1}^m \left[(\mu_{1,j}-u)^2 + (\mu_{2,j}-v)^2 \right]^{\frac{1}{2}} (\mu_{1,j}-u) \hat{H}_j \\ -\sum_{i=1}^k h_i \int_{y_i-r_i}^{y_i+r_i} \int_{x_i-\sqrt{r_i^2-(\omega-y_i)^2}}^{x_i+\sqrt{r_i^2-(\omega-y_i)^2}} \left[(z-u)^2 + (\omega-v)^2 \right]^{\frac{1}{2}} (\omega-v) dz d\omega - \sum_{j=1}^m \left[(\mu_{1,j}-u)^2 + (\mu_{2,j}-v)^2 \right]^{\frac{1}{2}} (\mu_{2,j}-v) \hat{H}_j \end{pmatrix} \quad (7)$$

Note, that we must apply gradient method carefully, because our objective function has not continuous on borders of areas. Therefore we have included some changes in the standard algorithm of a gradient method. As we cannot trust a gradient method completely, we have also checked it up by means of Mathcad minimization procedures. Experiments have shown the efficiency of the method at a small step. Further we will describe the program used to solve the problem.

4. The software environment of the problem and numerical results

Special software in Mathcad 14 has been developed. The complex of the programmes allows calculating the population density of Latvia for the given coordinate (x, y) . For example, for some coordinates (x, y) these densities are given in Table 4.

Table 4

Population densities for coordinates (x,y) and values of the objective function (6)

Coordinate, (x,y)	190, 150	360, 80	320, 160	40, 100	100, 180
Density, $f(x,y)$	52.216	33.16	12.82	29.04	16.99
Objective function, $g(u, v)$	$1.856 * 10^8$	$3.946 * 10^8$	$3.184 * 10^8$	$4.284 * 10^8$	$3.228 * 10^8$

Thus, we can see that the weakest coordinate on density is (320, 160). It is situated on the borders of Madona and Gulbene districts.

Further, the programmes allow one to calculate values of the objective function for different coordinates (u, v) . The Table 4 contains such values for some coordinates.

This complex of programs allows defining the weighted average coordinate of Latvia by using a gradient method too. Calculations have shown that the weighted average coordinate of Latvia is (189.94, 149.80). These coordinates correspond to the city of Riga.

5. Conclusions

The paper results allow calculating population density for the specified coordinate of Latvia territory. It can be used for solving many practical problems. For example, the problem of weighted average centre of Latvia was considered. The necessary software was elaborated. Experiments show that the suggested algorithms and software perform well.

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Kašurins Andrejs. Iedzīvotāju blīvuma sadalījuma statistiskais apraksts Latvijas teritorijā

Ļoti daudz praktiski uzdevumi prasa iedzīvotāju izvietojuma teritorijā apskatu. Tika apskatīts iedzīvotāju izvietojanas Latvijas teritorijā datu analītiskā apraksta uzdevums. Lietderīgi lielu pilsētu un rajonu iedzīvotāju skaitu izskatīt atsevišķi. Mūsu aprēķina nosacījumos ir pieņemts katru rajonu aprakstīt ar apli. Apļa rādiuss

aprēķina pēc nosacījumiem, ka apļa laukums sakrīt ar rajona laukumi un viņu kopējā sakritība maksimāla. Tiek pieņemts ka iedzīvotāju blīvums visā rajonā ir vienmērīgs. Pilsētas teritorijas iedzīvotāju blīvums aprakstīts ar divdimensiju vienmērīgu blīvumu. Iegūta izteiksme, kas noteiktām Latvijas teritorijas koordinātēm ļauj aprēķināt atbilstošu iedzīvotāju blīvumu. Pamatojoties uz to, ir risināts sekojošs uzdevums: tiek atrasts vidēji svērtais Latvijas centrs, t.i., tādas koordinātes, līdz kurām ir minimāls summārais attālums no visiem Latvijas punktiem (ņemot vērā iedzīvotāju blīvumu). Minimālā punkta meklēšana tiek veikta ar gradienta metodi. Tika apskatīts skaitliskais piemērs. Izstrādājot speciālas programmas nodrošinājumu Mathcad valodā. Izstrādātais programmu komplekss ļauj izskatīt Latvijas iedzīvotāju blīvumu dotajām koordinātēm, kā arī noteikt Latvijas centra vidēji svērtas koordinātes.

Kashurin Andrey. Statistical description of a distribution of population density over the Latvian territory

A number of vital practical problems require the description of a distribution of population over the considered territory. The problem of the analytical description of the available data on population allocation over the territory of Latvia is examined. It is reasonable to examine the population of big cities and areas separately. We assume that every district is represented by a circle. The circle radius is calculated judging by a condition that the square of the circle and area coincide, and their joint part is maximal. We assume that population of the area is distributed uniformly over its territory. The density of distribution of the population of city over the territory is described by two-dimensional normal density. The expression making it possible to calculate corresponding population density for the specified coordinates of the Latvian territory is received. On the basis of it the following solution is made: the weighted average centre of Latvia is found, i.e. coordinates, to which the total distance from all points of Latvia (subject to density) is minimal. It is possible to find the minimal point using gradient method. Special software has been developed in Mathcad 14. According to the task description, the following functions were implemented - calculation of population density by given coordinates and, detection of the weighted average coordinates of Latvia using gradient method.

Кашурин Андрей. Статистическое описание распределения плотности населения на территории Латвии

Многие важные практические задачи требуют описания распределения населения на рассматриваемой территории. В данной статье рассмотрена задача аналитического описания имеющихся данных по распределению населения по территории Латвии. Целесообразно население крупных городов и районов рассматривать по отдельности. Предполагаем, что каждый район представлен кругом. Радиус круга вычисляется из условия, что площадь круга и района совпадают, а их общая часть максимальна. Принимаем, что плотность населения района распределяется равномерно по его территории. Плотность распределения населения города по территории описана двумерной нормальной плотностью. Получено выражение, позволяющее для заданных координат территории Латвии вычислять соответствующую плотность населения. На этой основе решается следующая задача: находится средневзвешенный центр Латвии, то есть такие координаты, для которых суммарное расстояние от всех точек Латвии (с учетом плотности населения) минимально. Нахождение минимальной точки осуществляется градиентным методом. Разработано специальное программное обеспечение на языке Mathcad. Разработанный комплекс программ позволяет вычислять плотность населения Латвии для заданных координат, а также определять средневзвешенные координаты центра Латвии.