

Accuracy Estimation of GPS Receiver Parameters with Re-Reference System in Static Mode

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Abstract: This article describes some of GPS receiver testing results. Our attention was focused mainly on radial error estimation (in horizontal plane) that might be the most interesting for users of GPS devices.

Keywords: GPS receiver, Re-reference system, position fix, radial error, accuracy estimation.

INTRODUCTION

Accuracy index of civil GPS receivers is often represented by DRMS parameter (Distance Root Mean Square) and its doubled value (2DRMS), as well as CEP – Circular Error Probability. DRMS is equal to square root of latitude and longitude deviation mean squares. The CEP parameter indicates that 50 percent of the position fix errors don't exceed respective CEP value in meters. Contrary to DRMS (and 2DRMS) the CEP parameter is less affected by large gross errors.

By using DRMS, 2DRMS and CEP it is possible to define the confidence region with the centre in GPS receiver's true location point (reference point). The radius of this confidence region matches the accuracy of receiver's position fix in horizontal plane with specified probability of accurate estimation. Therefore the DRMS is sometimes referred to HRMS (Horizontal RMS) in literature. Fig. 1 illustrates an example of radial error estimation over 24 hours.

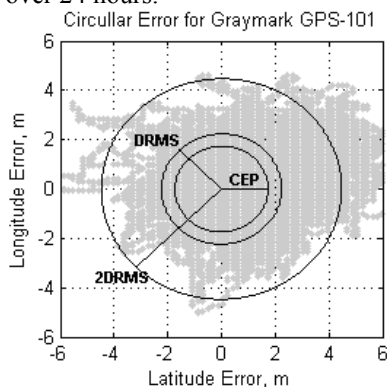


Fig. 1. Graymark GPS-101 receiver's radial error CEP, DRMS and 2DRMS estimated over 24 hours

The purpose of the testing is accuracy estimation of GPS receivers over various time intervals by using Re-Reference system described in [1] and [2]. After the converting coordinates by using standard methods described in [4], [6], it is possible to estimate the radial (horizontal) error of position fix in meters. From here, given the array of radial deviations (in meters) from reference point, we can estimate the statistical characteristics of the random process samples. Examples of such statistical characteristics are estimations of expectation (mean), standard deviation, probability density function (histogram) and cumulative probability function (probability that error doesn't exceed the fixed value) etc.

Similar researches were made before, for example in [7]. However we have analyzed different GPS receivers over various time intervals. We have used additional information about number and arrangement of satellites as well.

WGS 84 Ellipsoid

Results from [2] allow comparing the position fix accuracy between different GPS receivers in angular units. However, these means aren't sufficient for user, since it is necessary for him to determine the accuracy of position fix for individual receiver in linear radial units. In order to estimate the deviation of receiver position fix from its true location, it is advisably to convert receiver's angular coordinates to the estimation of radial error.

It is necessary to obtain the exact data about Earth ellipsoid before converting coordinates. GPS satellite navigation systems are based on WGS 84 Ellipsoid described in [3], [4].

According to these documents, Table 1 contains values of semi-major axis (equatorial Earth radius) and flattening:

Table 1. WGS 84 Ellipsoid data

Semi-major Axis	$a = 6378137.0 \text{ m}$
Flattening	$f = 1/298.257223563$

Based on the data from Table 1 it is possible to calculate semi-minor axis (polar radius) according to the following expression

$$b = (1 - f) \cdot a = 6356752.314245179 \text{ m.} \quad (1)$$

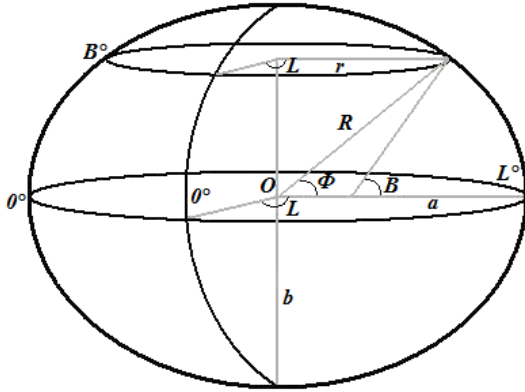


Fig. 2. WGS 84 Ellipsoid model

The earth ellipsoid's axes (equatorial radius a and polar radius b) are shown in Fig. 2. Furthermore, Fig. 2 illustrates angular coordinates of GPS receiver – latitude B and longitude L . Precise (reference) coordinates of Re-Reference system antenna are listed in [1], [2]. Table 2 contains latitude and longitude values in degrees.

