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Mathematical modeling of multipolar double fed induction generator with two phase secondary winding

G.Dilevs, E.Jakobsons

Riga Technical University, Faculty of Power and Electrical Engineering
1 Kalku Str., LV-1050, Riga, LATVIA
Tel.: +371 67089333
Fax.: +371 67089039
E-Mail.: guntis.dilevs@gmail.com, edgars.jakobsons@inbox.lv
URL: www.rtu.lv

Abstract: *This paper presents the results of mathematical modelling for the double fed induction generator, which has the ability to operate at a low rotation speed. The generator is multi pole with all of the windings placed on the stator. Rotor is tooth-like and has no windings on it. Primary winding is three phase, secondary winding is two phase. This generator can be applied for directly driven turbine without using the gearbox.*

Keywords: induction generator, double fed, slow speed.

1. Introduction

This paper presents mathematical modelling and its results of multipolar double fed induction generator with two phase secondary winding. The modelling was done with the target to design such a generator and to evaluate its generated power to the grid and to the secondary load. The research was targeted to the slow speed induction generator which can be applied as a directly driven double fed induction generator for wind turbnes. Part of the research was based on the prototype induction machine with a single phase secondary winding [7].

2. Special features of the multipolar double fed induction generator with two phase secondary winding

The construction diagram of the double fed induction generator is shown in the Fig. 1. Primary winding (A-X, B-Y, C-Z) is three phase and secondary winding (a-x, b-y) is two phase to keep the electromagnetic symmetry of the system. Both of the windings are placed in the slots of the stator. Rotor is tooth-like and has no windings on it. The number of rotor teeth Z_R is 25 and it corresponds to 25 pole pairs. For creation of the required electromagnetic link between windings it is necessary to have appropriate phase shifts between the processes going in pole extensions. For this purpose the pole extensions are divided into four groups. Each pole extension from the same phase has a phase shift of 90^0 between two groups. Two groups are embraced with the one coil of one secondary phase winding. This feature gives the phase shift for the currents i_a and i_b of 90^0. The pole extensions in the same group are divided with grooves where the primary windings A-X, B-Y and C-Z are placed. The step between the mixed pole extensions belonging to one group equals $t_1 = 2t_Z/3$, and that between the mixed pole extensions belonging to mixed groups equals $t_2=23t_Z/12$. This provides a phase shift of the pole extensions of 240 degrees within the same group, and of 30 degrees for mixed group [6].

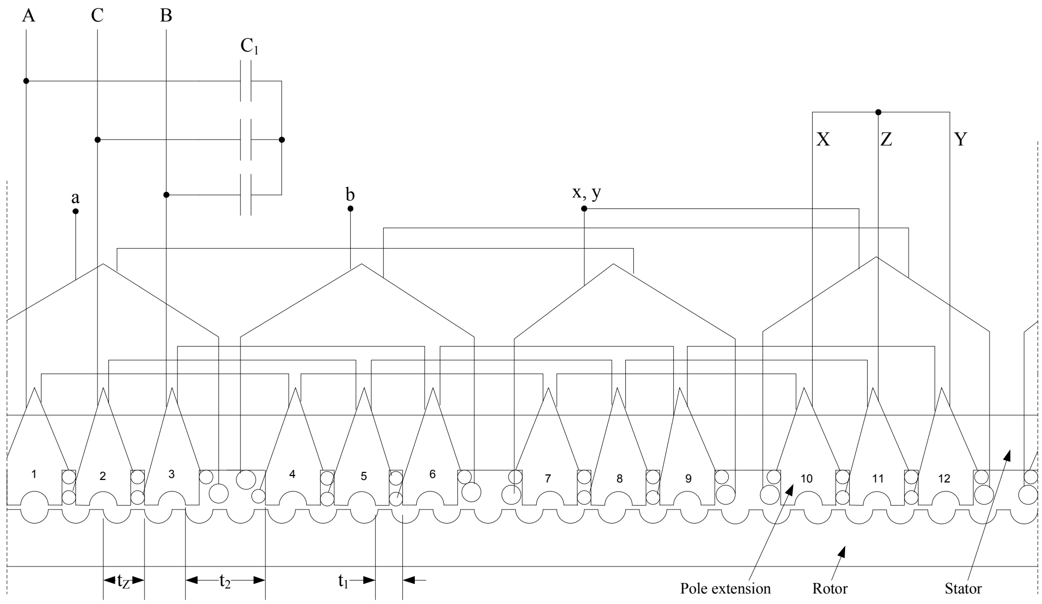

Figure 1: The construction diagram of the multipolar induction generator with two phase secondary winding; t_Z – tooth step of the rotor; t_1 – tooth step between pole extensions; t_2 – tooth step between pole extension groups.

