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OPTIMAL DESIGN OF FIBERGLASS PANELS WITH PHYSICAL VALIDATION

STIKLPLASTA PANEĻU OPTIMĀLA PROJEKTĒŠANA AR EKSPERIMENTĀLU VALIDĀCIJU

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Introduction

Fibre reinforced composites have many physical properties that makes them increasingly attractive in structural design applications, however manufacturing costs and quality robustness are issues which should be further improved. Traditionally, composite manufacturing methods are very labour intensive, thus development of automated manufacturing processes can reduce the manufacturing costs and increase quantities of finished products. To meet demand for diverge applications cases manufactured products should be optimized a priory. Common optimum design practice of stiffened structures under any loading combinations should involve extensive finite element analysis with complimentary experimental validation. However, such a procedure is only partly efficient as besides trade-off design the designer is seeking for alternatives in the overall perspective. Despite the advances in rapidly growing computational capacity, as in High Performing Computing or in GRID technologies, the enormous computational cost of complex engineering simulations makes it impractical to rely exclusively on simulation for the purpose of design optimization [1]. As a good practice, one could use mathematical approximations instead of full-scale analyses, thus reducing the level of numerical optimization complexity. Metamodels, also called surrogate models, are constructed from response approximations extracted from actual simulation models. In particular, for determination of the most suitable metamodeling technique fitting to deck under the bending load design procedure a different parametric and non-parametric approximations have been compared – low order global polynomials, locally weighted polynomials, partial polynomials, and Multivariate Adaptive Regression Splines.

Important research issue associated with metamodeling is how to achieve good accuracy of a metamodel with reasonable number of sample points. The sampling techniques, often referred to as design of experiments, should be implemented to reduce the number of simulation runs without decreasing the accuracy of the metamodel. The differences between sampling strategies for physical experiments and for computer experiments should be noted. Whilst physical experiments have statistical experimental errors, numerical analyses are deterministic and results are obtained with

100% repetition and no statistical variance of model parameters [2,3,4]. Currently, there is a wide range of literature concerning different methods for DACE [5], which include many approaches for space-filling designs. It should be noted that the first space-filling design criterion [2,3] for numerical experiments was proposed at Riga Technical University by Audze and Eglajs. While the accuracy of a metamodel is directly related to the approximation technique used and to properties of the problem itself, the type of sampling approaches6 have a direct influence on the approximation performance. It is generally accepted that space-filling designs, for example the Latin Hypercube design, are preferable for building of metamodels. In the current study, to reduce the number of computations, experimental design optimized according to Mean Square Error (MSE) criterion [6,7] has been selected.

1. Pultruded glass fiber reinforced plastic structures

One of the most efficient glass fibre reinforced plastic (GFRP) manufacturing technique could be outlined a pultrusion. The pultrusion is a continuous moulding process utilizing glass or other fibrous reinforcement in a polyester or other resin matrix. Pre-selected reinforcement materials like fibreglass roving, matt or cloth, are drawn through a resin bath where all the material is thoroughly impregnated with a liquid thermosetting resin. The wet fibrous laminate is formed to the desired geometric shape and pulled into a heated steel die. Once in the die, setting of the resin is initiated by controlling precise elevated temperatures. The laminate solidifies in the exact shape of the cavity of the die as the pultrusion machine is continuously pulling it. The haul-off equipment used is either a conventional belt puller (as used in extrusion) or a hand-over-hand reciprocating clamp type. The caterpillar type is a cheaper solution because no motion sequencing is involved but it can be expensive to produce the large number of gripper pads necessary for each profile. The caterpillar type also has the disadvantage that the gripping force cannot be isolated from the pulling force and for large profiles this can damage the profile. Continuous manufacturing process can produce the product of any span length. However, major limitations are associated with quality assurance of the slenderness of produced profiles and cross-sectional tolerances.

One of reason for utilisation of fiberglass products is high chemical corrosion, electrical isolation and low thermal resistance properties embedded in lightweight however stiff material. Most frequent application of fibeglass panels can be identified as sidewalls, roofs, floors for building structures as well as bridge decks, platforms, walkways and in other civil engineering areas. Most of the GFRP bridges constructed up to now [9] used multi-cellular pultruded deck systems. In principle, two construction forms are used: multi-cellular deck panels from adhesively bonded pultruded shapes and sandwich panels with different core structures [10,11].

In present work, design of GFRP composite stiffened panel structures has been considered similar to ones currently manufactured by Rishon-Inter.Ltd (http://www.rishon-inter.lv). Eleven I-type stiffener deck design, as shown in Figure 1, has been elaborated in parallel with experimental tests performed at Riga Technical University, Institute of Materials and Structures using dedicated test equipment. Three-point bending test case has been considered for numerical analyses and physical tests along with uniformly distributed load test case that has been elaborated only numerically.

