Impact of sampling of load stochastic process on probabilistic loss of load assessment for a power system

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Abstract - The loss of load probability (LOLP) is an appealing means used in operational reliability and capacity planning studies to evaluate the installed reserve margins and incentivize generation capacity investments in electrical power systems (EPSs) since it can express reliability explicitly and visually. LOLP recommendations can significantly differ depending on how the stochastic properties (variability and uncertainty) of the load are represented in the analysis. The stochastic load properties are represented by a continuous random process. This approach's difficulty with LOLP evaluation problems creates a huge computational burden. A simplified approach discretizes the load stochastic process into random variables (RVs) indexed by time. However, there is still a fundamental question: on which time intervals are RVs considered, hourly, daily, weekly, monthly, or seasonally? and based on which stochastic properties are considered, peak or average load variation in the supposed time interval? Therefore, this paper examines different approaches to modeling and quantifying the load's stochastic properties and their impact on the calculated LOLP values. Moreover, analytical expressions are derived to incorporate them into EPS's LOLP assessment efficiently. Besides, the approximation errors of the analytical LOLP expressions are discussed.

Keywords — Loss of load probability, Monte Carlo simulation, power system reliability, probabilistic evaluation, uncertainty, stochastic process.

I. INTRODUCTION

Reliability is one of the most important factors to consider when planning an electrical power system (EPS) [1]. The planning process uses reliability indices as criteria to decide on new investments in generation capacities to increase the installed reserve margins (IRM), i.e., the ratio of installed capacity to peak load. The capacity expansion model incorporates reliability indices, such as loss of load probability (LOLP) and/or expected energy not supplied (EENS), as constraints [2–3]. The LOLP is the most widely used reliability index to enforce the IRM requirement for planning and generation adequacy evaluation [4–6]. The LOLP measures the amount of load loss that occurs when the electric load exceeds the available generation capacity of EPS. Using the explicit and visual reliability index (exact probabilities), rather than the implicit expected value or variance, strikes a balance between economy and reliability, which is the main advantage of using LOLP [5]. However, the calculation of LOLP is the major impediment for the entire IRM optimization, as the LOLP value can significantly differ depending on the load power representation included in the analysis [7–8]. Thus, an EPS can significantly underperform, carrying a much higher or lower risk of not serving load than what the planner believes that the system LOLP will be, leading to failure to meet the planned reliability and generation capacity installation targets.

In order to reduce the computational burden of LOLP evaluation, most applications of industry planning studies and capacity market designs usually consider the brief length of load data by condensing long historical records into a typical load profile [9–15]. Data aggregation techniques capture load variations due to weather, time of day, and season. For instance, rather than use 8760 hours for one or a sample of years, loads for typical weekdays and weekends might be used to represent each month or season [9–11]. Another well-intended simplicity is the consolidation of multiple years of data by averaging across the different realizations, preserving seasonality, day of the week, and hourly trends [12–15]. All types of data aggregation of historical records can capture load variability; however, due to parameter estimation error (from using small or unrepresentative samples) and non-stationarities (e.g., economic growth and technological innovation), there is always uncertainty associated with the use of such load profiles [16–19]. Additional techniques, such as scenario development, were applied to capture uncertainties concerning potential non-stationarities [20].

The load uncertainties were represented by two approaches [21–26]. The main and most accurate one is simulating the load stochastic process by several random time trajectories over a chronological time span to incorporate load time-varying characteristics. A huge computational burden is the first approach's difficulty with LOLP evaluation problems concerning many other random variables (RVs), such as the probable states of generation capacity and the number of scenarios considered in terms of the optimization procedure [21–23]. In practical applications, another approach involves discretizing the stochastic load processes into time-indexed RVs. The transition from the analysis of random processes to the analysis of RVs paves the way for applying analytical approaches in the LOLP calculation procedure. The analytical approaches analyze the RVs using mathematical expressions, so they are efficient [24–27]. This work is motivated by a fundamental question: on which time interval is the discretization assumption valid and accurate enough? and based on which stochastic properties are considered, peak or average load variation in the supposed time interval?

