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ABSTRACTS

**LATVIJAS MATEMĀTIKAS BIEDRĪBA
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CALCULATION OF A COIL'S IMPEDANCE FOR THE CASE WHERE ELECTRIC AND MAGNETIC PROPERTIES OF A CONDUCTING HALF-SPACE DEPEND ON A VERTICAL COORDINATE

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External electromagnetic field can introduce permanent changes in a surface layer of a conducting material. As a result, electric and magnetic properties of the material may change with depth. In such cases, mathematical models of the process should be modified in order to take this variability into account (examples include surface hardening and decarbonization of metals). Conducting medium with constantly varying electric conductivity and magnetic permeability can be approximated by a large number of layers with constant electric and magnetic properties using, for example, the solution [1] for multilayer media with constant properties. Such an approach is considered in [2] where up to 50 layers are used to approximate varying electric conductivity and magnetic permeability profiles.

A different approach is followed in the present paper. Consider a circular single-turn coil carrying an alternating current. The coil is located above a conducting half-space with electric conductivity σ and magnetic permeability μ , where both σ and μ are exponential functions of the vertical coordinate z of the form

$$\sigma = \sigma_m e^{\alpha z}, \quad \mu = \mu_m e^{\beta z}. \quad (1)$$

Here σ_m , μ_m , α and β are constants.

For this case the system of equations for the amplitude of the vector potential is solved analytically. The solution for the change in impedance of the coil due to eddy currents induced in the conducting half-space is obtained in terms of improper integral containing Bessel's function of variable order and complex argument. More precisely, the order of the Bessel's function depends on the variable of integration λ , and the argument is a complex constant (independent of λ).

The change in impedance of the coil is computed for different values of the parameters of the problem. Calculations are done using Mathematica since it has built-in routines for numerical integration and also can effectively evaluate Bessel's functions of complex argument and variable order.

REFERENCES

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- [2] E. Uzal, and J.H. Rose. The impedance of eddy current probes above layered metals whose conductivity and permeability vary continuously. *IEEE Transactions on Magnetics*, **29**: 1869–1873, 1993.