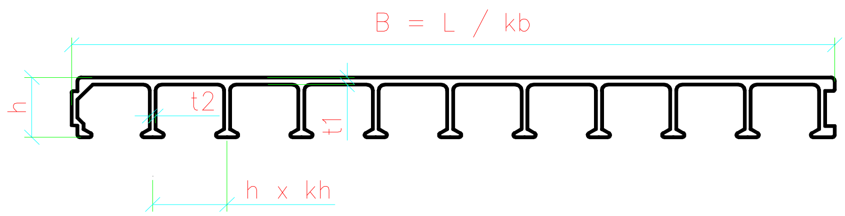

Fig.1. GFRP Deck geometry with parametrical variables

The flexural stiffness response of GFRP pultruded deck structure has been evaluated numerically by finite element method commercial software ANSYS [12] employing SHELL 181 4-node shell element. The mechanical properties for glass fiber composite used for full-scale analysis have been extracted by small coupon tests in tension and bending. Numerical values of load-deformation curve, stress, and strain distribution over the tested deck structure were extracted and incorporated in design of strain gauge locations for experimental validation. The numerical deflection and stress graphs of three-point bending test are shown in Figure 2. A comparative study between numerical and experimental results will be given in the 3. Chapter.

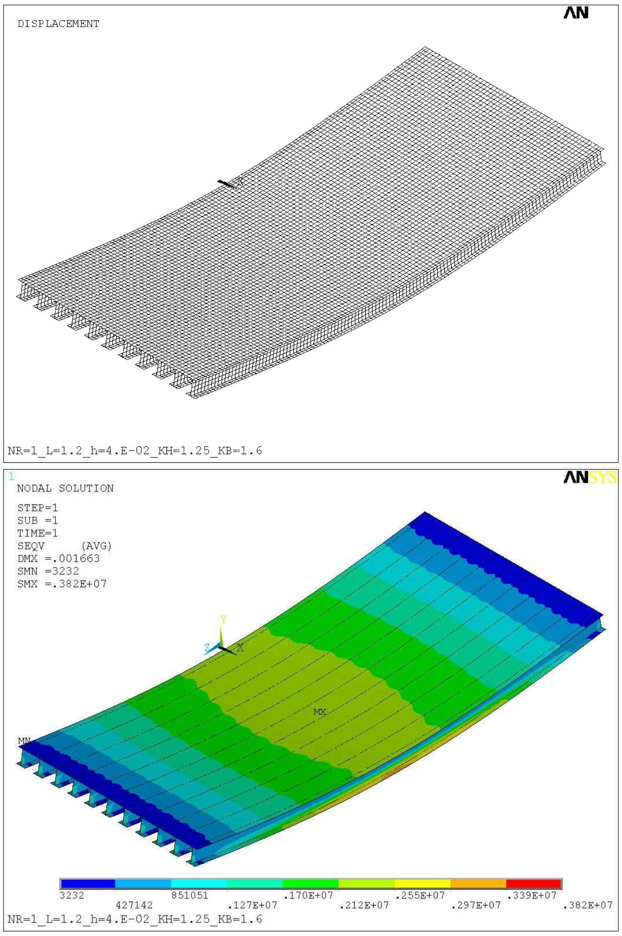

Fig.2. Three point bending numerical test – deflection (left) and stress-distribution (right) graphs

The choice of design variables should represent all geometrical parameters in optimum design procedure. However, most of those variables are rationally interconnected. Exploration of non-rational variable combination mostly leads to relatively high approximation errors. In particular, when non-proportional ratios of span and height are used for bending problems this usually causes singularity in approximations. Therefore, geometrical design variables with corresponding ratio variables have been proposed [11] for metamodeling procedure. As geometrical variables the panel length parameter L and the panel height parameter h along with two plate thicknesses have been taken: the cover plate thickness t_1 and the stiffener thickness t_2. Moreover, rational design variables as kb (ratio between the panel length and width) and kh (stiffener spacing parameter ratio between I-stiffener foot width and panel height) have been proposed. Such a procedure is required to restrain the combination where stiffener spacing is narrower than the deck height h. The numerical bounds of design variables are given in Table 1.

Table 1. Design space for deck structure

Name and notation		Lower bound	Upper bound	Units
Deck length	*L*	0.6	4	m
Deck stiffener height	*h*	0.03	0.10	m
Plate thickness	t_1	0.003	0.006	m
Stiffener thickness	t_2	0.003	0.006	m
Deck length to width ratio	*kb*	1.5	3	
Stiffener spacing ratio	*kh*	2	3	

In order to achieve the best performance (minimal prediction error of metamodels), a space-filling design of five hundred sample points optimized according to the Mean Squared Error [6,7] uniformity criterion has been selected. A numerical sampling procedure involves a large amount of analyzed numerical data that are unacceptably time demanding. Therefore, all numerical data sets have been analyzed in parallel, exploring the LatvianGrid (http://grid.lumii.lv) computing capabilities, thus reducing the necessary time and outsourcing the computational capacities.

2. Physical experiments at Riga Technical University, Institute of Materials and Structures

Three three-point bending tests until collapse of the structure for real deck panels have been performed. Adding the validation test that has been outfitted with strain-gauges and loaded until 60% of the total collapse load. The experimental setup of a one-meter span length and corresponding deflected collapse mode shape is shown in Figure 3. During the tests, the load-versus-deflection curves (outlined in Figure 4) and strains have been recorded by means of load cell and the strain gauge readings. Thus the data from physical experiments has been implemented in validation procedure of ANSYS [12] finite element model as summarized in Table 2.

