

# Simulation of Pessimism Influence on Choice of Farming Strategy under Uncertainty

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**Abstract** - Weather-related risks can substantially affect the crop volume depending on time and intensity of agricultural operations performed by the farmer. However, decisions under uncertainty are in some way also liable to another risk – that of excessively optimistic or pessimistic estimation of the decision maker. The paper presents results of optimistic and pessimistic farmer's agricultural operations efficiency based on the analysis of the developed discrete-event stochastic simulation model. The aim of the model is to obtain quantitative estimation of optimistic and pessimistic farmer's choice of agricultural operation strategy efficiency under fluctuation of weather conditions.

**Keywords** – farming strategy, uncertainty, crop, simulation, decision making, criterion

## I. INTRODUCTION

Under the present global crisis media often uses expressions like 'investors are pessimistic' or 'pessimistic'. The game of roulette where the number of red and black sectors which brings (in an unbiased situation) to an equal probability of scoring either colour, there are always some 'optimists' due to some reason believing that one of the colours appears more frequent. A classical example illustrating the difference in an optimist's and a pessimist's perceptions is the half-full glass of water paradigm; an optimist would describe the glass as being half-full whereas a pessimist would consider the glass half-empty. Taking into consideration dynamics of the process, in the eyes of a pessimist a glass is becoming empty whereas from the point of view of an optimist it is becoming full. Thus becomes obvious the difference in subjective estimation of the scope of probabilities of a certain event. The optimist tends to allocate a higher probability of favourable events, believing in stroke of good luck, unlike the pessimist who is more interested in securing his position rather than seeking additional gain.

Optimism and pessimism are important features of a person's attitude towards uncertainty. The decision weight of an event, measured by the willingness to bet on this event, differs usually from the probability of the event. Tversky and Wakker [12] study the relationship between decision weights and attitudes towards risk and characterize the possibility and certainty effects. Wakker [13] defines optimism and pessimism in terms of decision weights. That article contains also a brief survey of the relevant experimental literature. Kilka and Weber [4] demonstrate how decision weights and subjective probabilistic beliefs can be distinguished in experiments.

Within the scope of the present paper, the variations of distribution of subjective probabilities of optimistic and pessimistic farming strategies will be assessed using the weight functions. Using weight functions allows a qualitative analysis of influence of the degree of pessimism or optimism on the choice of an optimal strategy under the risks of weather.

It may be considered that a pessimistic farmer assigns higher importance (weights) to values of the expected crop value lying below some **cFk** value, which shall be defined as the crop sensitivity threshold both for a pessimistic and an optimistic farmer.

Let the degree of attitude (pessimism vs optimism) be defined as a relation of weights of the weight function beyond the **cFk** threshold, whereas the weights will remain constant over the interval of  $[0, \mathbf{cFk}]$  as well as constant in the range of  $[\mathbf{cFk}, 1]$ .

Two-dimensional histograms present a graphical representation of the frequency distribution of the selected variable of the crop in which the columns are drawn over the class intervals and the heights of the columns are proportional to the class frequencies (see Fig. 1).

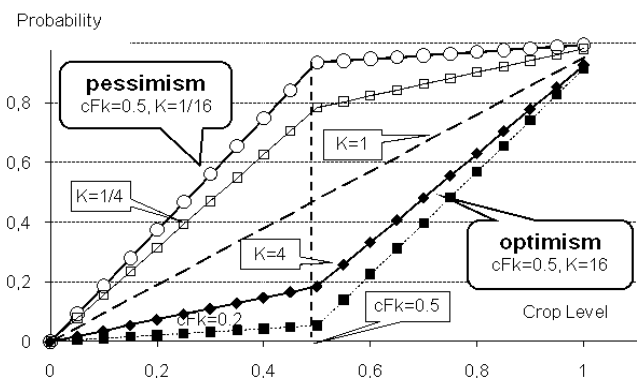


Fig. 1. Optimism and pessimism probability functions

The influence of farmers attitude in terms of pessimistic or optimistic expectation of the crop formally represents that the initial distribution of possible crop values  $H(\text{Cr})$  is multiplied with the weight functions of attitude (degree of pessimism or optimism)  $W(\text{Cr})$ . These weight functions are normalized so that the weights total one.

The function to consider is the step weight function defined by two parameters: the **cFk** (crop sensitivity threshold, in this study assumed  $\mathbf{cFk}=0.5=\text{const}$ ) and the value of the increment **K** ("degree of optimism").