Table 2. Angular coordinates of reference point in degrees

Latitude, B	56.941337861111°
Longitude, L	24.15578969694444°

Geocentric and geodetic latitude

GPS receivers calculate the geodetic latitude (to use it with maps) shown in Fig. 2. Geodetic latitude is defined as the angle between equatorial plane and perpendicular to Earth surface in position fix point. In Fig. 2 the geodetic latitude stands for B (in geodetic coordinate system). We need to know the geocentric latitude Φ to calculate radial errors (deviations). Geocentric latitude is defined as the angle between equatorial plane and straight line connecting position fix point on Earth's surface with WGS 84 Ellipsoid centre. Geodetic and geocentric latitudes are related by expression

$$b^2 \cdot \text{tg}(B) = a^2 \cdot \text{tg}(\Phi), \quad (2)$$

where a and b are equatorial and polar radii, respectively.

Fig. 3 shows graph of geodetic and geocentric latitude difference (equal to $B - \Phi$) depending on geodetic latitude B .

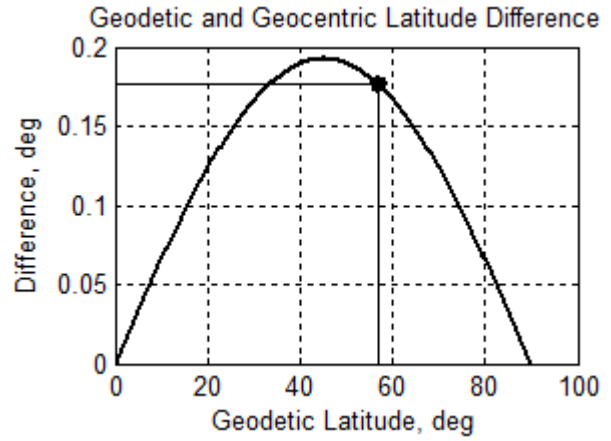


Fig. 3. Graph of geodetic and geocentric latitude difference

Latitude of reference point is shown in Fig. 3 by thick marker. Table 3 contains the values of geodetic and geocentric latitudes, as well as value of latitude difference.

Table 3. Re-Reference system antenna geodetic latitude, geocentric latitude and latitude difference

Geodetic latitude, B	56.941337861111°
Geocentric latitude, Φ	56.765150954050°
Latitude difference, $B - \Phi$	0.176186920612°

Geodetic and geocentric longitudes are identical and further will be referred to as longitude L .

Radial error

We have used formulas (with formula derivation) from [5] and [6] to calculate the radial error. Note that height of antenna (approximately 30.4 m) of horizontal error can be ignored in calculations. Our calculations show that height's influence on horizontal error in our case is insignificant. However the height cannot be ignored in such cases as accuracy estimation in mountains or on flying aircraft, as well as in case of precise phase measurements. Conversion formula from [6] can be used for these examples.

The radial error is defined as the length of vector with orthogonal projections – latitude error and longitude error:

$$ERR_R = \sqrt{ERR_{LAT}^2 + ERR_{LON}^2}. \quad (3)$$

Each of these projections is angular error in radians relative to reference point (in geodetic coordinate system). After multiplying each of them by Earth radius in this point, these angular errors are converted to arc lengths (the leg of right-angled triangle opposite to very small angle):

$$ERR_{LAT} = R \cdot \Delta\Phi, \quad (4)$$

$$ERR_{LON} = r \cdot \Delta L = R \cdot \cos(\Phi_0) \cdot \Delta L, \quad (5)$$

where r is the radius of parallel circle and Φ_0 is geocentric latitude in reference point. R is radius drawn from Earth's centre to reference point on the surface. This radius can be calculated by following formula:

$$R = \frac{a \cdot b}{\sqrt{a^2 \sin^2(\Phi_0) + b^2 \cos^2(\Phi_0)}}, \quad (6)$$

where Φ_0 is geocentric latitude in reference point.