3. The basic equations of the mutlipole induction generator with two phase secondary winding

The equations are similar to those for the conventional induction machine having two phase secondary winding on the rotor. The equations given below describe the electromagnetic processes in the multipolar induction machine. Magnetic flux linkage can be obtained with the matrix equation

$$[\Psi_{ik}] = [w_{ik}] \times [\lambda_{ik}] \times [w_{ik}]^T \times [i_{ik}], \qquad (1)$$

where

$[\Psi_{ik}] = \begin{bmatrix} \Psi_A \\ \Psi_B \\ \Psi_C \\ \Psi_a \\ \Psi_b \end{bmatrix}$ is the column matrix of magnetic flux linkages of the windings;

$[i_{ik}] = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \end{bmatrix}$ is the column matrix of currents in the windings;

$$[w_{ik}] = \begin{bmatrix} w_k & 0 & 0 & w_k & 0 & 0 & w_k & 0 & 0 & w_k & 0 & 0 \\ 0 & w_k & 0 & 0 & w_k & 0 & 0 & w_k & 0 & 0 & w_k & 0 \\ 0 & 0 & w_k & 0 & 0 & w_k & 0 & 0 & w_k & 0 & 0 & w_k \\ w_k & w_k & w_k & 0 & 0 & 0 & -w_k & -w_k & -w_k & 0 & 0 & 0 \\ 0 & 0 & 0 & w_k & w_k & w_k & 0 & 0 & 0 & -w_k & -w_k & -w_k \end{bmatrix}$$

is the rectangular matrix of the winding turns per each coil according to Fig. 1, where the rows represent the phases and the columns represent the pole extensions. The sign "–" shows the secondary winding direction according to the primary winding direction [1].

$[\lambda_{ik}] = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \lambda_{12} \end{bmatrix}$ is the diagonal matrix of magnetic conductivities, where $\lambda_k = a_0 + a_1 \cos(z_R \alpha - \gamma(k-1))$ is magnetic conductivity of K-th pole extension represented by the Fourier series as a function of the turning angle α of the rotor, a_0 – constant component, a_1 – component of the first harmonic [6]. The equations, which express magnetic flux linkage for each phase, are

$$\Psi_A = w_{k1}\begin{pmatrix} \lambda_1\left(w_{k1}i_A + w_{k2}i_a\right)+ \\ +\lambda_7\left(w_{k1}i_A - w_{k2}i_a\right)+ \\ \lambda_4\left(w_{k1}i_A + w_{k2}i_b\right)+ \\ +\lambda_{10}\left(w_{k1}i_A - w_{k2}i_b\right)+ \end{pmatrix} =$$

$$= 4a_0 w_{k1}^2 i_A + 2a_1 w_{k1} w_{k2} i_a \cos\left(Z_R\alpha\right) + ;(2)$$

$$+ 2a_1 w_{k1} w_{k2} i_b \cos\left(Z_R\alpha - 90^0\right)$$

$$\Psi_B = w_{k1}\begin{pmatrix} \lambda_3\left(w_{k1}i_B + w_{k2}i_a\right)+ \\ +\lambda_9\left(w_{k1}i_B - w_{k2}i_a\right)+ \\ \lambda_6\left(w_{k1}i_A + w_{k2}i_b\right)+ \\ +\lambda_{12}\left(w_{k1}i_B - w_{k2}i_b\right)+ \end{pmatrix} =$$

$$= 4a_0 w_{k1}^2 i_B + 2a_1 w_{k1} w_{k2} i_a \cos\left(Z_R\alpha - 120^0\right) + ;$$

$$+ 2a_1 w_{k1} w_{k2} i_b \cos\left(Z_R\alpha - 210^0\right)$$

$$\Psi_C = w_{k1}\begin{pmatrix} \lambda_2\left(w_{k1}i_C + w_{k2}i_a\right)+ \\ +\lambda_8\left(w_{k1}i_C - w_{k2}i_a\right)+ \\ \lambda_5\left(w_{k1}i_C + w_{k2}i_b\right)+ \\ +\lambda_{11}\left(w_{k1}i_C - w_{k2}i_b\right)+ \end{pmatrix} =$$

$$= 4a_0 w_{k1}^2 i_C + 2a_1 w_{k1} w_{k2} i_a \cos\left(Z_R\alpha - 240\right) + ;$$