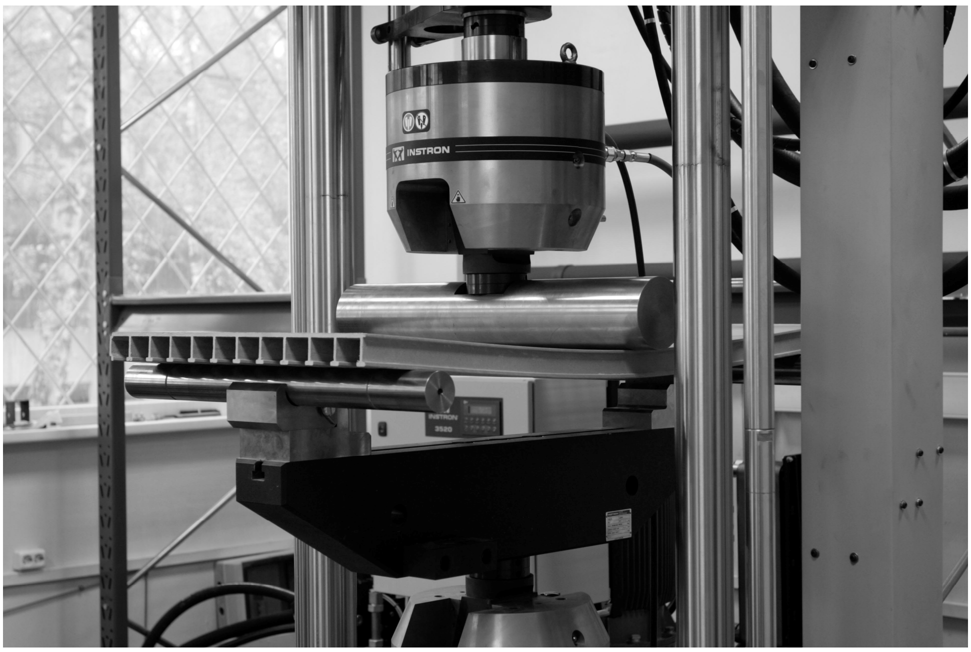

Fig.3. The experimental three-point bending test setup and collapse mode of the stiffened deck structure

The test results obtained experimentally for four tested deck structures have been compared with values obtained numerically by ANSYS [12]. One could observe from the Figure 4 and the Table 2 that all panels have practically the same loading stiffness. However, there is a certain divergence between obtained critical load levels. Nevertheless, in validation procedure the numerical deflection results have about 10% discrepancy with physical test results what should be considered as a good agreement between actual (manufactured) and numerical model. Moreover, numerical stress threshold value practically corresponds to the value obtained by small specimen tension tests. Validation procedure outlined that the load level corresponding to deflection limit legislated by building codes [1/250 to 1/150 of the deck span] are practically one fifth of the ultimate stress value.

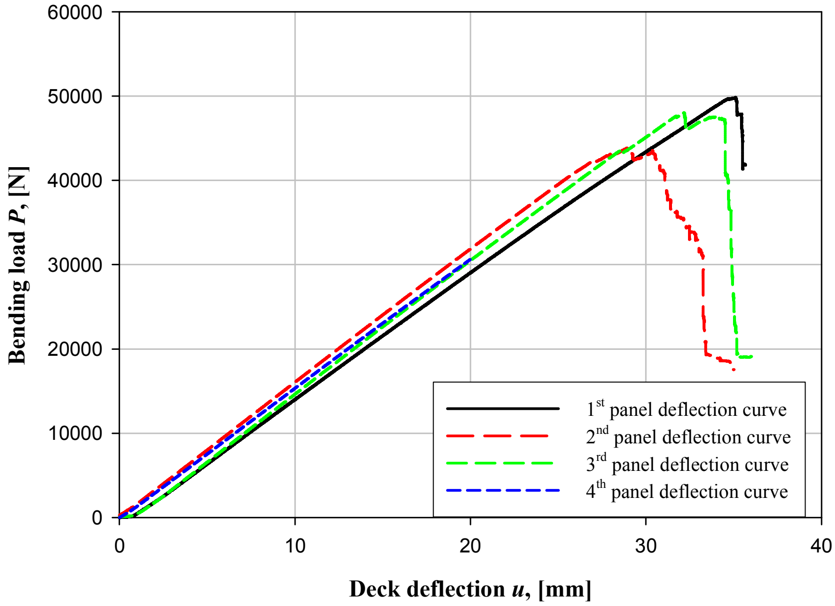

Fig.4. Load-versus-deflection curves obtained experimentally

Table 2. Validation between experimental and numerical results

Load(N)	1st panel test, deflection (mm)	2nd panel test, deflection (mm)	3rd panel test, deflection (mm)	4th panel test, deflection (mm)	ANSYS, deflection (mm)	ANSYS stresses (MPa)
5000	4.1	3.1	4.1	3.4	3.5	23.5
10000	7.4	6.3	7.1	6.6	7.1	47.1
15000	10.7	9.4	10.2	9.8	10.6	70.6
20000	14.0	12.5	13.4	13.0	14.2	94.1
25000	17.3	15.6	16.5	16.3	17.7	118
30000	20.6	18.8	19.7	19.6	21.2	141
35000	24.1	22.1	22.9	N.A.	24.8	165
40000	27.5	25.3	26.2	N.A.	28.3	188
43500	30.1	30.4	28.5	N.A.	30.1	205
48000	33.4	-	32.1	N.A.	34.0	226
49800	35.1	-	-	N.A.	35.3	234

3. Employed metamodeling techniques

This section briefly overviews the employed metamodeling techniques: full global polynomials and locally weighted polynomials of 2nd, 3rd, and 4th order, Multivariate Adaptive Regression Splines (MARS), and partial polynomials constructed using Adaptive Basis Function Construction (ABFC) approach.