ΔL and $\Delta\Phi$ in formulas (4-6) are corresponding increments in radians of angular coordinates.

Results of accuracy estimation over 24 hours for Graymark GPS-101 receiver

Results from [2] are based on the process samples taken over 1-2 hours. In order to get more reliable statistical estimations (assuming the process is stationary and ergodic) it is necessary to collect as much latitude and longitude deviation data as possible. However, in case of satellite navigation systems there is no strict stationary of time series. Assuming deviation fluctuations to have hidden periodicity, we suppose that process is stationary. The presence of hidden periodicity was confirmed by means of narrow-band filtration and spectral analysis of errors (see example in Fig.16). Thus, accuracy estimation results will be better averaged over long-term random process samples rather than short-time ones.

So we have collected latitude, longitude and height data from several GPS receivers over 24 hours in the following time interval (UTC time):

- the start of data taking: 19.03.2008, 12:53:32;
- the end of data taking: 20.03.2008, 12:54:36.

Fig. 4 shows radial error of Graymark GPS-101 receiver over 24 hours.

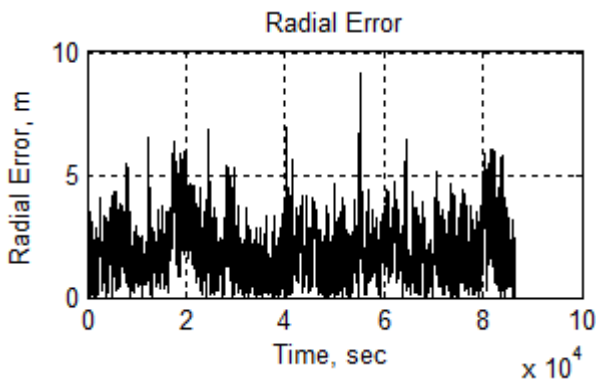


Fig. 4. Graymark GPS-101 receiver radial (horizontal) estimated over 24 hours

Fig. 5 illustrates graph of normalized estimated autocorrelation for Graymark GPS-101. This estimation was calculated by using the following expression

$$\rho[m] = \frac{1}{N\sigma^2} \cdot \sum_{n=0}^{N-|m|-1} (X[n+m] - \mu)(X[n] - \mu), \quad (7)$$

where σ^2 is the estimated variance and μ is estimated expectation (average) of random variable $X \in [0, N)$, where $N = 86464$ is a number of radial error's samples (each sample is 1 sec). Correlation window for data in Fig.4 is 2352.4 seconds. This window was estimated by width of rectangle with height 1 and area equivalent to absolute normalized autocorrelation curve occupied area.

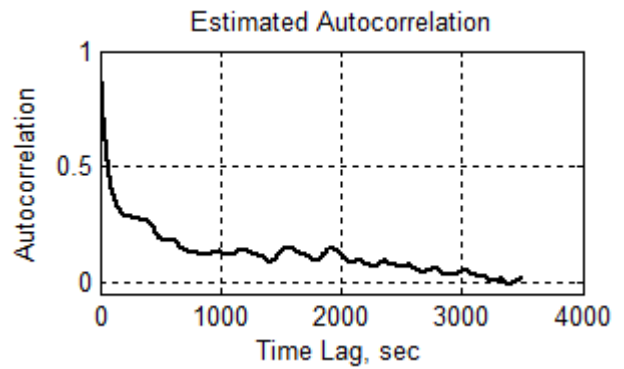


Fig. 5: Normalized estimated autocorrelation for Graymark GPS-101 receiver's radial error over 24 hours