$$+ 2a_1 w_{k1} w_{k2} i_b \cos\left(Z_R\alpha - 330^0\right)$$

$$\Psi_a = w_{k2}\begin{pmatrix} \left(\sum_{k=1}^{6}\lambda_k\right) w_{k2} i_a + \left(\lambda_1 - \lambda_7\right) w_{k1} i_A + \\ +\left(\lambda_3 - \lambda_9\right) w_{k1} i_B + \\ +\left(\lambda_2 - \lambda_8\right) w_{k1} i_C \end{pmatrix} = ;$$

$$= 6a_0 w_{k2}^2 i_a + 2a_1 w_{k1} w_{k2} i_A \cos\left(Z_R\alpha\right) +$$

$$+ 2a_1 w_{k1} w_{k2} i_B \cos\left(Z_R\alpha - 120^0\right) +$$

$$+ 2a_1 w_{k1} w_{k2} i_C \cos\left(Z_R\alpha - 240^0\right)$$

$$\Psi_b = w_{k2}\begin{pmatrix} \left(\sum_{k=1}^{6}\lambda_k\right) w_{k2} i_b + \left(\lambda_4 - \lambda_{10}\right) w_{k1} i_A + \\ +\left(\lambda_6 - \lambda_{12}\right) w_{k1} i_B + \\ +\left(\lambda_5 - \lambda_{11}\right) w_{k1} i_C \end{pmatrix} =$$

$$= 6a_0 w_{k2}^2 i_b + 2a_1 w_{k1} w_{k2} i_A \cos\left(Z_R\alpha - 90^0\right) +$$

$$+ 2a_1 w_{k1} w_{k2} i_B \cos\left(Z_R\alpha - 210^0\right) +$$

$$+ 2a_1 w_{k1} w_{k2} i_C \cos\left(Z_R\alpha - 330^0\right)$$

The equations of voltage for this type of generator are the same as for the conventional induction machine:

$$\begin{aligned} u_{AB} &= i_A R_1 + \frac{d\Psi_A}{dt} \\ u_{BC} &= i_B R_1 + \frac{d\Psi_B}{dt} \\ u_{CA} &= i_C R_1 + \frac{d\Psi_C}{dt} \\ u_a &= i_a R_2 + u_{Ca} + \frac{d\Psi_a}{dt} \\ u_b &= i_b R_b + u_{Cb} + \frac{d\Psi_b}{dt} \end{aligned} \qquad (3)$$

where (2)

u_{AB}, $u_{BC,}$ u_{CA} - the grid line voltages;
i_A, i_B, i_C - the phase currents of the generator;
i_a, i_b - the currents in secondary circuit;
R_1, R_2 - the active resistance of primary and secondary windings;
u_{Ca},u_{Cb} is the voltage drop on capacitor C_2 of the secondary winding.
Self-inductances L_1, L_2 and mutual inductance L_m can be calculated according to equation (4) [3].

$$\begin{aligned} L_1 &= 4a_0 w_{k1}^2 \\ L_2 &= 4a_0 w_{k2}^2 \\ L_m &= 2a_1 w_{k1} w_{k2} \end{aligned} \qquad (4)$$

where

L_1 – self inductance of one phase of primary winding;

L_2 – self inductance of one phase of secondary winding;

L_m – mutual inductance between primary and secondary windings.

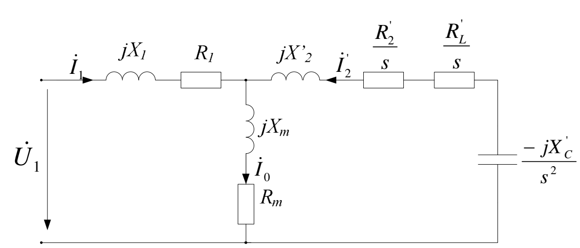

Figure 2: The circuit diagram of the multipole induction machine with two phase secondary winding [6].