Fig. 1 portrays the probability functions obtained through integration of the respective weight functions. In the graph, the X-axis scale represents crop level expressed in relative units ranging from 0 to 1. The Y-axis defines subjective probability of obtaining respective crop level estimated by an 'optimist' and a 'pessimist' farmer with different  $K$  values ('magnitude of optimism') with  $cFk$  ('crop sensitivity threshold') being constant at  $cFk=0.5$ .

An 'optimist' farmer assigns smaller weights to lower crop values whereas the relative importance of the higher weights is higher for him, so the respective probability function lies below the 45-degree line. On the contrary, a 'pessimist' does not expect a high crop volume rather hoping for a somewhat average crop level, therefore his expectation curve lies above the 45-degree straight line meaning high importance for relatively low crop values. The break point is named  $cFk$  whereas the ratio of the tangents of the approximation line left of break point to the tangent right of the breakpoint is referred to as  $K$ .

From here on we can describe the degree of optimism with two parameters. The first parameter is the location of the optimism point on the scale of crop value, we might refer to this as the "crop sensitivity threshold". If this parameter lies around zero, it means that any crop value is taken with the same degree of optimism, whereas the closer this parameter lies to one, the farmer is optimistic only to the maximum crop. The second parameter of the degree of optimism is the "magnitude of optimism", which is defined by the  $K$  value; the greater the  $K$  value, the more powerful is farmer's reaction. Obviously,  $K=1$  is the point of neutrality; there is neither pessimism nor optimism present. For  $K>1$  range of values, the magnitude of optimism is proportionate to the  $K$  (the higher the  $K$ , the higher the optimism level). On the contrary, for  $K<1$  range of values, the lower is the  $K$  value, the higher is the magnitude of pessimism.

The paper presents the results of optimistic and pessimistic decision makers' agricultural operations efficiency based on the application and analysis of the developed discrete-event stochastic simulation model. The aim of the model is to obtain quantitative estimation of farmer's choice of agricultural operation strategy efficiency under unanticipated fluctuation of weather conditions. According the model, farmer's strategies are described by two main parameters: 1) agricultural operation start time and 2) intensity of the operation.

Should a crop model be realistic enough, such method can be used for the analysis of crop sensitivity to changes in nature parameters, and for choosing an optimal farming strategy. For the time being, however, such research has not been known to the authors of the present paper.

Recently there has been presented methodical approach for finding an optimal farming strategy under conditions of uncertainty [5, 6] which relies on the concept of agricultural functions [6, 7]. Agricultural function in its simple form stands for the dependence of crop volume on agricultural operation time. It reflects farming experience in cultivation of a specific crop under specific conditions [9].

Following this approach any deviation in weather conditions affecting the crop will be reflected in the space of parameters of agricultural function. This space has an essentially smaller number of variables, such as the parameters defining the function shape. The aggregate farming experience for various crops allows defining the typical agricultural function shape along a time scale, which is a trapezoid of unequal lateral sides and slightly oblique top which is mostly considered constant; this is the maximum efficiency area the farmer would be willing to achieve. It should be noted that the change of weather conditions might drastically affect duration of the highest efficiency area and its start time [1].

Thus, in the considered aggregated approach, the variety of randomly variable parameters of weather conditions is reflected as alteration of small number of essential function parameters. Such an approach allows quantitative estimation of influence of random fluctuations in functions' basic parameters both on the average crop, and the histogram of crop distribution which gives essentially more information for decision-making when deciding for the best strategy [2, 3].

Therefore the present paper concentrates on the analysis of farmer's decision-making when choosing the best-performing strategy under conditions of weather uncertainty.

The structure of the model for simulation runs of farmer strategy efficiency (meaning total harvest value of the area) is illustrated in Fig. 2.

## II. BRIEF MODEL DESCRIPTION

The structure of the model graphically presented in Fig. 2 includes the following general sub-models and blocks:

- Sub-model of agricultural function **AF**, start time input block **Taf**, **AF** duration **durAF** input block and additional parameters **Pa**;
- Sub-model of farmer activity, operation intensity input block **durFR**, and farmer's start time input block **Ts**;
- Convolution of agricultural function **AF** and farmer activity function **FR**, yielding the total crops from the given farmland plot;
- Random generators **RG<sub>u<sub>i</sub></sub>** of random values  $u_1, u_2$ , that deliver time uncertainty **Taf** of agricultural function start time  $Taf - U_{1min} \leq Taf \leq Taf + U_{1max}$  and randomly change duration of agricultural function **durAF** over value range  $durAF - U_{2min} \leq durAF \leq durAF + U_{2max}$ ;
- Sub-model of farmer optimism with **cFk** and **K** parameters;
- Computation block **H**: calculation of values of histograms **h<sub>i</sub>** of randomly distributed crop;
- Computation block **W**: calculation of weight functions **W(cFk, K)** weight values **w<sub>i</sub>** with the parameters of optimism **cFk, K**;