Fig. 6,8,10 illustrate graphs of estimated probability density functions (histograms) and Fig. 7,9,11 illustrate estimated cumulative probability functions. It is considered that latitude and longitude error distributions are similar to the normal (Gauss) distribution. At the same time the radial error is defined as length of vector with orthogonal projections – latitude error and longitude error. That means the radial error can be represented by geometric sum of latitude and longitude errors, so its distribution is similar to Rayleigh distribution [7]. Graphs mentioned above show that experimental estimations of probability densities in Fig. 6,8 and probabilities themselves in Fig. 7,9 are indeed very similar to normal distribution's probability density function and cumulative density function. Graphs of radial error estimated probability density in Fig. 10 and radial error estimated probability in Fig. 11 is similar to Rayleigh distribution corresponding graphs. These conclusions were confirmed by calculating accuracy parameters for all GPS receivers (see Rayleigh distribution source data in [7] for information).

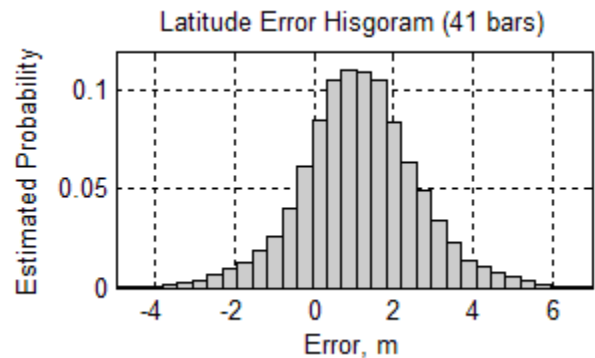


Fig. 6. Latitude error histogram for Graymark GPS-101 receiver latitude over 24 hours

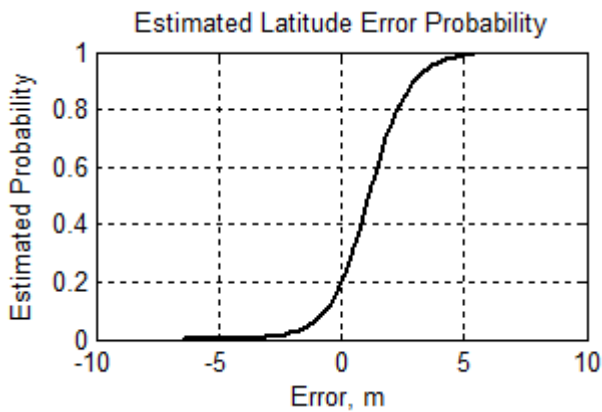


Fig. 7. Estimated latitude cumulative distribution function for Graymark GPS-101 receiver over 24 hours

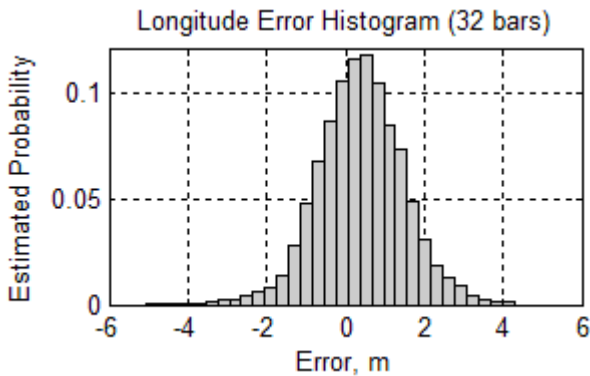


Fig. 8. Longitude error histogram for Graymark GPS-101 receiver latitude over 24 hours

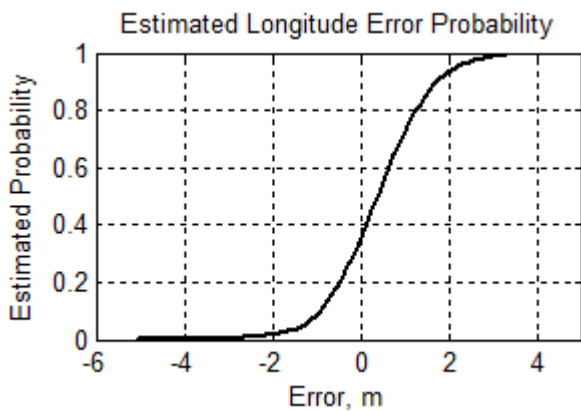


Fig. 9. Estimated longitude cumulative distribution function for Graymark GPS-101 receiver over 24 hours