According to the construction diagram in Figure 1 and the equivalent circuit diagram in Figure 2 it is possible to write the complex equations [3].

$$\dot{U}_1 = -\dot{E}_1 + \dot{I}_1 \hat{Z}_1 ;$$
$$\dot{U}_2^{'} = -\dot{E}_2^{'} - \dot{I}_{21}^{'} \hat{Z}_{2S}^{'} ; \quad (5)$$
$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2^{'} ,$$
$$\dot{E}_1 = \dot{E}_2^{'} = -j\sqrt{2}\pi w_1 \Phi f_1 k_w .$$

where
E_1 = *emf* generated in the windings,
w_1 – number of windings in the phase coil;
Φ – magnetic flux;
f_1 – frequency;
k_w – winding factor.
$\dot{I}_1$ – primary current;
$\dot{I}'_2$ – secondary current;
$\dot{I}_0$ – magnetizing current.

$$\hat{Z}_1 = R_1 + jX_1 ;$$
$$\hat{Z}_{2S}^{'} = \frac{R_2^{'}}{s} + jX_2^{'} - j\frac{X_{C2}^{'}}{s^2} \quad (6)$$

where
R_1- active resistance of the primary winding,
R_2' – active resistance of the secondary winding, which is reduced to the stator,
X_1 – reactive impedance of the primary winding,
X_2' – reactive resistance of the secondary winding,
s – slip,
X_{C2}' – impedance of the capacitors in the secondary winding;

Voltage of the secondary load equals

$$\dot{U}'_L = \frac{R_L}{s}\dot{I}_2^{'} . \quad (7)$$

where
R_L – secondary load resistance.

To calculate the output power of the induction machine it is necessary to obtain the currents $\dot{I}_1$, $\dot{I}'_2$, $\dot{I}_0$ according to the slip s. There fore the power S_1 generated to the grid and active power P_2 generated to the secondary load R_L equals

$$\dot{S}_1 = m_1 \dot{U}_1 \dot{I}_1^{*} = P_1 + jQ_1 ; \quad (8)$$
$$P_2 = m_2 I_2^{'2} R_L , \quad (9)$$

where
m_1 – number of phases in the primary winding;
m_2 – number of phases in the secondary winding.

The efficiency factor η equals

$$\eta = \frac{P_1 + P_2}{P_1 + P_2 + \Delta P_1 + \Delta P_m + \Delta P_2 + \Delta P_{mech}} , \quad (10)$$

where
$\Delta P_1 = m_1 I_1^2 R_1$ - electrical losses in the primary windings;
$\Delta P_m = m_1 I_0^2 R_m$ - magnetizing losses;
$\Delta P_2 = m_2 I_2^2 R_2$ - electrical losses in the secondary windings;
ΔP_{mech} – mechanical losses in the shaft and bearings [3].

Within the increase of the slip s of the generator, it is important to keep the same value of the current I_2. This can be done by changing the load resistance R_L. Within the increase of the slip s, increases the load resistance R_L and the power P_2 generated in the secondary circuit.

4. The results of the mathematical modelling

The mathematical modelling was done in two parts. In the first part the magnetic field of the multipolar double fed induction generator magnetic system was analysed. In the second part the results were used for generated power calculations. The analysis of magnetic field was done for one pole extension to obtain the magnetic flux Φ value. The components a_0 and a_1 correpsonds to the magnetic conductivity of the one pole extension. According to that, there was established model of the magnetostatic field for calculating magnetic flux through the air gap of the pole extension.

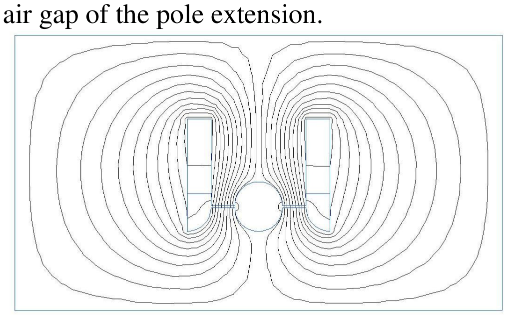

Figure 3: Magnetic circuit of $\Lambda_{max.}$

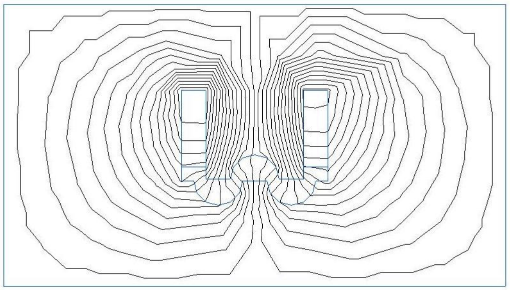

Figure 4: Magnetic circuit of Λ_{min}.

The calculations was done for two states – maximum, where rotor tooth is against tooth of one pole extension and the magnetic flux has the maximum value and minimum state, where rotor tooth is against the gap of the pole extension and the magnetic flux has the minimum value [2].