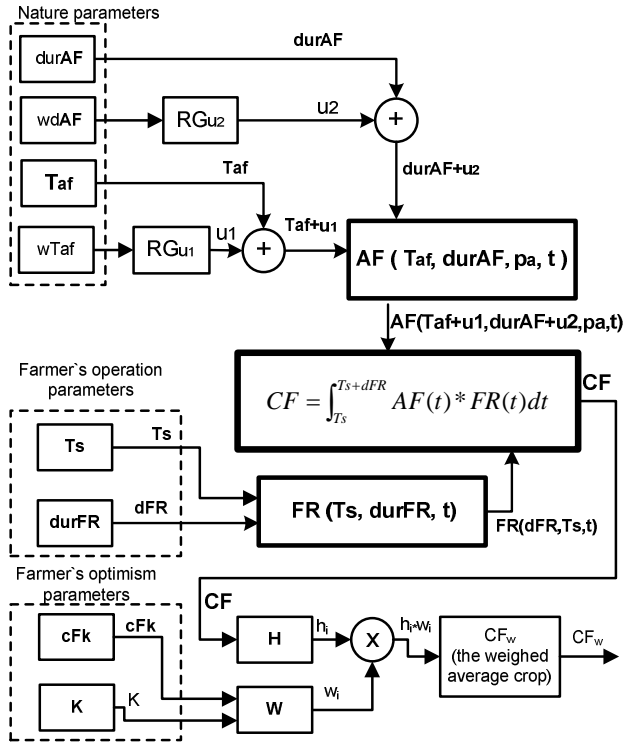


Fig. 2. Overview of agricultural operation model

- Computation block  $CF_w$ : calculation of average weighted crop values  $CF_w$ .

### III. FORMAL MODEL DESCRIPTION

The efficiency of agricultural operation is estimated through the efficiency criterion **Crop**:

$$\int_{Ts}^{Ts+durFR} AF(Taf+u_1, durAF+u_2, pa, t) * FR(durFR, Ts, t) dt. \quad (1)$$

The following function possesses relatively large degree of flexibility yet being simple enough for the purpose of the research above:

$$AF(Taf+u_1, durAF, t) = (\text{UnitStep}[t-Taf+dur1AF] - \text{UnitStep}[t-Taf]) / (dur1AF * (t-Taf+dur1AF) + (\text{UnitStep}[t-Taf] - \text{UnitStep}[t-Taf-dur2AF]) + (\text{UnitStep}[t-Taf-dur2AF] - \text{UnitStep}[t-Taf-dur2AF-dur3AF]) / (dur3AF * (Taf+dur2AF+dur3AF-t))$$

It should be noted that **UnitStep(x)** represents the unit step function, equal to 0 for  $x < 0$  and 1 for  $x \geq 0$ .

Modelling farmer activity (which is assumed constant) performing agricultural operations involved the following expression:

$$FR(t, Ts, durFR) = (\text{UnitStep}(t-Ts) - \text{UnitStep}(t-Ts-durFR)) / durFR$$

Thus, intensity is the inverse value of the duration **durFR** of the operation. The start time **Ts** of an agricultural operation is determined by the farmer. In simulation experiments the deviation of optimal time durations **dur2AF** and agricultural function start time **Taf** varied significantly.

#### A. Agricultural trapezoidal function

Let us consider a trapezoid function, relatively well describing most of the farming cultures. By changing the duration **dur1AF** of velocity of the front of agricultural function, we might affect the approach of the most favourable time for performing a specific operation. The duration **dur2AF** of the most favourable time span of performing the operation will be referred to as the duration of the agricultural function with start time **Taf** (see Fig. 3).

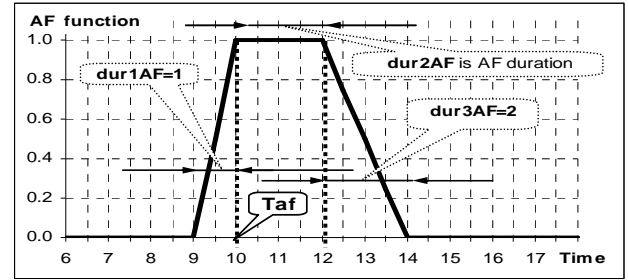


Fig. 3. Typical agricultural trapezoidal function chosen for research,  $Taf=10$

First, three cases of uncertainty (or forecast errors) of agricultural operation start time for uniformly random distribution within intervals  $9 \leq Taf \leq 11$ ,  $8 \leq Taf \leq 12$  and  $7 \leq Taf \leq 13$  are considered. The duration of the function remains unchanged: **AF** duration = 2 days.