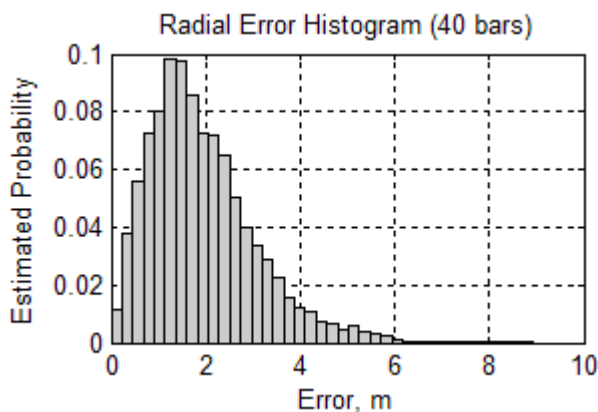


Fig. 10. Radial error histogram for Graymark GPS-101 receiver latitude over 24 hours

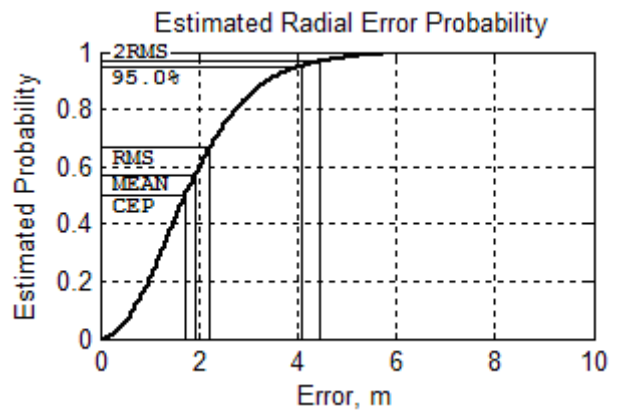


Fig. 11. Estimated radial error cumulative distribution function for Graymark GPS-101 receiver over 24 hours

Experimentally estimated accuracy parameters of various GPS receivers

Horizontal and vertical lines in Fig. 11 denote accuracy parameters of Graymark GPS-101 receiver. Table 4 contains these statistical characteristics for all GPS receivers which were used in experiment.

Table 4. Accuracy parameters of GPS receivers estimated over 24 hours process samples

	Graymark GPS-101	Garmin eTrex	Garmin GPSMAP 296	Magellan eXplorist 500
2DRMS	96.623% 4.4705 m	96.984% 4.3356 m	97.049% 4.4644 m	99.337% 2.6474 m
95%	95.0% 4.0753 m	95.0% 3.9762 m	95.0% 4.0832 m	95.0% 1.6186 m
DRMS	66.965% 2.2353 m	67.376% 2.1678 m	66.67% 2.2322 m	64.962% 1.3237 m
MEAN	57.282% 1.9281 m	57.404% 1.8668 m	57.023% 1.9447 m	64.683% 1.2735 m
CEP	50.0% 1.7343 m	50.0% 1.6736 m	50.0% 1.7376 m	50.0% 1.2153 m

After comparing results from Table 5 we can see that Magellan eXplorist 500 receiver has severely better accuracy parameters than ones of other GPS receivers. The reason for it is the fact that this GPS receiver supports SBAS (Satellite Based Augmentation System) mode and at the moment of experiment it was activated. This receiver used EGNOS satellites (European Geostationary Navigation Overlay Service), similar to WAAS satellites over North America.

Graph of radial error for Magellan eXplorist 500 over 24 hours is shown in Fig. 12.