Our main contributions are (a) the creation of new statistical tests that give us new information about how to show the random aspects of the load variation process and (b) the suggestion of mathematical expressions that let us figure out the LOLP analytically with different load random representations. The models and methods used in this study are described in Section III, after Section II, where the problem of time-variant reliability evaluation is highlighted. Section IV presents an example case study, and the results are analyzed to address the objectives of the work. In Section V, conclusions are drawn.

II. MATHEMATICAL FORMULATION OF THE TIME-VARIANT RELIABILITY ANALYSIS

Power system reliability refers to the power source's ability to provide consumers with the necessary electrical power within a specified time interval, even when the characteristics of the required load and available generation capacity change. Two factors determine the uncertainty of a power shortage between load and generation: probabilistic load $L(t) = L_{max} - \Delta L(t) \pm \delta L(t)$ and probabilistic available generation $G(t) = G_{inst}(t) - G_{pl}(t) - G_{emg}(t)$, where L_{max} – annual peak load forecast; $\delta L(t)$ – load forecast error; $\Delta L(t)$ – the load profile deviation from the annual peak value; G_{inst} – installed capacity of generating system; $G_{pl}(t)$ and $G_{emg}(t)$ – planned and emergency disconnected power of generating units. The power shortage between generation and load observed at an arbitrary time $t \in [0, T]$ can be expressed as follows: $Z(t) = G(t) - L(t)\ \forall t \in [0, T]$, where $L(t)$ and $G(t) = [G_1(t) \dots \dots . G_g(t)]^T$ are stochastic processes. The required load is considered as a time-continuous stochastic processes. While the available capacity of generation could be deterministic or random variable or stochastic process depending on the type of generation units. For example, wind or solar power generation is expressed as a stochastic process. However, the conventional power generation is written as a vector of random variables $G = [G_1 \dots \dots . G_g]^T$ provided that the failure probability of generators is independent of time.

The failure probability, $LOLP$, is complementary to reliability, which is defined as the probability that the required load doesn't exceed the generation capacity at any arbitrary time point within the considered interval $[0, T]$. Mathematically, it follows,

$$LOLP = \mathcal{P}\{L(t) > G(t) \forall t \in [0, T]\}$$
$$= \frac{1}{T} \int_0^T \int_{Z(t)<0} f_Z(z(t)) dz(t) dt \quad (1)$$

where $\mathcal{P}$ denotes the probability of the event in the brackets, $Z(t)$ is the power balance function at time point t, and $f_Z(z(t))$ is the joint PDF of $Z(t)$. Due to the introduction of the time scale, reflecting the accumulation of EPS failure risks during a period of interest, Eq. (1) is complicated. Moreover, the complexity can be even more dependent on how the stochastic properties of the load process are represented. The next section is on how to represent the load stochastic process.

A. Stochastic properties of Load variation

The required load is represented by a power curve which must incorporate both the variability $\Delta L(t)\ \forall\ t \in [0, T]$ and the uncertainty of power demand $\delta L(t)$ within the considered time interval. The load value at any time point can be expressed as follows: $L(t, \varepsilon) = L_{max}\big(1 - \Delta L(t, \varepsilon)\big) = L_{max}\, \Delta \tilde{L}(t, \varepsilon)$ (3) in which $\Delta \tilde{L}(t, \varepsilon)$ is a load stochastic process (i.e., a random function of time) and ε is a RV represents the uncertainty of load value. For example, weekly process of load variation as a percentage of annual peak load can be represented as follows: $\Delta \tilde{L}(t, \varepsilon) = 0.82 + 0.15 \cos(\frac{2\pi t}{52} + \varepsilon)$, where ε is a uniform RV in the interval (0, 2π). A possible collection of time trajectories as shown in Fig. 1 can be generated randomly for the weekly load process.

III. LOLP ASSESSMENT PROCEDURES

First, for simplicity, we can assume that the available capacity of generation G is deterministic (g) i.e., neglecting the probabilistic nature of generation capacity within the considered interval $t \in [0, T]$ and considering only the load is random.