As described by Simpson et al.[13], it is assumed that the inputs to the actual computer analysis are supplied in vector **x**, and the outputs (or responses) from the analysis – in vector **y**. Then the true computer analysis code evaluates

$$y = f(x) \tag{1}$$

where $f(x)$ is a complex engineering analysis function. The computationally efficient metamodel approximation is

$$\hat{y} = g(x) \tag{2}$$

such that

$$y = \hat{y} + \varepsilon \tag{3}$$

where ε includes both approximation and random errors.

3.1 Full global polynomials

Low-order polynomials are the most widely used metamodels [1,13,14]. For example, second-order polynomial can be defined as follows:

$$\hat{y} = \beta_0 + \sum_{i=1}^{d} \beta_i x_i + \sum_{i=1}^{d} \sum_{j=i}^{d} \beta_{ij} x_i x_j \tag{4}$$

where d is the number of input variables; β_0, β_i, β_{ij} are coefficients usually determined by the ordinary least squares method minimizing

$$\beta = \arg\min_{\beta} \sum_{i=1}^{n} \left(\hat{y}_{(i)} - y_{(i)}\right)^2 \tag{5}$$

where β are the calculated coefficients; n is the number of sample points; $\hat{y}_{(i)}$ is the value of the metamodel's response for the i-th sample point; $y_{(i)}$ is the actual value of the response of the computer analysis code in vector **y**. A more complete discussion on the polynomial metamodels and least squares method can be found in Myers & Montgomery [14]. In the present study full polynomials of 2^{nd}, 3^{rd}, and 4^{th} order have been employed.

3.2 Locally weighted polynomials

Locally weighted polynomial approximation was originally proposed by Cleveland [15]. It was designed to address situations in which the global polynomials do not perform well or cannot be effectively applied without undue effort. The approximation is carried out by pointwise fitting of low-order polynomials to localized subsets of the data. The advantage of this method is that the analyst is not required to specify a global function of the data. However, the method requires considerably higher computational resources.

The assumption of the local polynomial approximation is that near the query point the value of the actual response changes smoothly and can be approximated using a low-order polynomial. The coefficients of the polynomial are then calculated using the weighted least squares method giving the largest weights to the nearest (usually according to the Euclidian distance) sample points and the lowest or zero weights to the farthest sample points.

The coefficients β are calculated by the weighted least squares minimizing

$$\beta = \arg\min_{\beta} \sum_{i=1}^{n} w(x_{query}, x_{(i)})\left(\hat{y}_{(i)} - y_{(i)}\right)^2 \tag{6}$$

where w is a weight function; x_{query} is the query point nearest neighbors of which will get the highest weights; $x_{(i)}$ is the i-th point in vector **x**. The weight function w depends on the Euclidean distance (in scaled $[-1,1]^m$ space) between the point of interest x_{query} and the points of observations x. One of the most widely used weight functions is the Gaussian weight function [11]:

$$w(x_{query}, x_{(i)}) = \exp\left(-\alpha\mu^2\right) \tag{7}$$

where α is a coefficient and the μ can be calculated as

$$\mu = \left\|(x_{query} - x_{(i)})\right\| / \left\|(x_{query} - x_{farthest})\right\| \tag{8}$$

where $\|\cdot\|$ is the Euclidian norm; $x_{farthest}$ is the farthest point in the neighborhood of the point x_{query}. In general, the Gaussian weight function with constant value $\alpha = 1/2$ is used in local approximations varying only the value of the considered nearest neighbors (unlike in equation (6) where all the sample points are used) [11]. However, in the present study all the sample points have been used and the locality of the approximation has been controlled by varying the value of the coefficient α. If α is equal to zero then local approximation transforms into global approximation. The best value of α is found using the leave-one-out cross-validation technique [16]. In the present study locally weighted polynomials of 2nd, 3rd, and 4th order have been employed.

3. 3 Multivariate Adaptive Regression Splines

Multivariate Adaptive Regression Splines [17,18] was proposed as a method for flexible regression modeling of high dimensional data (i.e., a large number of input variables). The model takes the form of an expansion in product spline basis functions, where the number of basis functions as well as the parameters associated with each one (product degree and knot locations) are automatically determined by the data through a forward/backward iterative approach. Compared to polynomial approximations, the use of MARS for engineering design is relatively new. However, its application is drawing an increasing attention of the researchers (e.g., Jin et al. [1]).