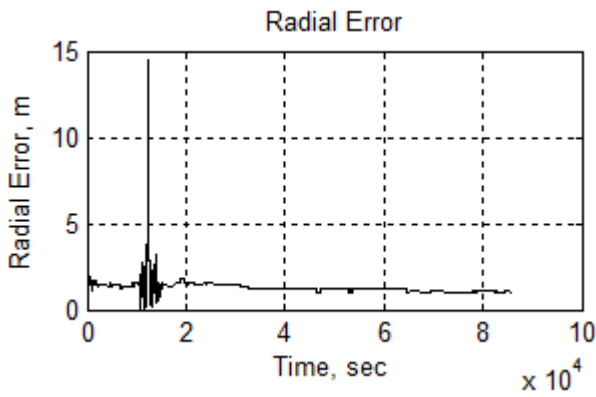


Fig. 12. Radial error for Magellan eXplorist 500

In Fig. 12 we can see that position fix accuracy was significantly worse in short interval (about 1 hour interval after approx. 3 hours of data taking).

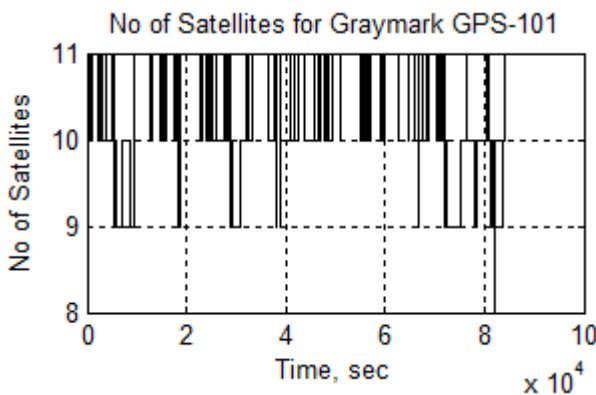


Fig. 13. Number of active satellites over 24 hours for Graymark GPS-101 receiver

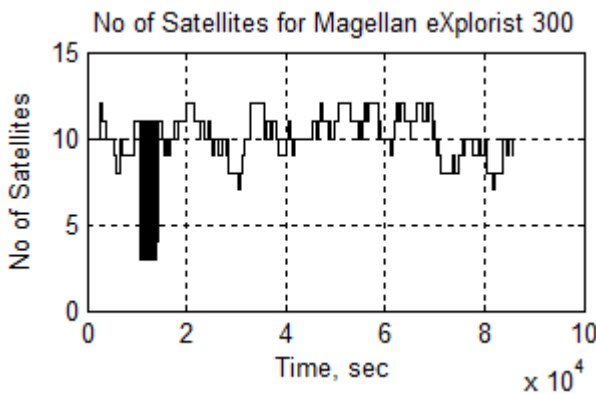


Fig. 14. Number of active satellites over 24 hours for Magellan eXplorist 500 receiver

During research it was determined that this degradation of accuracy was caused by changes in number of active satellites. GPS receiver uses only active satellites in position fix calculation process and outputs number of these satellites in NMEA 0183 protocol format described in [8]. Number of active satellites for Magellan eXplorist 500 over all 24 hours is shown in Fig. 14. This graph shows that number of active satellites was continually changing from 11 to 3 satellites exactly in same time interval when the accuracy degradation was observed. Note, that 3 satellites is minimal number of satellites that makes it possible to calculate position fix on horizontal plane (it is necessary to track at least 4 satellites for normal

navigation). We should add that area in accuracy decrease interval is not solid as it can be seen in Fig 14. At a larger scale of this time interval number of active satellites is severely unstable and it is constantly changing from 11 to 3 satellites.

At the same time, other GPS receivers did not detect such instability of active satellites number. For example, Fig. 13 shows number of active satellites for Graymark GPS-101 over 24 hours.

Note, that number of active satellites is one of the key factors that affects accuracy of GPS receiver.

Accuracy of GPS receiver is affected also by positional relationship of active satellites, not just their number. The volume of figure formed by constellation of satellites on sky and GPS receiver on surface is inversely proportional to GDOP parameter (Geometric Dilution of Precision). GPS receivers output various GDOP numbers (PDOP, HDOP, VDOP) in NMEA 0183 protocol format. For small GDOP values, when elevation angles of satellites are different, GDOP is considered to be "good". Pseudorange errors calculated from satellite signals are multiplied by respective GDOP to estimate horizontal and vertical location of receiver or timing errors. Accordingly there is Horizontal DOP (HDOP), Vertical DOP (VDOP), Position DOP (PDOP) and Time DOP (TDOP).

