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DEDUCTIVE ENUNCIATION OF KINEMATICS OF THE CLASSICAL MECHANICS: GEOMETRICAL PROBLEMS AND PASSAGE FROM GEOMETRY TO KINEMATICS

KLASISKĀS MEHĀNIKAS KINEMĀTIKAS DEDUKTĪVS IZKLĀSTS: ĢEOMETRISKĀS PROBLĒMAS UN PĀREJA NO ĢEOMETRIJAS PIE KINEMĀTIKAS

Viktors Labendiks, researcher, Mr.sc.ing.
Riga Technical University, Aviation Institute
Address: 1, Lomonosov Street, Riga, LV-1019, Latvia
Phone: 28334004
E-mail: lvp@junik.lv

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The volume of all knowledge is constantly incremented; it is the most appreciable in the latest decades. In order that teaching was qualitative enough, the new knowledges is necessary add constantly to a material given to students. However the amount of the hours assigned even for the main disciplines, is not increased, and sometimes decreases because of occurrence of new disciplines which become also the cores for any specialty. In these conditions „old" knowledges should be presented more compactly, however their scientific level should not decrease. For disciplines with a physical and mathematical content the application of vectors (which have appeared thanks to needs of physics) allows to enunciate a material more compactly, and also the deductive method, created by an ancient Greek science, allows to achieve this objective (see about it in publications [1, pp.130-131; 2; 3, pp.19-22]).

Sentences to apply a deductive enunciation to kinematics of a classical mechanics have appeared in publications [4-11].

However, both for teachers, and for students, here there are some difficulties related to that circumstance, that school teaching of mathematics corresponds insufficiently to needs of the higher school. Disputes do not stop about how that to overcome, including how should be mathematical school education [see, for example, 12-16].

As an example what there are difficulties for the former schoolboys who have become by students of technical colleges, we will reduce school definition of a vector as „parallel transposition or the detrusion of space spotted by the assignment of steam of points" [17, p.9; 18, Part 1, p.72]. Knowingly the academician of L.S. Pontrjagin has termed such definition as "ugliness" [19, p.6] (he has not explained such title, probably, he meant, that it is not false, but at the same time partially and one-sidedly, and such it is really ugly). Such definition of a vector hampers to understand, that such a vector of a velocity, acceleration, force (about difficulties of formation of first two of these concepts before occurrence of concept of a vector see in [20])[1].

[1] It and other definitions of a vector are considered by the author of the given paper in [29, p.98].

As other example we will mention one of the basic concepts of a mechanics – an angular velocity. At first sight it seems very simple – judging by a title it is a velocity of a change of an angle that is the first derivative of this variable –“angle”.

Similarly, the rotary acceleration is the first derivative of an angular velocity and a flexon from magnitude “angle”. Similarly, the rotary acceleration is the first derivative of an angular velocity, and it is the flexon from magnitude „angle”. However in the school mathematics the angle is not at all an analytical variable which can be differentiated, but a geometrical figure [1, p.598; 16, pp.16, 102, 157, 163; 18, Part 2, p.284; 23, pp.10-11]. In the publication [14, p.99] it is scored, that „the formalized definition of an angle under the scheme by Hilbert is so difficult, that hardly pupils can understand it without the explanation by means of drawing”. We see even in the calculus of vectors manual for students of technical colleges such definition of an angle [21, p.15]: “As the angle between two vectors term the smaller part of a plane restricted to two rays, starting with one point and guided equally with the given vectors” (it corresponds to school geometrical concept of an angle).

Next without any definition there there is an angle between a vector and an axis (p.34), a rotation angle of rigid body, a vector of finite turn and a vector of an angular velocity of a rigid body (pp.157-159). Further the author of this mathematical work needed ascertainment of „mechanical sense” of angular velocity vector, next he reduces „the proof of existence of an angular velocity of a solid body” (pp.161-162).