MARS model can be defined as a sum of basis functions [17,18]:

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i f_i(x) \tag{9}$$

where $f_i(x)$ is a basis function; k is the number of basis functions in the model except for the constant basis function $f_0(x) = 1$ coefficient of which is the β_0. The basis functions are of the form

$$f_i(x) = \prod_{j=1}^{d_i} \left[s_{ji}(x_{v(j,i)} - t_{ji})\right]_+ \tag{10}$$

where d_i is the number of variables (interaction order) in the i-th basis function; $s_{ji} = \pm 1$; $x_{v(j,i)}$ is the v-th variable, $1 \le v(j,i) \le d$; t_{ji} is knot location on each of the corresponding variables. The subscript "+" means that the function is a truncated power function [17,18]. The coefficients β are again determined by the ordinary least squares method (equation (5)). In the present study piecewise-cubic MARS version 3.6 without a specific restriction of the number of basis functions or interaction orders has been employed.

3. 4 Partial polynomials

Low-order global polynomial approximations have been well accepted in engineering practice, as they require low number of sample points and are computationally very efficient. On other hand they can not approximate highly nonlinear behavior. Instead, higher-order polynomials can be employed. However, if no special care is taken they tend to overfit the data and produce high errors in regions where the sample points are relatively sparse. One possible remedy for the overfitting problem is employment of the subset selection (also called model building) techniques [13,14]. The techniques are aimed to identify the best subset of polynomial terms (or basis functions) to include in the model and to remove the unnecessary ones, in this manner creating a partial polynomial model (in lieu of "full" model) of increased predictive performance. However, the approach of subset selection assumes that the chosen fixed full set of predefined basis functions (usually just by choosing a fixed maximal order of the polynomial) contains a subset that is sufficient to describe the target relation sufficiently well. Hence, the efficiency of subset selection largely depends on whether or not the predefined set of basis functions contains such a subset.

There exists a different approach for polynomial model building which does not assume a predefined set of basis functions – Adaptive Basis Function Construction [19,20]. The approach allows generating polynomials of arbitrary complexity and order without the requirement to predefine any basis functions or to set the maximal order of the polynomial (or any other hyperparameters) – all the required basis functions are constructed adaptively.

Generally, a polynomial model can be defined by a linear summation of basis functions:

$$\hat{y} = \sum_{i=1}^{k} \beta_i f_i(x) \tag{11}$$

where the coefficients β are still calculated by the ordinary least squares method (equation (5)); $f_i(x)$ is a basis function which generally can be defined as a product of the input variables each raised to some order:

$$f_i(x) = \prod_{j=1}^{d} x_j^{r_{ij}} \tag{12}$$

where r_{ij} is the order of the j-th variable in the i-th basis function (a non-negative integer). It should be noted that when all r_j's of a basis function become equal to 0, the basis function becomes equal to 1, ergo it is the intercept term.

The matrix r completely defines all the basis functions in the model – each row corresponds to one basis function with all of its orders. Construction of the model is carried out in an iterative manner directly with the matrix r using five simple so-called model refinement operators which allow adding, copying, modifying, and deleting the rows of r, i.e., adding, copying, modifying, and deleting the basis functions of the model [19,20]. As a search procedure a modification of the Sequential Floating Forward Selection [21] algorithm is employed while models are evaluated using the Corrected Akaike's Information Criterion [22]. Additionally, in order to lower the model building issues of selection bias and selection instability a technique of model averaging (also called ensembling or combining) is carried out[20].

3. 5. Metamodel evaluation

To evaluate the metamodels, 5-fold cross-validation technique [16] has been used where the full data set is divided in five equally-sized subsets. In each of the five cross-validation iterations, four of the subsets are used for metamodel building and one subset is used as an independent test set for evaluation of the metamodel. As the metamodel accuracy measure the Relative Root Mean Square Error has been used:

$$RRMSE = 100\% \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n_t}(y_i - \hat{y}_i)^2}}{STD} \tag{13}$$

where y_i is the response value of the i-th test point; $\hat{y}_i$ is predicted value of i-th test point; n_t is the number of test points; STD is the standard deviation in test sample:

$$STD = \sqrt{\frac{1}{n}\sum_{i=1}^{n_t}(y_i - \bar{y})^2} \; . \tag{14}$$

It should be noted that RRMSE and STD are calculated using strictly only the test sample and averaged over all the cross-validation runs.

4. Metamodeling results

As structural response parameters the following entities have been taken: global deflection of the deck panel *u*, the relative deflection ratio between the deck length and the plate deflection Δu, the maximum equivalent stress at the upper plate and the stiffeners σ_{top} , σs_{tift} and the maximum shear stresses in stiffeners τ. A total of 500 sampling points have been generated and a cross-validation procedure together with the RRMSE measure has been carried out comparing the different metamodeling techniques. Two loading scenarios have been selected for metamodeling: the concentrated three-point bending load case and uniformly distributed load case with simple support boundary conditions. The obtained results are summarized in Tables 3 and 4.