We found no obvious bond between HDOP and radial error; however it is possible they are correlated. There are smoothed curves of HDOP and radial error for Graymark GPS-101 (compare it with Fig. 4) in Fig. 15. These curves were obtained by calculating moving average (MA) of respective data arrays (see [9] for details). This can be done by using low-pass filter with 2700 seconds long impulse response. Fig. 15 illustrates that there is tendency for radial error to increase at greater HDOP values; however this tendency is not constant in time.

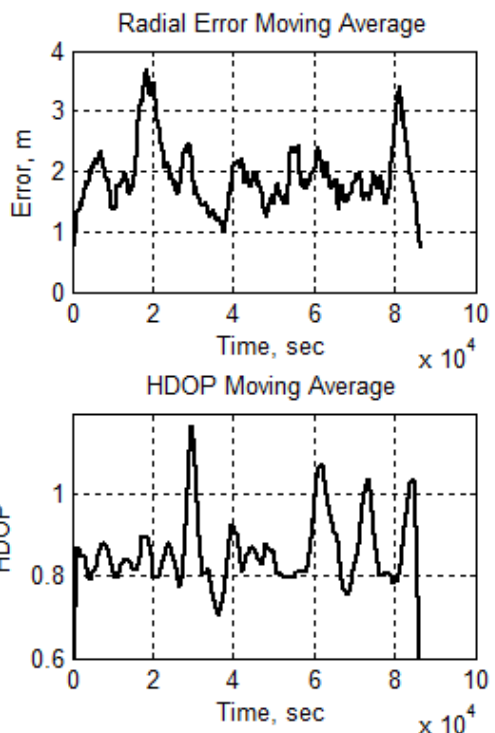


Fig. 15. Curves of HDOP and horizontal radial error for Graymark GPS-101 receiver smoothed by low-pass filter (impulse response is 2700 samples long)

We should note that smoothing of 24 hour process samples shows the presence of periodical sine-type trend in the longitude error with period of 24 hours. This trend corresponds to the peak in the spectral domain of longitude error data in Fig. 16. In order to achieve such smoothing we used low-pass filter with at least 6 hours long impulse response. There is also information about this trend on the web page [7].

For spectral power density estimation we use periodogram (Welch) method described in [9]. However, in order to find 24 hours spectral component in process sample, we need to have at least one full period of that spectral component in data. Since 24 hours of data (86465 samples) is all we have, all we can do is to estimate power spectrum over single section – without periodogram averaging procedure. Data sampling frequency of all GPS receivers except Garmin eTrex is 1 sample/sec. Garmin eTrex receiver's data sampling frequency is 0.5 samples/sec (1 data sample per 2 seconds). Discrete power spectrum of data section was calculated by applying DFT to it and squaring resulting magnitude values. We have used weighting by “hemming” window for data section. After weighting data section was padded with 15 additional sections of zeroes of the same length (86565 zeroes in each).

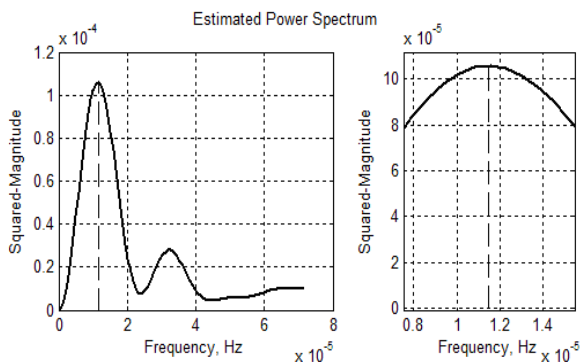


Fig. 16. Estimated power spectrum of longitude error. The frequency of peak is $F = 1.1565 \cdot 10^{-5}$ Hz and it corresponds to spectral component with period of $T = 864677.9$ sec = 24.02 h

Results of accuracy estimation over 2 hours for various GPS receivers

For comparison, Table 5 contains accuracy parameters for same GPS receivers estimated over last 2 hours of 24 hours data mentioned above. The time interval is following (UTC time):

- the start of data taking: 20.03.2008, 10:54:36;
- the end of data taking: 20.03.2008, 12:54:36.