A. Simulation approaches

Monte Carlo simulation (MCS) is a robust and powerful approach to assess the EPS reliability especially for high-dimensional problems. It uses many sampled time trajectories of $L(t)$ to approximate the reliability (or failure probability). In each simulation run, a trajectory of stochastic process $L(t)$ for $\in [0, T]$, denoted by $L(t)$, is generated, and is compared against available power generation. If $L(t) \le g$ holds for all $t \in [0, T]$, then the EPS is deemed as success and otherwise failure. Guaranteed by the strong law of large numbers [21], the average of these simulation replications converges to the EPS reliability (i.e., the solution to Eq. (2)). Mathematically, for a reference period of $[0, T]$, $LOLP$ is obtained by (2).

$$LOLP = 1 - \mathcal{P}\{L(t, \varepsilon) \le g\} = 1 - \mathbb{E}[\mathbb{I}(L(t, \varepsilon) \le g) = 1]$$
$$\approx 1 - \frac{1}{N} \sum_{s=1}^{N} \mathbb{I}(L^{(s)}(t) \le g\ \forall t \in [0, T]) \quad (2)$$

where g is the available capacity of generation, $\mathbb{E}[.]$ denotes the mean value of the variable in the brackets, N is the number of simulations or scenarios or time trajectories, and $\mathbb{I}[.]$ is an indicator function, which returns 1 if the statement in the bracket is true and 0 otherwise.

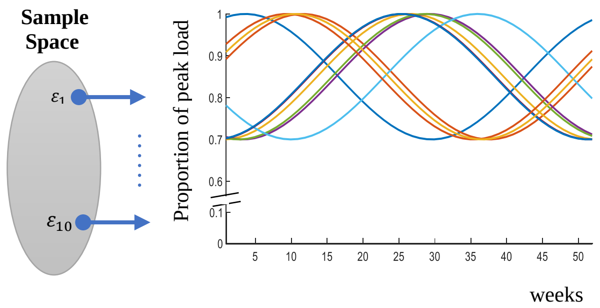

Fig. 1. *A possible ensemble of realizations of random weekly load process*

To get samples $L^{(s)}(t)$ in each simulation run, the matrix $L^{(s)}(t) = \left[L^{(s)}(t_1), \dots, L^{(s)}(t_k), \dots, L^{(s)}(t_K)\right], s = 1, \dots, N$ is formed. The LOLP in Eq. (2) can be rewritten as

$$LOLP \approx 1 - \frac{1}{N} \sum_{s=1}^{N} \{\mathbb{I}\big(L^{(s)}(t_1) \le g\big) \cap .. \cap \mathbb{I}\big(L^{(s)}(t_K) \le g\big)\}$$

$$= 1 - \frac{1}{N}\sum_{s=1}^{N} \mathbb{I}(\max_{k=1,..,K} L^{(s)}(t_k) \le g)$$

Considering random conventional power generation, the generation capacity is calculated as a sum of each generating unit's available power, taking into account its availability probabilities. By the convolution method, the probabilistic series of the available generation capacity $\{G_j, \mathcal{P}_j,\ j = 1,\dots,N_c\}$ is formed. Thus, LOLP can be formulated as follows:

$$LOLP = \sum_{j=1}^{N_c} p_j \cdot \left[1 - \frac{1}{N}\sum_{s=1}^{N} \mathbb{I}(\max_{i=1,..,n} L_i^{(s)} \le g_j)\right] \quad (4)$$

B. Analytical approaches

The main drawback of simulation approach is that it imposes a heavy computational burden for low failure probabilities, since the number of required simulation runs is in proportion to $1/LOLP$. By considering the load stochastic process as a sequence of load RVs indexed by time $L_i, i = 1,\dots,n$, LOLP can be calculated analytically. Assuming that load is stationary normal random process i.e., the statistical properties do not change by time and each random variable for any fixed time (t) is statistically independent normal RV, the LOLP is expressed as follows:

$$LOLP = 1 - \prod_{i=1}^{n} \mathcal{P}\{L_i \le g\} = 1 - \prod_{i=1}^{n} \left[F_L(g, m_{L_i}, \sigma_{L_i})\right]$$

where $F_L(.\,, m_{L_i}, \sigma_{L_i})$ is the cumulative distribution function (CDF) of load RV at fixed time point i, and m_{L_i} and σ_{L_i} are the load mean and standard deviation, respectively, of L_i provided that one load event within the time interval (t_{i-1}, t_i). Considering random power generation, LOLP formulation follows

$$LOLP = \frac{1}{T}\sum_{j=1}^{N_c}\sum_{i=1}^{n} T_i p_j \left(1 - F_L(g_j, m_{L_i}, \sigma_{L_i})\right) \quad (5)$$