It is necessary to note, that in the publication [1, p.598] it is written also: “Sometimes the angle is considered as a measure of rotational displacement of a ray round its beginning to the given standing. … In geometrical systems in which the pointwise-vector axiomatics is bottom supposed, under an angle understand defined metric magnitude related to two vectors …”. Jean Dieudonne Jean Dedonne about it has written: „I think …, in particular, about improbable misunderstandings and the paralogisms arising already for so simple concept as „an angle” when it is considered from the traditional point of view, meanwhile as from the point of view of a linear algebra it is not different, as key parameter in a plane rotation group” [12, p.18]. And further [12, p.22]: “It would be desirable to liberate as soon as possible the pupil from a strait jacket of traditional figures, mentioning them as seldom as possible (except, of course, such as points, straight lines and planes) and to use instead geometrical transformations of all plane or all space”.

Transformations about which here there is a speech, it is possible to present in the form of a composition of the basic four aspects of transformations: 1) parallel transpositions; 2) rotational displacements round an origin of coordinates; 3) symmetry (specular reflexions) to an origin of coordinates; 4) homotheties (tension-compression) with centre in an origin of coordinates. The idea that various geometrical systems can be interpreted, as theories of invariants of these transformation groups, was developed in 1872 in own „The Erlangen’s program” by Felix Klein (1849-1925) [15, pp.201-244; 22].

Population of first two kinds of transformations form a subgroup [1, pp.167-168] of eigen movements (the improper do not conserve orientation, i.e. is contain symmetry) [1, p.169; 15, p.247]. F. Klein has very clearly featured in 1908 how to construct an Euclidean geometry on a plane, which includes the affine geometry constructed on the basis of parallel transpositions [15, pp.247-256], and the metric geometry constructed on the basis of turns [15, pp.256-264].

He begins exposition of build-up of metric geometry with a phrase: „If we wish to get now further into field of metric concepts of geometry, in particular to erect concepts *of an angle* between two straight lines and *distance* between any two points (so far we could speak only about distance between two points, lying on an axis x or an axis y) we should study a complete group of motions” [15, p.256]. Further he notes, what „eventually with each rotational displacement there is match of some real number – *an angle of this rotational displacement*” [15, p.257]. And further: „Having erected thus common definition of an angle, we will make now also *distance* definition *between any two points* whereas till now we could compare only distances on the same straight line by means of parallel transpositions” [15, p.259]. In the same place he has offered also the second variant of axiomatics of geometry with words: „… And now let me specify in brief the basic thoughts of this second build-up of geometry. 1) Here begin, as before, with introduction *of points* and *straight lines* and with the sentences (postulates), concerning their *junction, a disposition, and continuity*. 2) But here inject – and it here is new – as new basic concepts, on the one hand *distance between two points* *(segment length)*, and on the other hand – *the angle between two straight lines,* and is erected concerning them by the postulates which essence consists in the statement of that *segments and angles can be measured by a well-known method by means of numbers*. 3) As characteristic (for the second build-up) postulates which substitutes … postulates of group of movements, the first sentence appears here about a congruence of triangles …" [15, pp.267-269].

Let's pay attention to what F. Klein does not notice in the first variant of build-up of metric geometry that he has two concepts of an angle, in the beginning as *an angle between two straight lines* (as well as in the second variant) and then as *an angle between initial and final positions of one straight line*, and he note before introduction of such concept: „We will imagine by analogy with parallel transpositions all turns also continuous, starting from an initial positions, and we will speak again about *trajectories* which describe thus by each point" [15, p.257].

In our opinion in it there is no contradiction, and, on the contrary, the idea is included implicitly here that we can consider an angle between two straight lines as initial value prior to the beginning of turn, then to add (or to take away) an angle of rotation and to gain final value of an angle between two straight lines after turn. In such a way it is possible to introduce concept of a motion of one straight line concerning another at first in one plane, and then and in space. Similarly for points it is possible to introduce concept of a motion of one point concerning another, and to gain operation of addition of distances between points (or segments) in the beginning along one straight line, and then under an angle to each other (after there is turn of segment). We gain a direct passage from geometry into kinematics on the basis of these considerations and with an allowance for thought reduced in the beginning of this paragraph about the continuous motions and trajectories.