Table 3

Cross-validation RRMSE errors of different metamodels in the case of three-point bending load

Metamodels	Parametric polynomial approximations				Locally weighted polynomials			MARS
	2nd	3rd	4th	ABFC	2nd	3rd	4th	
Panel deflection, *u*	27.89	13.04	6.71	0.81	16.75	9.90	6.12	3.00
Comparative deflection, Δu	14.09	3.95	1.57	0.45	7.09	2.74	1.47	2.03
Max_stresses, σ_{top}	14.36	4.74	2.32	1.33	7.35	3.69	2.29	2.03
Max_stresses, σ_{stif}	12.56	5.45	3.93	3.00	7.28	4.55	4.03	3.92
Max_shear_stresses, τ	10.32	7.42	7.73	2.18	7.91	7.11	7.71	5.82

Table 4

Cross-validation RRMSE errors of different metamodels built in the case of uniformly distributed load

Metamodels	Parametric polynomial approximations				Locally weighted polynomials			MARS
	2nd	3rd	4th	ABFC	2nd	3rd	4th	
Panel deflection, *u*	34.23	18.04	11.54	1.07	23.15	14.15	10.47	5.26
Comparative deflection, Δu	43.93	30.19	27.36	9.33	23.18	31.65	26.15	26.57
Max_stresses, σ_{top}	16.94	6.36	5.59	3.56	7.88	5.28	5.52	4.94
Max_stresses, σ_{stif}	16.18	7.61	4.59	1.67	10.01	5.97	3.84	3.54
Max_shear_stresses, τ	10.36	8.09	11.38	7.32	8.30	8.05	11.56	11.28

The conventional 2nd order polynomials, which are mostly associated with engineering problems of the response surface methodology, gave the worst approximation results for almost all the response values. It has been noted that locally weighted polynomials of the 2nd order considerably increased the predictive performance. As overall observation could be stated that, by increasing the order of polynomials, the approximation performance rose, however the higher was the order the smaller was the improvement of the locally weighted polynomials over the global ones. Additionally, it should be expected that decreasing the number of the sampling points would lead the full polynomials of higher orders to overfitting the data thus rapidly reducing their predictive performance. The best results were obtained using the ABFC approach, leaving the MARS technique as the second best. One can conclude that, although using higher order global polynomials or locally weighted polynomials can improve the predictive performance, an elaborated adaptive search for partial polynomials or regression splines has capabilities to provide an even further performance boost.

Moreover, three-dimensional graphical validations of the developed metamodels for the deck panel deflection *u* versus the panel length *L* and the panel height *h* parameters in the case of concentrated three-point bending load with 500 sample points were carried out as presented in Figure 5 and 6. By graphical validation of panel deflection and shear stresses one can easily identify that low order global polynomial approximations at the maximum height and the minimum length behave differently than expected. In particular the second order and to a lesser extent also the forth order polynomial surface plots show a decrease of stiffness when increasing the panel height. On the other hand, the third order polynomial function shows non-negative deflection at the boundaries, which would indicate bending against gravity. The MARS and partial polynomials reduce this unwanted behaviour.

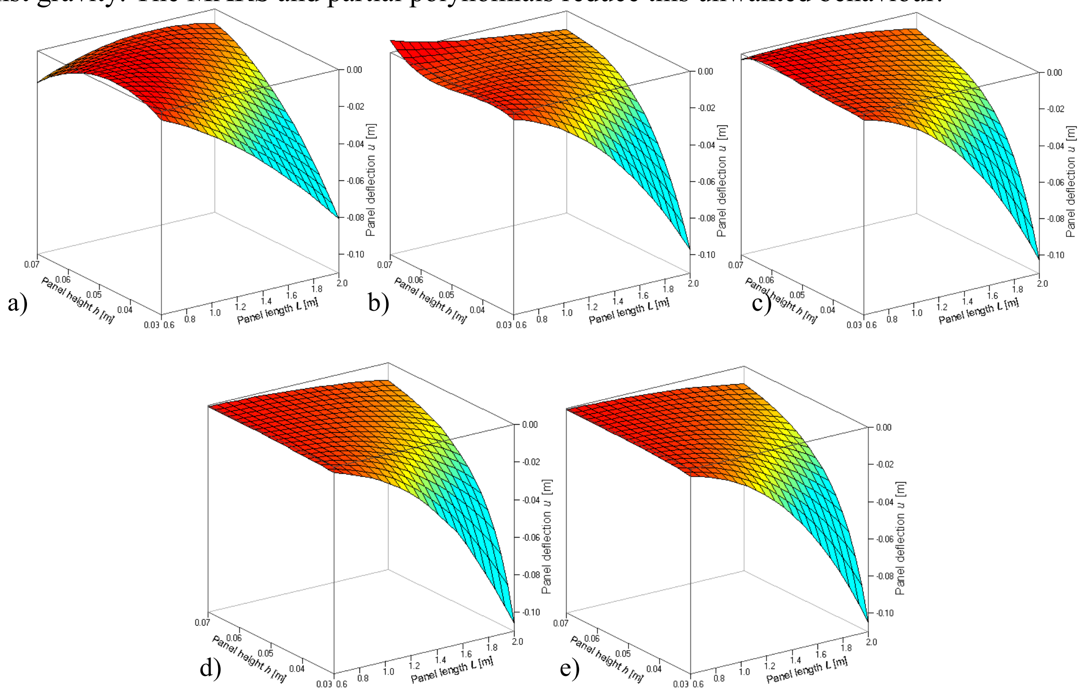

Fig.5. Graphical validation of surrogate models for panel deflection in the case of concentrated load. Full global polynomials of 2nd (a), 3rd (b), and 4th (c) order and MARS (d); ABFC (e)