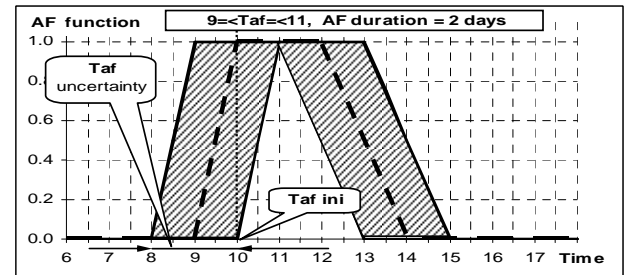


Fig. 4. Area of uncertainty of agricultural function at  $9 \leq Taf \leq 11$

The area of uncertain values of agricultural trapezoidal function will be altered as intervals increase. The areas of uncertainty of the agricultural functions appear on Fig. 4 and Fig. 5 as shaded regions.

The area of the uncertainty of agricultural values at  $7 \leq Taf \leq 13$  represents the shaded area from Fig. 4, expanded one day in both directions of x-axis.

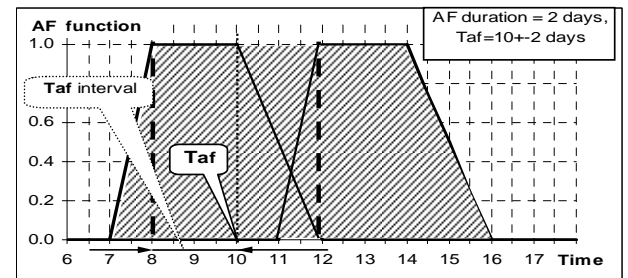


Fig. 5. Area of uncertainty of agricultural function at  $8 \leq Taf \leq 12$ .

All of these agricultural functions have their maximum value of 1.

#### IV. METHODOLOGY OF RESEARCH

Uncertainty in agricultural function start time is modelled through time scale shifts of 1, 2, 3, etc. number of days. This shift however remains unknown to the farmer, and therefore the state of nature is modelled in this manner.

According to the conventional terminology, risks arise when probabilities of approach specific states of nature are known in advance. However, under circumstances when the states of nature remain unknown to the farmer, we have an uncertainty context.

Uncertainty of agricultural duration start time **Taf** is modelled as randomly distributed value **u1** such as:  $(-wTaf) \leq u1 \leq (+wTaf)$ . The beginning of agricultural function now is unified distributed within the flexible interval  $(Taf - wTaf) \leq Taf \leq (Taf + wTaf)$ . Thus, the higher value of **wTaf**, the less definite is the beginning of agricultural function **Taf**.

For instance, if **Taf** = 10 and **wTaf** = +/-1 day, the real start of **AF** might be equally expected at any moment of the time interval **Taf** ∈ [9, 11] with duration of 2 days with the average value of **Taf** = 10. In case of **Taf** = 9 and **wTaf** = +/-3 days, the start of **AF** might eventually fall equally expected at any moment of **Taf** ∈ [6, 12] which is 6 days long.

The variables of simulation experiment comprise two groups of parameters: the first group is parameters of the states of nature **Taf**, **wTaf**, **wdAF**, where the notions stand for:

- **Taf** - agricultural function start time on the calendar scale
- **wTaf** - uncertainty of the date
- **wdAF** - variability of agrifunction duration.

The second group includes farming strategy parameters **Ts**, **durFR** also determined in the calendar time units, where the notions stand for:

- **Ts** - operation calendar start time and
- **durFR** - duration of completing operation **AF**.

#### V. CRITERIA AND CHOICE OF OPTIMAL STRATEGY

Decision criteria, or factors, are the specific measures that one uses to determine which alternative is the best choice. It is important to identify all the relevant factors to arrive at the most efficient solution. One should be careful about the number of factors as too many factors may make the decision process too cumbersome and inefficient. However, irrelevant factors have a way of exposing themselves pretty early in the process. They may not be truly measurable, or the information required may not be available [10].

Thus, the influence of farming strategies **Xi** (**Tsi**, **durFRi**) parameters on the average crop volume **MCF** calculated for weighted crop histograms (Fig. 2) with diverse combinations of parameters (**Tafi**, **wTafi**, **wdAFi**) states of nature **Sj** has been researched.