We note, that the physicist and the physiologist Hermann Helmholtz (1821-1894) [16, c.267] was the first (in 1868) who used concept „motion" for geometry. Mario Pieri (1860-1913) has offered in 1899 geometry axiomatics in which was only two basic concepts – a point and a motion (transition) [16, c.267]. In the same year appeared David Gilberta's axiomatics (1862-1943 [23] with the basic concepts a point, direct, a plane and a congruence instead of a motion. As noted in [16, c.267], M. Pieri's axiomatics was gained bulky and it becomes difficult to viewing because of smaller number of the basic concepts.

Friedrich Schur (1856-1932) has offered in 1904 axiomatics in which instead of one of D.Gilberta's basic concepts „congruence" has been introduced the concept „superposition" [1, p.214; 16, pp.159-162], similar to concepts „motion" or „transition". As noted in [16, p.159], mathematicians did not wish to apply last two terms, as „these terms contact representation about motion process, extraneous for geometry". F.Klein has noted much earlier: „Were afraid what together with motions the alien element will enter into geometry – time" [15, p.268]. These fears should advise who is engaged an enunciation's of kinematics, that the variant of axiomatics of geometry with motions is the most suitable.

It is necessary to underline, that with the advent of concept of group of transformation of space though have become clear, that „superposition's" by F. Shur organize group [16, p.160], but only F. Klein has shown clearly, that this group consists of two movements – detrusion and the rotational displacement which standard are distances and angles. We will note still, that at Euclid „the plane angle is an obliquity to each other two lines" [16, p.253], that is closer to definition of an angle by F. Klein (in the beginning of the first and in second variants of his axiomatics). A.D. Aleksandrov comes eventually after the definitions of an angle as geometrical figure [16, pp.16, 102, 157, 163] to conclusion: „Angle definition as obliquities of one line to another expresses essence of this concept better, than its definition as steams of rays, or the part of a plane restricted to two rays. An angle in a delta circuit, an inscribed angle in a circle, etc. understand actually on Euclid" [16, p.255]. For some reason till now nobody recollected definitions of an angle by F. Klein.

Let's return now to concept „angular velocity". Publication [21] where „the mechanical sense" (in publication [9, p.80] „physical sense") an angular velocity is become clear was above mentioned and the proof of existence of an angular velocity of a solid body is reduced. There are also other operations where furnish similar proofs [for example, 7, p.4], and in [24, p.41] is directly told, that „a vector $\vec{\omega}$ – not a primitive magnitude and cannot be introduced by definition, and appears by reviewing of allocation of the linear velocities in a skew field". And it in spite of the fact that in earlier operation of the first of authors of this book by an angular velocity for a point it is termed derivative of an angle of polar coordinates [25, p.80] and the first derivative of an angle of cylindrical co-ordinates, and the flexon of this angle is termed by a rotary acceleration [25, p.97]; further for a solid body the concept about a dihedral angle of rotation of a skew field [25, p.102], and then similar titles for an angular velocity and a skew field rotary acceleration [25, pp.103-104]) is introduced.

Proceeding from stated above, it is necessary to become clear up to the end the following: if straight lines are not point sets with certain properties, and self-maintained initial plants of structure of space, angles – the necessary metric magnitudes characterizing an incongruity and motions of these plants; and if thus angles – analytical variables angular velocities and accelerations is also magnitude „primitives", equivalent with the linear velocities and accelerations and not spotted by them (by the way, Euler's formula, on the contrary, spots the linear velocities of any point of a solid body through an angular velocity of this skew field).