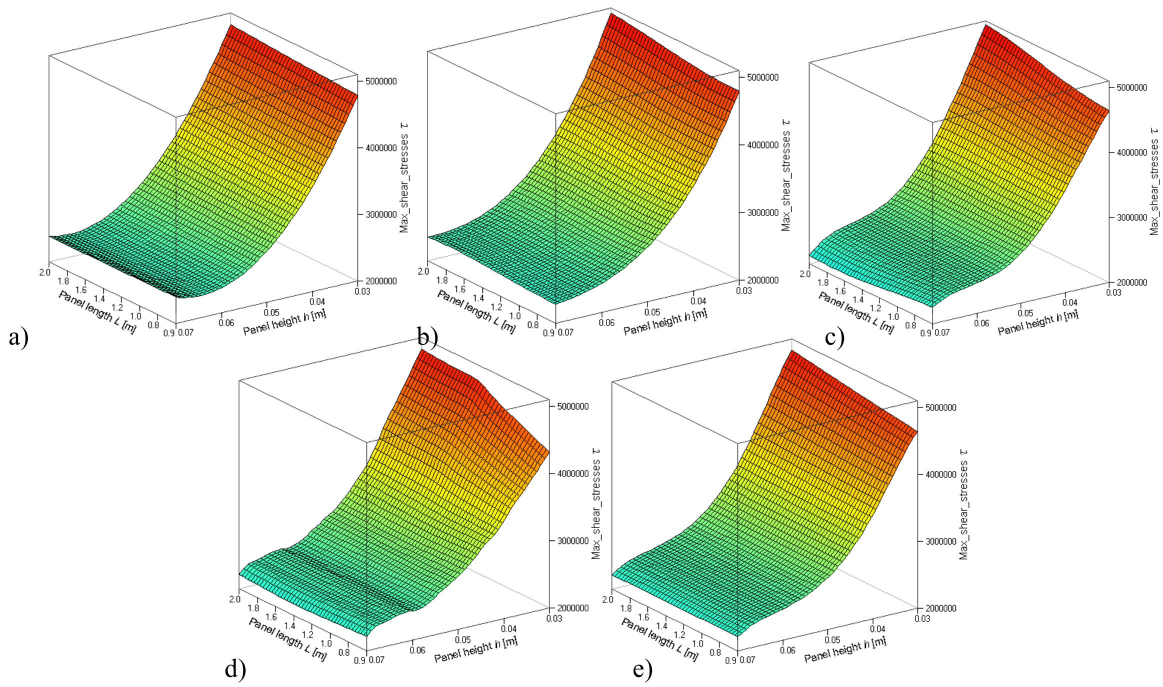

Fig.6. Graphical validation of surrogate models for panel shear stresses in the case of concentrated load. Full global polynomials of 2nd (a), 3rd (b), and 4th (c) order and MARS (d); ABFC (e)

Overall, the low-order locally weighted polynomial approximation, MARS, and the ABFC gave the best overall perspective of structural behaviour by creating a plateau-like surface for the height of stiffened panel designs. It seems that here for an optimization procedure all the three methods can be used with some confidence however for what-if analysis the ABFC would be the most accurate.

Conclusion

The comparison study between parametric and non-parametric metamodels has been elaborated for design of pultruded GFRP deck structures under the bending load. Two loading scenarios have been selected to investigate the metamodeling efficiency and have been validated with physical experiments. It has been concluded that the partial polynomials and MARS are capable to improve the prediction accuracy compared to conventional 2nd order polynomials, which frequently are associated with engineering problems of the response surface methodology. In particular, the bending deflection responses could be improved by an order of magnitude compared to the 2nd order polynomials. In contrary, the improvement in approximation prediction for equivalent stresses and shear stresses are less efficient. Elaborated metamodels have the capability to be used in implementation of optimum design methodology for the bended deck structures.

Acknowledgments

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References

[1] Jin R., Chen W., and Simpson T., “Comparative studies of metamodeling techniques under multiple modelling criteria”, Journal of structural and Multidisciplinary Optimization, Vol. 23:1, 2001, pp. 1-13.
[2]Audze P., and Eglais V., “New approach for planning out of experiments”, Problems of Dynamics and Strength, Vol. 35, Zinatne¸ Riga (in Russian), 1977, pp. 104-107.
[3] Eglajs V., “Approximation of data by multi-dimensional equation of regression”, Problems of Dynamics and Strength, Vol. 39, Zinatne, Riga (in Russian), 1981, pp. 120-125.
[4] Simpson T. W., “A Concept Exploration Method for Product Family Design”, Ph.D. Dissertation, Georgia Institute of Technology, 1998.
[5] Santner T. J., Williams B. J., and Notz W. I., The Design and Analysis of Computer Experiments, Springer, 2003.
[6] Auzins J., “Direct optimization of experimental designs”, 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conf., AIAA paper, No.2004-4578, 2004.
[7] Fang K. T., and Wang Y., Number-Theoretic Methods in Statistics, Chapman & Hall, London, 1994.
[8] Williams B., Shehata E., and Rizkalla S., “Filament-Wound Glass Fiber Reinforced Polymer Bridge Deck Modules”, ASCE Journal of Composite for Construction, Vol. 7 No. 3, 2003, pp. 266-276.
[9] Keller T. “Use of fiber reinforced polymers in bridge construction”, Structural Engineering Documents, No. 7, International Association for Bridge and Structural Engineering IABSE, 2003
[10] Kalnins K., Eglitis E., Jekabsons G., and Rikards R., “Metamodels for optimimum design of laser welded sandwich structures”, Design, Fabrication and Economy of Welded Structures, edited by K. Jarami and J. Farkas, Horwood, Chichester, UK, 2008, pp. 119-126.