The higher the average value **MCFij** = **MCF(Xi, Sj)** the more efficient **Xi** strategy at the given state of nature **Sj**. On conditions of probability risk  $P(Sj) > 0$  the probabilities of

states of nature **j**=1,..., **Ns** are known in advance. This information is used in order to pick the optimal strategy using probability-weighted criteria of efficiency. On uncertainty conditions we do not know anything on probability of either state of nature **Sj**.

However, even on uncertainty conditions it might be assumed that if the probability of either state of nature is very low, such condition would not be considered as a state of nature **S**. On contrary, should probabilities of other states of nature be high enough, these would be considered as a state of nature **S**, since any analysis of hardly probable states would not be satisfactory even for a pessimistic farmer.

Consequently, even on conditions of complete uncertainty there is a probability proportion comprising variety of states of nature **S**, namely, all states of nature are approximately equally probable at a rational level, i.e.,

$$P(S1) \sim P(S2) \sim \dots \sim P(Sj) \sim \dots \sim P(SNs), \\ P(Sj) > 0; j=1, \dots, Ns. \quad (2)$$

Let us form the **XS** matrix, where rows **Xi** correspond to farmer strategies with specific combinations of parameters (**durFR**, **Ts**) and columns **Sj** featuring states of nature with respective combinations of parameters (**wTafi**, **wdAFi**). Thus, if possible farmer strategy is arranged in the rows and parameters of the nature arranged in columns, the intersection of *i*th row and *j*th column will contain the average value of crop **MCFij**, obtained through the application of *i*th strategy at *j*th state of nature.

Modelling included farmer strategies with agricultural operation durations **durFR** ranging 1 to 10 days with the step value of one day. The uncertainty of the beginning of agricultural operation **Ts** was modelled using normal probability distribution; **Ts** took values from 7<sup>th</sup> to 13<sup>th</sup> day of a month with one day increment. The states of nature were modelled through regular distribution within the boundaries of the parameters **wTaf**, **wdAF** for all the possible combinations of values **wTaf**=1,2,3,4,5; **wdAF**=0,1,2. Thus, the dimension of the **XS** matrix is equal to 70×15.

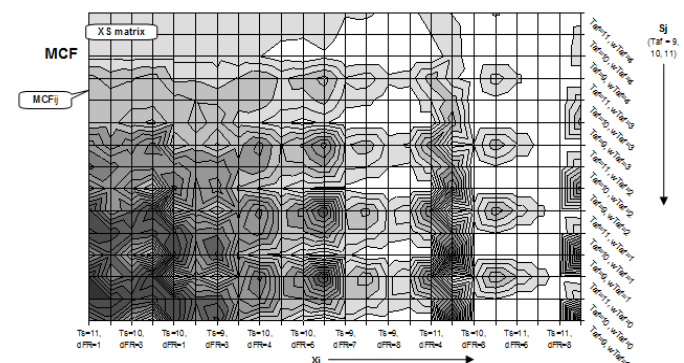


Fig. 6. 2D contour graph of XS matrix

Thus, for the range of values of the “magnitude of optimism” parameter we have a standard initial matrix ready for application of some techniques of decision-making theory and the theory of operations research. A part of the resulting

matrix is presented in a contour graph in Fig. 6. The shaded shapes mean greater crop values. Farming strategies are plotted along the X-axis while Y-axis reflects states of nature. The Z-scale corresponds to the average crop values  $MCF_{ij}$ , i.e., elements of  $XS$  matrix.

As it follows from the graph, the farming strategies lead to approximately equal results with no evidence of a clearly better strategy for all states of nature.

The peak efficiency shifts from one farming strategy to another even at light changes in the states of nature. Choosing an optimal strategy becomes even more challenging.

The choice of the best farming strategy basically depends on farmer preferences which can be mathematically expressed by means of a suitable optimality criterion.

#### A. Optimal choice algorithm: Laplace criterion

Let us consider what kind of strategies will be considered optimal if the decision-maker adheres to Laplace optimality criterion.

The Laplace criterion applies all the available information on the  $XS$  strategy-state of nature matrix and explicitly uses probabilities of states of nature which are assumed being equal  $P(S_j) = 1/N_s, j = 1, \dots, N_s$ .

At the same time, the Laplace criterion assumes application of probability theory in order to deal with uncertainty. This implies that for each state of nature  $S_j$  in  $S$ , the farmer as the decision maker should assess the probability of  $P(S_j)$  that  $S_j$  state will occur.

The Laplace criterion relies on the *Principle of Insufficient Reason* which states that if no probabilities had been assigned by the decision-maker (who is assumed to be rational and capable of handling basic probability theory), it follows that there was insufficient reason for the farmer to evaluate that any state  $S_j$  was more or less likely to occur than any other state.