Let's note that not only M. Pieri tried to do without straight lines, as basic concepts. In mentioned above is pointwise-vector axiomatics straight lines also are not the basic concepts and are spotted by means of guiding unit vectors. Authors of operation [11, p.7] state: „The usual space surrounding us has structure of only pointwise space. But the possibility of introduction of the linear structure is necessary for constructive build-up of kinematics in it. Therefore as initial space in which all reviewings are spent, the certain three-dimensional Euclidean (is pointwise-vector) space" is taken axiomatic. From this statement the deduction directly follows, that adding to structure of pointwise space of other structures, in particular straight lines, happens subjectively, only in heads of people. Hence, in this case it is gained, that the space existing round us objectively does not possess property of an isotropy. But Anri Poincare has written in 1906: „What, first of all, properties of space in the true sense? … Here some of its most essential properties: 1. It is continuous. 2. It is perpetual. 3. It has three measuring. 4. It is homogeneous, i.e. its all points are identical among themselves. 5. It is isotropic, i.e. all straight lines which transit through the same point, are identical among themselves" [26, p.42]. And he has written in 1910: „That such geometry for the philosopher? This study of some group? Which? Groups of movements of solid bodies" [26, p.364].

Let's pay attention to two facts, linking geometry and kinematics with dynamics and confirming, that presence of straight lines and the isotropy related to them are attributes (essential properties) spaces.

The first can mention that fact, that according to theorem by Emmi Noether (1882-1935) of symmetry of space directly are related to conservation laws: the momentum conservation law is a consequence of homogeneity of space, and the conservation law of a moment of momentum is a consequence of an isotropy of space[2]. Under homogeneity still understand invariancy concerning transposition or detrusion transformation, as an isotropy – invariancy concerning rotational displacement transformation [27, p.41].

The second fact – the amount of the differential equations for exposition of translational and rotational movements of the free solid body in three-dimensional space (three equations of forces and three equations of the moments) corresponds to number of degree of freedoms of such skew field (3 of linear and 3 angular); the equations of the moments are separate of the equations of forces and are not reduced to them.

The mathematics is uniform, therefore methods of the linear algebra grounded on is pointwise-vector axiomatics by Hermann Weyl (1885-1955) with concept „vector space"[3], it is necessary to use more widely as utility of application of such methods for kinematics is shown in [11]. However, in our opinion, it is necessary to make passage to these methods, having reconsidered the basic concepts, proceeding from the axiomatics grounded on ideas by F. Klein, and introducing concept of a vector not by means of system of postulates, and as the mathematical plant, uniting two basic metric magnitudes – distance (segment length) and an angle, characterizing standards of an incongruity (or motions from each other) points and straight lines, in the form of a segment of a straight line of certain

[2] Later it has been erected, that requirements of the theorem by Emmi Noether are only sufficient, but not necessary, but it has not changed an essence of the matter. Necessary and sufficient conditions of existence of conservation laws have been discovered by N.H. Ibragimov [27, pp.42-43].

[3] H. Weyl considered that the concept „vector space" belongs to algebra, instead of geometry [28, p.23].

length with a direction characterized by an angle, digitized from other given straight line. Then it is possible to relate easily concept of a mathematical vector to physical vectors (velocities, accelerations, forces, etc.), historically arisen because of necessity to add in some cases to a scalar (size) concept „direction”[4].

So, we see, that for a deductive account of kinematics it is necessary to be converted not only to the basic geometrical concepts, but also there is a necessity of a select of the most suitable variant of axiomatics of geometry. And it is necessary to mean, that there were also other variants of axiomatics of geometry[5], the indisputable axiomatics still is not present, and consequently there will be new variants. In this connection it is possible to tell, that the phrase of academician A.Ju. Ishlinsky said at lecture, read for science officers, engineers and teachers of a mechanics (Kiev, 1968): „You know, that the mechanic by Galilee-Newton till now in a due standard not axiomatic account unlike the geometry which axiomatization has been terminated by great mathematician D. Hilbert” [31, p.473], was fallacy (and has misled many other things).