[11] Kalnins K., Skukis E., and Auzins J., "Metamodels for I-core and V-core sandwich panel optimization", Proceedings of 8th Intern. Conf. Shell Structures: Theory and Applications, edited by W. Pietraszkiewicz and C. Szymczak, Taylor & Francis, London, 2005, pp. 569-572.
[12] ANSYS, software package, Ver. 11, Canonsburg, USA, 2008.
[13] Simpson T. W., Peplinski J., Koch P. N., and Allen J. K., "Metamodels for Computer-Based Engineering Design: Survey and Recommendations", Engineering with Computers, Vol. 17, No. 2, 2001, pp. 129-150.
[14] Myers R.H., and Montgomery D.C., Response Surface Methodology: Process and Product Optimization Using Designed Experiments, 2nd ed., John Wiley & Sons, New York, 2002
[15] Cleveland W. S., and Devlin S. J., "Locally weighted regression: An approach to regression analysis by local fitting", Journal of the American Statistical Association, 83, 1988, pp. 596-610.
[16] Kohavi R., "A study of cross-validation and bootstrap for accuracy estimation and model selection", Proceedings of IJCAI-95, edited by C. S. Mellish, Morgan Kaufmann, 1995, pp. 1137-1143.
[17] Friedman J. H., 1991, "Multivariate Adaptive Regression Splines", The Annals of Statistics, Vol. 19, No. 1, pp. 1-141.
[18] Friedman J. H., "Fast MARS", Department of Statistics, Stanford University, Tech. Report LCS110, 1993.
[19] Jekabsons G., and Lavendels J., "Polynomial regression modelling using adaptive construction of basis functions", IADIS International Conference, Applied Computing 2008, Algarve, Portugal, 2008, p. 8
[20] Jekabsons G., "Ensembling adaptively constructed polynomial regression models", International Journal of Intelligent Systems and Technologies (IJIST), Vol. 3, No. 2, Paper 11, 2008, pp. 56-61. URL: http://www.waset.org/ijist/v3/v3-2-11.pdf
[21] Pudil P., Novovicova J., and Kittler J., "Floating search methods in feature selection", Pattern Recognition Letters 15, 1994, pp. 1119-1125.
[22] Hurvich C. M., and Tsai C-L., "Regression and time series model selection in small samples", Biometrika 76, 1989, pp. 297-307.

Kalniņš K., Jēkabsons G., Beitlers R., Ozoliņš O. Stiklplasta paneļu optimāla projektēšana ar eksperimentālu validāciju.
Pētījuma mērķis ir izvērtēt dažādu metamodeļu pielietošanas iespējas stiklplasta paneļu optimālas projektēšanas metodoloģijas izstrādē, veicot eksperimentālu validāciju. Raksts sastāv no ievada, četrām daļām un secinājumiem. Pirmā daļa ietver pultrūzijas procesa un stiklplasta īpašību un aprēķina īpatnību aprakstu. Otrā daļa sastāv no Rīgas Tehniskās universitātes, Materiālu un konstrukciju institūta laboratorijā veikto eksperimentu rezultātu apstrādes un validācijas. Trešā daļa ietver parametrisko un bez parametrisko aproksimācijas metamodeļu izveides stratēģijas un metamodeļu precizitātes novērtējuma kritēriju aprakstu. Ceturtajā nodaļā apkopoti aproksimāciju kļūdu, kā arī grafiskās validācijas rezultāti. Rakstu noslēdz secinājumi un ierosinājumi turpmākiem pētījumiem.

Kalniņš K., Jēkabsons G., Beitlers R., Ozoliņš O. Optimal design of fiberglass panels with physical validation.
The aim of conducted research was to evaluate the efficiency of metamodels for elaboration in the optimum design methodology for fiberglass panels with physical validation. The paper consists of introduction, four parts devoted for discussion and conclusions has been drawn. The first part describes pultrusion process, fiberglass properties and numerical analysis background. Second part consists of processing the laboratory experiments performed at Riga Technical University Institute of Materials and Structures and validation with numerical results obtained by ANSYS. Third part includes theoretical background of metamodeling. In forth chapter the results have been shown and the graphical validation performed. The conclusions and references for the future research has been drown at the end of the article.

Калниньш К., Екабсон Г., Беитлерс Р., Озолиньш О. Оптимальное проектирование стекло-пластиковой панели с экспериментальным подтверждением.
Цель данного исследования является оценка эффективности метамодели для разработки методологии оптимального проектирования стеклопластиковой панели с экспериментальным подтверждением.Статья состоит из введения, четырех глав и заключения. В первой главе

рассматривается получение одноосно ориентированного волокнистого пластика, его свойства и исходные данные модели для численного расчета. Вторая глава содержит результаты численного расчета и результаты эксперимента, который был выполнен в лаборатории Института Материалов и Конструкций Рижского Технического университета. Теоретическое описание метамодели находится в третьей главе. В четвертой главе показаны численные результаты ошибок аппроксимации и их графическое подтверждение. Список литературы и заключение для дальнейшего исследования располагается в конце статьи.