The annual time interval $[0, T]$ is divided into n identical sections, namely, $[(0, t_1); (t_1, t_2); \dots, (t_{n-1}, t_n = T)]$. These sections may be 4 seasons or 12 months or 52 weeks or 364 (52*7) days. Considering daily time resolution, the time interval (t_{i-1}, t_i) consists of 24-hourly load RVs as shown in Fig. 2. Here, each L_i is a random vector consisting of 24-hourly load RVs as follows: $L_i = [L_{i,j}, j = 1,..,24]$ $, i = 1,\dots,364$. Different approaches could be developed based on the choice of load RVs at each daily interval.

First approach (M1): Each day can be considered as sets of 24 independent normal load RVs. First, LOLP is calculated using hourly load distributions RVs. The probabilistic characteristics of each hourly load (expectation, standard deviation) are expected to be known. The LOLP is expressed as follows:

$$LOLP_{M1} = 1 - \frac{1}{364}\sum_{i=1}^{364}\left(\prod_{j=1}^{24}\left[F_{\mathrm{L}}\left(g_i, m_{L_{i,j}}, \sigma_{L_{i,j}}\right)\right]\right)$$

After that, LOLP distortions due to load variation selection (peak or average) are determined as follows. By considering the hourly peak load for each day, the LOLP can be written as follows:

$$LOLP_{M1-max} = \frac{1}{364}\sum_{i=1}^{364}\left(1 - F_{\mathrm{L}}\left(g_i, m_{L_i^{max}}, \sigma_{L_i^{max}}\right)\right) =$$
$$= 1 - \frac{1}{364}\sum_{i=1}^{364} F_{\mathrm{L}}\left(g_i, m_{L_i^{max}}, \sigma_{L_i^{max}}\right)$$

where $L_i^{max} = \max_{j=1,..,24} L_{i;j}$ is the hourly peak load within the day interval (t_{i-1}, t_i). Second, an aggregated daily profile is derived through averaging the hourly load across 24 hours for each day.

$$LOLP_{M1-av} = 1 - \frac{1}{364}\sum_{i=1}^{364} F_{\mathrm{L}}\left(g_i, m_{L_i^{av}}, \sigma_{L_i^{av}}\right)$$

in which

$$m_{L_i^{av}} = \frac{1}{24}\sum_{j=1}^{24}\left(m_{L_{i,j}}\right);\ \sigma_{L_i^{av}} = \sqrt{\left(\sum_{j=1}^{24}\left(m_{L_{i,j}}{}^2\right) - m_{L_i^{av}}\right)};$$
$$i = 1,\dots,364$$

Second approach (M2): To assess the effect of aggregation, the average load RV is calculated based on load levels. For example, as shown in Fig. 2, load RVs are aggregated based on three load levels, provided that each level has the same number of load RVs.

$$LOLP_{M2} = \frac{1}{3}\sum_{j=1}^{3}(1 - F_{\mathrm{L}}\left(g, m_{L_j^{av}}, \sigma_{L_j^{av}}\right)) \quad (7)$$

In which, the parameters of the average load RV for each level are

$$m_{L_j^{av}} = \frac{3}{8736}\sum_{k=1|k\in j}^{\frac{8736}{3}}\left(m_{L_k}\right); \sigma_{L_{av}} = \sqrt{\left(\sum_{k=1|k\in j}^{\frac{8736}{3}}\left(m_{L_k}{}^2\right) - m_{L_j^{av}}\right)}$$