So, it is considered that a rational farmer would be capable of assigning probability distributions  $P(S_j)$ . Since it is not the case, all the states of nature  $S_j$  must be considered equally probable. Therefore, the probability  $P(S_j)$  for each  $S_j$  must equal  $1/N_s$ , where  $N_s$  is the number of states of nature in  $S$ .

It is a fairly straightforward logic which tries to compensate for the lack of empirical knowledge on real distribution of probabilities through the principle of insufficient reason.

Thus, the Laplace criterion does not rely on any additional information on distribution of probabilities of states of nature, but it integrates this information through "rational" assumptions basing on the principle of the insufficient reason.

The optimisation algorithm might be formalised as follows:

- 1) Assign the constant probability  $P(S_j)=1/N_s$  for each state of nature  $S_j$  in  $S$ , for  $j = 1, \dots, N_s$ .
- 2) For each  $i$ th row of matrix  $MCF_{ij}$ , it is necessary to calculate the efficiency measure of  $i$ th strategy for all states of nature  $E(X_i)=\sum_j P(S_j)*MCF_{ij}$ .
- 3) Choose an action yielding the maximum  $E(X_i)$ , i.e., choose strategy  $X_{opt}$ , for which  $\max E(X_i)$ .

It would be worthwhile to consider strengths and weaknesses of the Laplace criterion. By assuming uniform

distribution of states of nature  $S$ , under this criterion it is assumed that all the elements  $MCF_{ij}$  of the strategy-state of nature matrix  $XS$  have equal weights of  $1/N_s$ .

There is a chance for improper use of the Laplace criterion which lies in the fact that the states of nature are not equally probable in most cases.

The weakness of the principle of insufficient reason lies in an implicit assumption that all decision makers who had not assigned a probability distribution to  $S$  had not done so because there is no reason to assume states of nature equally probable. However, in the real world human eventually deviate from the idealised decision maker: in a practical real world situation they will be unable to quantitatively estimate probabilities. Fig. 8 illustrates the change of parameters of optimal farming strategies as per the Laplace criterion with growing values of 'magnitude of optimism'  $K$  from  $K_{min}=1/16$  to  $K_{max}=16$ .

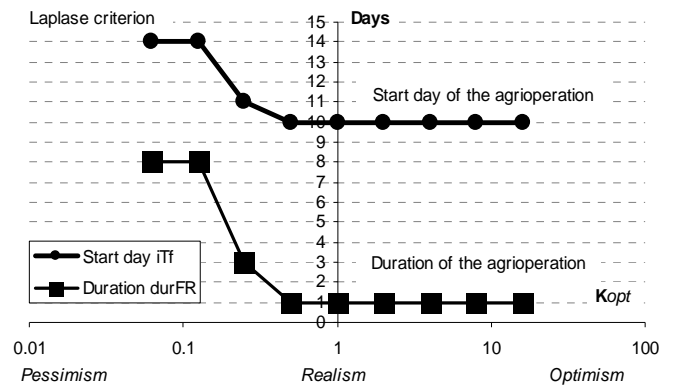


Fig. 8a. Nine best-performing strategies with 'magnitude of optimism' ranging from  $K_{min}=1/16$  to  $K_{max}=16$  considered optimal in accordance with the Laplace criterion.

As it might be inferred from the Fig. 8b, farmer's pessimism results in a blurred and prolonged area of the high values of the criterion (upper contour pane at  $K=1/16=0.0625$ ). For  $K=0.0625$  the contour area at 5% error level (of criterion's maximum value) embraces ten strategies. This area also includes the maximum value of the criterion at  $K=0.25$  which is marked by a circle in the middle contour graph. The higher the 'magnitude of optimism' value, the more concentrated becomes the area of efficient solutions (at  $K=16$ ). Therefore, an 'extreme' optimist has less winning strategies than a 'moderate' optimist.

In accordance with the criterion, we might conclude that at 5% error level a pessimist's choice problem is a more complicated one than that of an optimist, which indeed resembles real-life setting.

As the level of pessimism increases, the Laplace criterion tends to result in decreasing operation start time and its duration. Thus, for a pessimistic decision-maker the criterion proposes operation start on the 14<sup>th</sup> day whereas the realistic scenario requires start to occur on the 10<sup>th</sup> day. This implies that a pessimistic farmer is suggested a 4-day delay and stretching of operation time to 8 days compared both to the

realistic and the optimistic decision-makers who are suggested not to spend more than one day on carrying out the operation.