The the magnetic conductivity Λ_{max} and Λ_{min} can be obtained by

$$\Lambda_{max} = \frac{\Phi_{max}}{F}, \ \Lambda_{min} = \frac{\Phi_{min}}{F} \qquad (11)$$

where

Φ_{max}, Φ_{min} – magnetic flux,

F – magnetic force.

According to (11) the components a_0 and a_1 equals

$$a_0 = \frac{\Lambda_{min} + \Lambda_{max}}{2}; \ a_1 = \frac{\Lambda_{max} - \Lambda_{min}}{2} \qquad (12)$$

The parameters of the equivalent circuit diagram are showed in Table 1.

Table 1: The calculated parameters of the multipolar induction generator

Parameter	Value	Units
a_0	0,00000525	H
a_1	0,00000317	H
w_1	60	
w_2	60	
L_1	0,075	H
L_2	0,11	H
L_m	0,023	H
R_1	2,11	Ω
R_2	0,82	Ω
X_1	23,75	Ω
X_m	7,17	Ω
X_2'	53,43	Ω
R_2'	1,23	Ω
X_{C2}'	72,40	Ω
R_L	3,30	Ω

With the capacitance X_{C2}' generator achieves its maximum power when the slip reaches the value -1,11. The power curves of the generator are showed in the Figure 5.

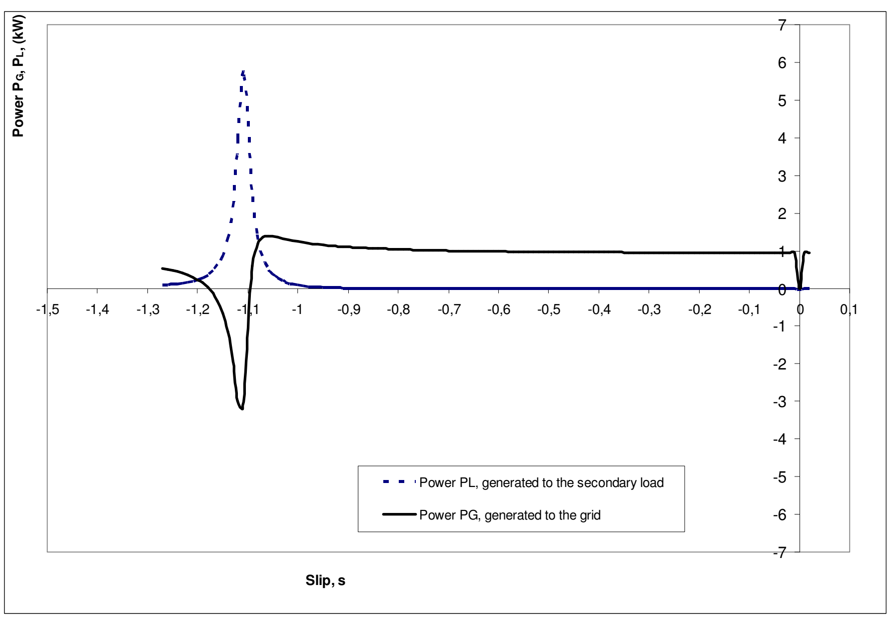

Figure 5: The generated power to the grid P_G and to the secondary load P_L.

According to the equation (10) the calculations for the generated power at the slip value -1,11 was done and the results are showed in the Table 2.

Table 2: The calculation results

No	Calculated parameters of the generated power	unit	Value
1	Primary line voltage, U_1	V	380
2	Secondary voltage, U_L	V	91
3	Grid frequency, f_1	Hz	50
4	Power transferred to the grid, P_G	kW	3,2
5	Power transferred to the secondary load R_L, P_L	kW	5,2
6	Slip, s	-	-1,11
7	Capacitors, C_2	μF	66
8	Electrical losses in the primary winding, ΔP_1	kW	0,13
9	Magnetizing losses, ΔP_m	kW	0,60
10	Electrical losses in the secondary windings, ΔP_2	kW	1,4
11	Efficiency factor, η		0,78

5. Conclusions

The results of the mathematical modelling show that multipole double fed induction generator is reliable for power generation in slow speed applications e.g. wind turbines.

The two phase secondary winding provides the electromagnetic symetry of the generator and allows to aplly the equivalent circuit diagram for generated power calculations.

Parameters of the eqivalent circuit diagram can be determined from the mangetic flux analysis for one pole extension.

The generator achieves its maximum generated power at the resonance mode, according to that the power factor control device must be applied in the secondary winding to achieve maximum generated power at any rotation frequency of the generator.

Acknowledgement

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