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INTERPOLATION OF THE MARKOV CHAIN AND ESTIMATION

ANDREJS MATVEJEVS

Riga Technical University

Kaļķu 1, LV-1628, Rīga, Latvia

E-mail: `Andrejs.Matvejevs@rtu.lv`

The possibility of identifying nonlinear time series using nonparametric estimates of the conditional mean and conditional variance is studied. Most nonlinear models satisfy the assumptions needed to apply nonparametric asymptotic theory. Sampling variations of the conditional quantities are studied by simulation and explained by asymptotic arguments for the first-order nonlinear autoregressive processes. The paper deals with the identification and prediction problems of the autoregressive models of nonlinear time series. We will remind that assumption about normal distribution of time series allows to calculate the conditional expected value of phase variable as linear functional of its past values $\{x_t, s < t\}$. We should deal with the estimation of unknown function in nonlinear difference equation of the first order with usual kind of information about the distribution law. In many applied problems of time series regression analysis one can write

$$x_{n+1} = f(x_n) + \sigma_n \xi_{n+1}, \tag{1}$$

where ξ_n is (i.i.d.)-random error, with zero average and variance one. For searching for the function $f(x_n)$ we consider the model in which unknown function depends on the elements of Markov chain. We can write that the conditional expected value of random variable looks like

$$\mathbf{E}\{x_{n+1}|\mathcal{F}^n\} = \mathbf{E}\{x_{n+1}|x_n\} = \sum_y p(x_n, y)y = f(x_n), \tag{2}$$

that determines non-linearity of functional dependence x_{n+1} from x_n . The next step is to investigate the Markov chain built on the equation $u_{n+1} = h(u_n) + g(u_n)\xi_{n+1}$, which is closely connected with equation (1). So we need to express the functions $h(u_n)$ and $g^2(u_n)$ through the transition probabilities. For this purpose the main task is to find the transition probabilities of Markov chain on the basis of observed values of the time series.

If we denote $R(l)$ as the set of matrix of Markov chain's observations $L = ||l_{kj}||, (k, j = 1, ..., m)$ having property $\sum\limits_{k,j=1}^{m} l_{kj} = l$ with the initial state U_k of the Markov chain, then the unbiased estimates of transition probabilities $P_{kj}^{(l)}$ from state U_k to state U_j for l steps are

$$\hat{P}_{kj}^{(l)} = \frac{\sum\limits_{L \in R(l)} P_{kj}^{(l)}(L) \cdot P_{kj_n}^{(n-l)}(N - L)}{P_{kj_n}^{(n)}}, N \in R(n), \tag{3}$$

where j_n - final state U_{jn} index .