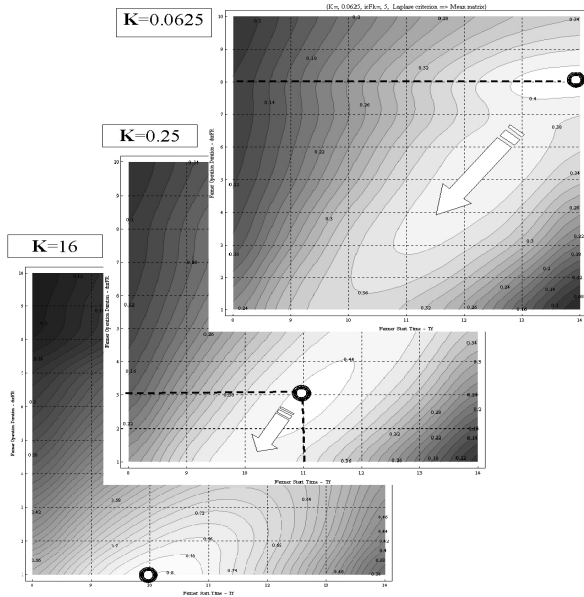


Fig. 8b. The Laplace criterion contour graphs for three values of the 'magnitude of optimism'  $K$ . The maximum values are marked with circles.

Due to the decision-maker's inability to assess the probabilities, all states of nature are assumed equally probable. Thus, absence or ignorance of the reasons is enough to assume equal distribution of states of nature probabilities weighted in accordance with the chosen parameters of degree of optimism or pessimism  $cFk=0.5=\text{constant}$ ,  $K=\text{variable}$ .

Let us consider different farmers' expectations. An overly optimistic would be the **maximax** criterion when one expects a good situation and the obtained crop will be the highest.

The **maximax** rule recommends to accept such a strategy, for which the condition  $\text{maxi}[\text{max}_j(\text{MCF}_{ij})]$  holds. Obviously, the **maximax** strategy is the most efficient at two equally probable states of nature, and as the number of states grows its average efficiency drops in direct proportion to the number of states of nature.

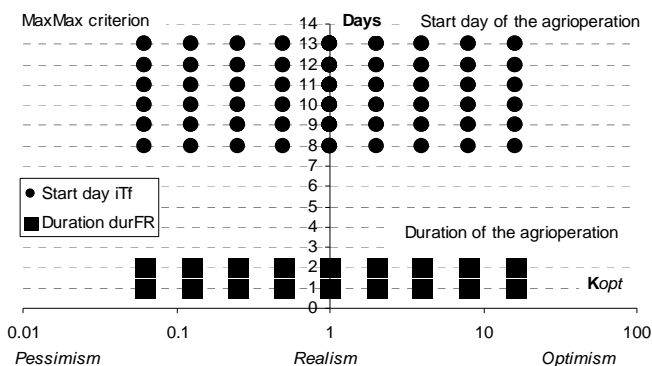


Fig. 9a. Best-performing strategies with 'magnitude of optimism' ranging from  $K_{\min}=1/16$  to  $K_{\max}=16$  considered optimal according to the **maximax** criterion.

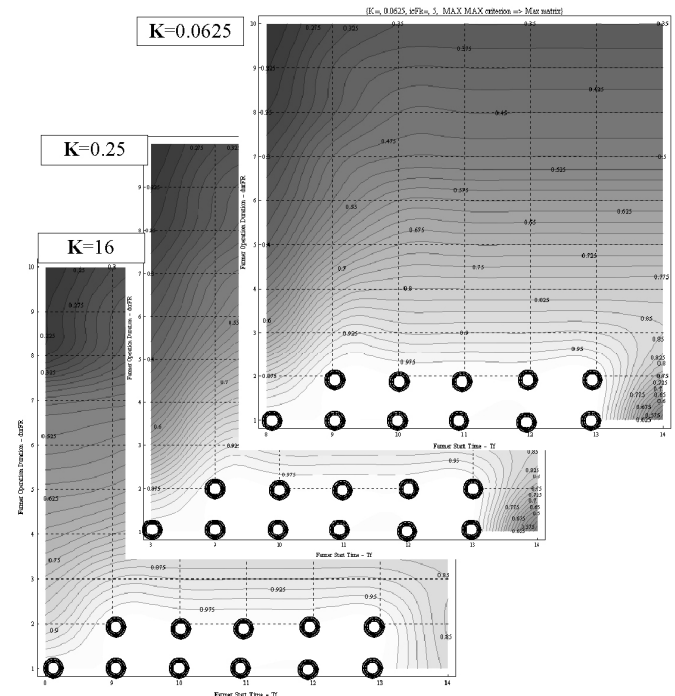


Fig. 9b. The **maximax** criterion contour graphs for three values of 'magnitude of optimism'  $K$ . The maximum criterion values are marked with circles.