Table 5. Accuracy parameters of GPS receivers estimated over 2 hours process samples

	Graymark GPS-101	Garmin eTrex	Garmin GPSMAP 296	Magellan eXplorist 500
2DRMS	97.792% 5.5454 m	97.333% 3.8381 m	96.069% 3.8176 m	100% 2.1166 m
95%	95.0% 5.0833 m	95.0% 3.5793 m	95.0% 3.6362 m	95.0% 1.0684 m
DRMS	63.347%	65.5%	69.889%	28.404%

	2.7727 m	1.919 m	1.9088 m	1.0583 m
MEAN	56.208% 2.4002 m	54% 1.7081 m	53.903% 1.6414 m	28.404% 1.0583 m
CEP	50.0% 2.1401 m	50.0% 1.6279 m	50.0% 1.4913 m	50.0% 1.0684 m

As we can see from Table 5, horizontal accuracy parameters can become worse for some GPS receivers, as well as improve for other ones when estimating over shorter process samples. There are several possible reasons for this. First of all, there are regular and random factors affecting accuracy, such as regular daily dislocation of satellites and random changes in troposphere and ionosphere conditions. Offset errors can also be caused by Selective Availability (SA), that is by intentional degradation of accuracy. SA hard error has period of few hours and it can't be averaged over short intervals.

CONCLUSIONS

This article reveals results of accuracy estimation for various GPS receivers over 24 hours. Results of our experiments show that distribution of latitude and longitude errors (in meters) is very similar to normal distribution. Distribution of radial error, which is geometric sum of these two errors, is very similar to Rayleigh distribution.

The accuracy of GPS receiver is much affected by the number of active satellites as well as their positional relationship. If the number of satellites is around minimal requirements the accuracy is worse. We didn't discover any obvious bound between radial error in horizontal plane and HDOP parameter, which describes positional relationship of satellites. However there might be some correlation. Further researches are required in this aspect.

In conclusion we should mention remarkable GPS Accuracy web page of David L. Wilson's [7], which is dedicated to accuracy estimation principles for GPS receivers. This web page contains tables of accuracy parameters and graphs of estimated probability distributions for various GPS receivers estimated over up to 30 days. Results of our experiments are similar to ones in [7].

REFERENCES

- [1] A.Kluga, M.Kulikovs, V.Semjonova, A.Zelenkovs. GPS user devices parameter control methods // Telecommunications and Electronics. - Riga: RTU, 2007. - Vol. 7, 45-48.
- [2] A.Kluga, J.Kluga, V.Semjonova, E.Grabs. Estimation of GPS Receiver Parameters with Re-reference System and Signal Simulator // Electronics and Electrical Engineering. - Kaunas: Technologija, 2008. - No. 6(78), 69-72.
- [3] How WGS 84 Defines the Earth // http://home.online.no/~sigurdhu/WGS84_Eng.html
- [4] NIMA Technical Report TR8350.2 // <http://earth-info.nga.mil/GandG/publications/tr8350.2/wgs84fin.pdf>
- [5] В.И.Пантелеев. Теория фигуры Земли // <http://www.pereplet.ru/pops/lectures/figura/tfe.html>
- [6] Dr. T.S. Kelso. Orbital Coordinate Systems, Part III // <http://www.celestrak.com/columns/v02n03> and http://space.kursnet.ru/ts_kelso/russian/vol02n03.sht
- [7] David L. Wilson's GPS Accuracy Web Page // <http://users.erols.com/dlwilson/gps.htm>
- [8] NMEA 0183 protocol // http://en.wikipedia.org/wiki/NMEA_0183
- [9] С.Л.Марпл. Цифровой спектральный анализ и его приложения: Пер. с англ. –М.: Мир,1990. – 584 с.