Third approach (M3): The whole annual period is considered. The unreliability of an EPS viewed as its inability to meet the annual peak load. Therefore, LOLP can be written as follows:

$$LOLP_{M3-max} = 1 - F_{\mathrm{L}}\left(g, m_{L^{max}}, \sigma_{L^{max}}\right)$$

where $L^{max} = \max_{i=1,..,364} L_i^{max}$. An alternative formula based on yearly average load RV is as follows:

$$LOLP_{M3-av} = 1 - F_L\left(g, m_{L^{av}}, \sigma_{L^{av}}\right)$$

$$m_{L^{av}} = \frac{1}{8736}\sum_{\substack{i=1,..363;\\ j=1,..,24}}\left(m_{L_{i,j}}\right);$$

$$\sigma_{L^{av}} = \sqrt{\left(\sum_{\substack{i=1,..363;\\ j=1,..,24}}(m_{L_{i,j}}{}^2) - m_{L^{av}}\right)} \quad (6)$$

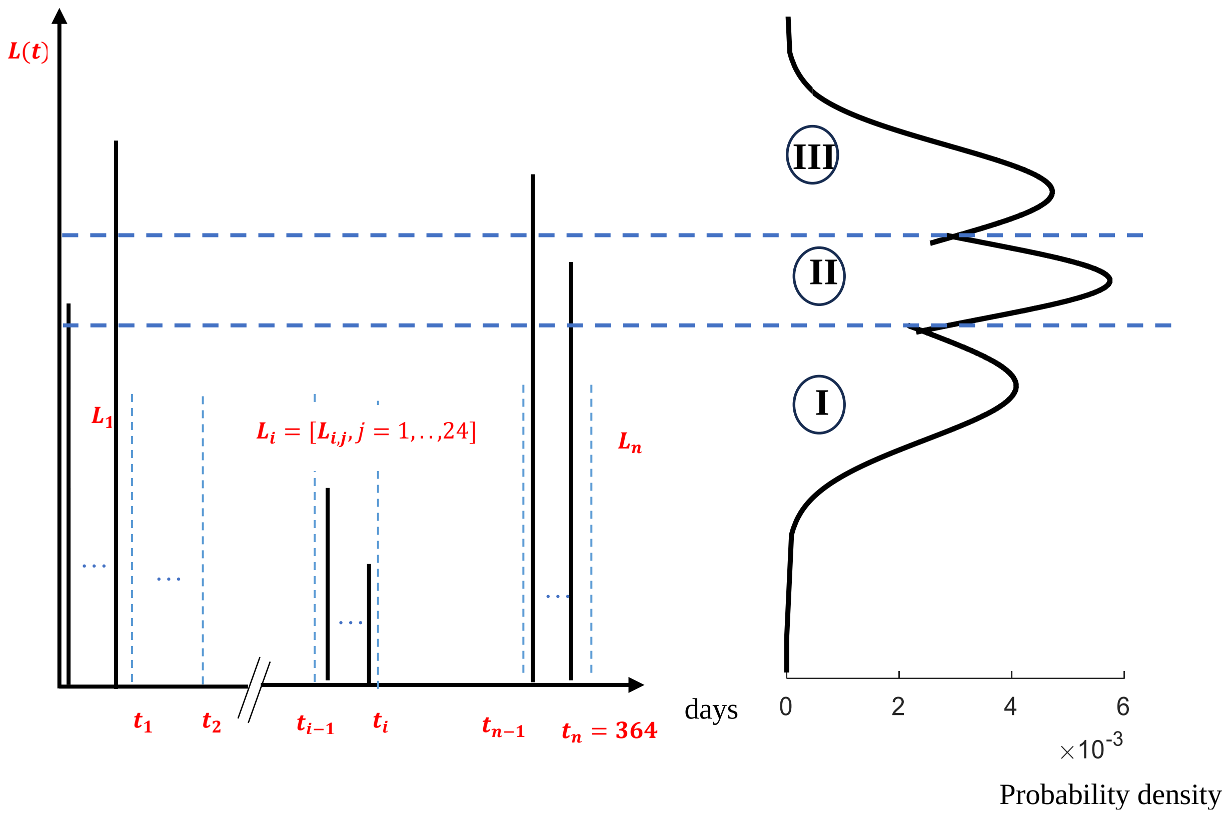

Fig. 2. *Derivation of PDF for aggregated load RVs based on three load levels.*

IV. Computational Results

A numerical study is performed to compare the proposed procedures. An EPS with a generating station consisting of 3 groups of generators (number, rated power (MW) and the failure probability of a generating unit): $\{n_G;\ P_G;\ q_G\} = [\{2;\ 300;\ 0.05\};\ \ \{5;\ 100;\ 0.04\};\ \ \{10;\ 25;\ 0.03\}]$ is considered and annual peak load $L_{max} = 0.8 \cdot G_\Sigma = 1080\ MW$ in which G_Σ is the total installed capacity of generation. Weekly process of load variation as a proportion of the annual peak load can be represented as follows: $L(t,\varepsilon) = 0.82 + 0.15\cos(\frac{2\pi t}{52} + \varepsilon)$, where ε is a uniformly distributed RV in the interval $(0,\ 2\pi)$. Daily load as a proportion of weekly load is considered as follows: $\{0.9, 0.95, 1, 1, 1, 0.7, 0.4\}$. Hourly load curve as a proportion of daily load and duration time is considered as follows: $\{0.5, 6;\ 0.8, 2; 1, 4; 0.8\ 4; 0.9\ 6; 0.7\ 2\}$.

The LOLP results are presented for different approaches. First, the MCS with 10^4 random time trajectories is used to obtain an accurate LOLP, which is 0.0049 for the test system. After that, the results of the analytical approaches are shown in Table. 1. First, LOLP is calculated using the first approach (M1-24 RVs). It produces almost identical results to the simulation method (MCS) with significantly lower computational time. The value of the resulting LOLP is considered the "base" for comparing other analytical approaches. Table I shows a comparison as a percentage of the base solution.

