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STOCHASTIC MODELING OF ANIMAL POPULATION DYNAMICS

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The most popular classical mathematical model for biological pest control may be given as a system of ordinary differential equations (see, for example, in A. D. Bazykin at all [1] and Yu. M. Svirezhef and D. I. Logofet [2] system of the equations (1)):

$$\begin{cases} \frac{dx}{dt} = xf(x) - g(x,y) \\ \frac{dy}{dt} = yh(y) + m(x,y), \end{cases} \tag{1}$$

where x is a density of a pest (or prey) population and y is a density of a predator population, functions $f(x)$ and $h(y)$ define the relative growth rates of populations without contacts, functions $g(x,y)$ and $m(x,y)$ are changes of populations, growth rates caused by a cooperative effect, called functional responses. In reality, even through all functions of the right part in (1) are derived in compliance with biological laws, the parameters of these functions are random and should be estimated by collecting and analyzing environmental data. Therefore, several papers propose and analyze mathematical models for interacting populations in a form of stochastic Ito differential equations (see, for example, the book of J. Murray [3] and references there):

$$\begin{cases} dx(t) = \big[x(t)f\big(x(t)\big) - g\big(x(t),y(t)\big)\big]dt + \sigma_1\big(x(t),y(t)\big)dw_1(t) \\ dy(t) = \big[y(t)h\big(y(t)\big) + m\big(x(t),y(t)\big)\big]dt + \sigma_2\big(x(t),y(t)\big)dw_2(t), \end{cases} \tag{2}$$

where $w_1(t)$ and $w_2(t)$ are correlated Wiener processes given on probability space $(\Omega, \mathfrak{F}, \mathbb{P})$.

The paper deals with population mathematical models given in a form of fast oscillating stochastic impulsive differential equations. The proposed approximative algorithm for quantitative analysis of population dynamics consists of two steps. First, we construct an ordinary differential equation for the mean value of population growth and analyse the average asymptotic population behaviour. Then, applying diffusion approximation procedure, we derive the stochastic Ito differential equation for random deviations on the above average motion and analyse probabilistic characteristics of possible stationary population state. The algorithm is applied to the model of biological pest control of Holling type II.

REFERENCES

[1] A. D. Bazykin, A. I. Rhibnik, B. Krauskopf. *Nonlinear Dynamics of Interacting Populations.* World Scientific, Singapore, 1998.

[2] Yu. M. Svirezhev, D. J. Logofet. *Stability of biological communities.* Moscow, MIR Publishers , 1983.

[3] J. Murray. *Mathematical Biology I: An Introduction* . Volume I. Springer-Verlag, 3rd edition, 2003.