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[4] In more details the concept „direction” is considered in operation of the author [29, pp.98-100].
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Labendiks V. Klasiskās mehānikas kinemātikas deduktīvs izklāsts: ģeometriskas problēmas un pāreja no ģeometrijas pie kinemātikas

Šajā pētījumā ir aplūkoti Eiklīda ģeometrijas izklāsta aksiomātiski dažādi varianti atkarībā no piemērotas versijas, kas palīdzētu atvieglot pāreju no šīs ģeometrijas pie klasiskās mehānikas kinemātikas. Tāpat ir izpētītas pretrunas un uztveres grūtības, kuras rodas skolēniem un studentiem, sastopoties ar tādu fundamentālu klasiskās mehānikas jēdzienu definēšanu, kā vektors, leņķis un leņķiskais ātrums. Autors piedāvā oriģinālu deduktīvu kinemātikas izklāsta ceļu, balstoties uz Feliksa Klejna ģeometrisku ideju bāzes par telpas transformācijām, kustībām un diviem galvenajiem metriskajiem lielumiem – attālumiem starp diviem punktiem (nogriežņa garumu) un leņķi starp divām taisnēm. Autors papildina šo pieeju, ieviešot savādāku mainīgā vektora un tā atvasinājuma definēšanu (ar attēlu palīdzību un formulām ar robežām). Autorā piedāvātās aprakstošās definīcijas (nosakāmā objekta īpašību apraksts, kas brīvā vektora gadījumā satur garumu un virzienu, bet kārtējiem atvasinājumiem – atbilstošos atvasinājumus pēc garuma un virziena). Tādas aprakstošās definīcijas kļūst iespējamas pēc tam, kad ir izvestas formulas, kurās atvasinājumi pēc garuma un virziena ietilpst tiešā veidā.

Labendik V. Deductive enunciation of kinematics of a classical mechanics: geometrical problems and passage from geometry to kinematics

In this publication are analyzed various variants of an axiomatic enunciation of Euclidean geometry from the point of view of selection of suitable version for passage from geometry to kinematics of classical mechanics. Too are examined the contradictions and arising because of them difficulties of perception by schoolboys and students of definitions of such basic concepts, as vector, angle and angular velocity. By author is proposed the way of a deductive enunciation of kinematics on the basis of F.Klejna's geometrical ideas about space transformations, the motions, and two basic metric variables – distance between two points (segment length) and an angle between two straight lines, and also on the basis of offered by the author of the present article of passage from constructive definitions of variable vector and its derivatives (by means of drawings and formulas with limits) to descriptive definitions (describing properties of the defined object, in case for free vector – in possession of length and a direction, and for following derivatives – in possession of corresponding derivatives of length and a direction).

Лабендик В. Дедуктивное изложение кинематики классической механики: геометрические проблемы и переход от геометрии к кинематике

В этой публикации кратко рассмотрены различные варианты аксиоматического изложения евклидовой геометрии с точки зрения выбора подходящей версии для перехода от геометрии к кинематике классической механики. Также исследованы противоречия и возникающие из-за них трудности восприятия школьниками и студентами определений таких фундаментальных понятий, как вектор, угол и угловая скорость. Автором предложен путь дедуктивного изложения кинематики на основе геометрических идей Феликса Клейна о преобразованиях пространства, движениях и двух основных метрических переменных – расстояние между двумя точками (длина отрезка) и угол между двумя прямыми, и также на основе предложенной автором данной статьи замены конструктивных определений переменного вектора и его производных (посредством рисунков и формул с пределами) дескриптивными определениями (описание свойств определяемого объекта, в случае свободного вектора – это обладание длиной и направлением, и для следующих производных – обладание соответствующими производными длины и направления). Такие дескриптивные определения становятся возможными после вывода формул, в которых производные длины и направления присутствуют в явном виде.