Next, 365-day profiles are derived based on peak and average variations. First, the daily load is considered the hourly peak load over 24 hours of the day. This is applied in the approach M1-Max. Second, an aggregated daily profile (M1-AVG) was derived by averaging the hourly load over 24 hours for each day. The calculated LOLP based on daily peak load RVs in M1-Max is an overestimate. While the approach M1-AVG is a reasonably good approximation of the base solution since its LOLP value is nearly 1.2 of its base method counterparts. The second approach (M2) computes LOLP using three aggregated load RVs, or the average load RVs derived from three load levels, to evaluate the impact of aggregation. Finally, we calculate LOLP based on some specified hours over the year: the annual peak load (M3-Max), the annual average load (M3-AVG), and the top 10% of peak load over the year (M3-top 10% Max). The choice of annual peak load in M3-Max and M3-top 10% Max can dramatically impact system LOLP. The approach M2 appears to be the most promising approach for calculating analytically the LOLP. The best in terms of accuracy and computation time is the M3-AVG which is useful in practical calculations.

TABLE I. LOLP Results

Approaches	LOLP	Proportion of base solution
M1-24-RVs (Base solution)	0.0046	1
M1-Max	0.0109	2.3696
M1-AVG	0.0056	1.2174
M2-levels	0.0051	1.1087
M3-Max	0.0225	4.8913
M3-AVG	0.0078	1.7391
M3-top 10% Max	0.0221	4.7826

V. Conclusion

In this paper, the problem of selecting a stochastic approach that can model the variability and uncertainty of power system loads has been investigated. The reported results indicate that the transition from LOLP simulation estimation, which uses a load random process, to LOLP analytical estimation, which uses a set of hourly load RVs, is valid, accurate, and requires minimal computational expenses. LOLP distortions caused by the choice of peak and average load variations based on both daily and yearly loads have been discussed. The daily average load RVs approach has superior LOLP results compared to its base method's (hourly load RVs) counterpart. For a trade-off between computation burden and accuracy, the yearly average load of RVs has a LOLP value of 1.7 of the base method value. While using the annual peak load can dramatically impact system LOLP (4 of the base method value), The LOLP calculation approach with aggregated load RVs based on load levels (e.g., three levels), which are the average load RVs calculated based on three load levels, has clearly superior LOLP results from both the accuracy and computation efforts.

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