The **maximax** criterion completely ignores the subjective attitude proposing the same recommendation, namely to delimit the duration of agricultural operation with two days ( $\text{durFR}_i \leq 2$ ) and start performing it at any day  $T_s$  starting with the 8th till 13th day of the month. Therefore, the area of maximum criterion values at any values of 'magnitude of optimism' is delimited only to farming strategies  $T_s=8-13$ ,  $\text{durFR}=1,2$  as it follows from Fig. 9a and Fig. 9b.

#### B. Optimal choice algorithm: Wald criterion

The Wald (or **maximin**) criterion, which might be considered as a criterion of an extreme degree of pessimism would be the next criterion to consider. According to the Wald criterion, the least risky strategy is chosen.

Technically, this means that the farmer cannot end up in a situation worse than the one chosen as per the Wald criterion. This fundamental property of the criterion makes it attractive for cautious decision makers and is applied in situations allowing zero riskiness. However, in practical situations excessive pessimism of the criterion might appear unprofitable.

The step sequence for the Wald criterion-based optimisation would be as follows:

- 1) For each strategy (rows in **strategy-state of nature XS** matrix) determine the minimum possible crop value. This represents the worst possible outcome for the given strategy. Should this strategy be pursued, no worse result at any state of nature would be possible.
- 2) The crop volume for the given strategy will not lie below the minimum possible. The highest crop has to be chosen from among low-yield outcomes for each of the farming strategies. In order to arrive at such a

solution one additional column is needed, the one that would contain the minimum values across rows. The maximum element in this column would represent the required solution.

The maximin/Wald criterion cannot be considered a rational approach for many cases as it ignores most of the information available in the **strategy-state of nature** matrix. The solution is searched only among cells of **XS** matrix that have extreme values. Information contained in other cells of **XS** matrix is absolutely ignored.

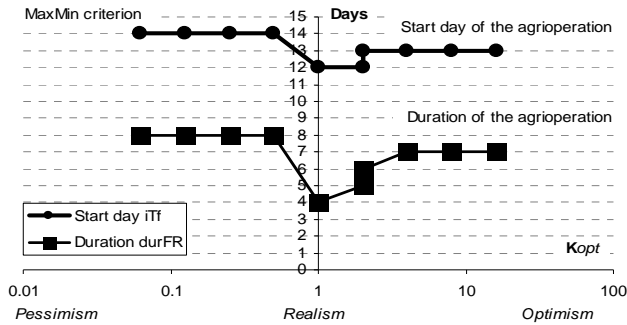


Fig. 10a. Ten best-performing strategies with 'magnitude of optimism' ranging from  $K_{min}=1/16$  to  $K_{max}=16$  considered optimal in accordance with the Wald criterion.

The Wald criterion shows somewhat odd results. At a high degree of pessimism ( $K < 0.1$ ) the criterion suggests undertaking a long agricultural operation of eight days' duration ( $durFR=8$ ) with a delayed start of  $T_s$  (i.e. delay of four days compared to the initial function).

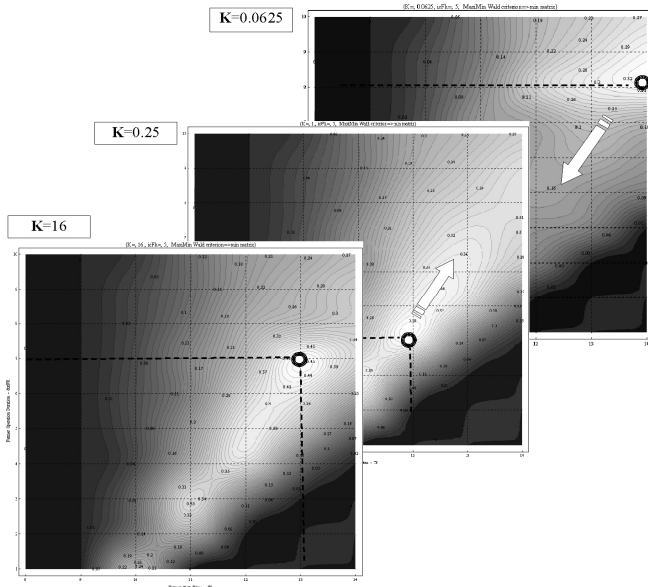


Fig. 10b. Wald criterion contour graphs for three values of 'magnitude of optimism'  $K$ . The maximum criterion values are marked with circles.

Assumed no specific preferences (i.e.  $K=1$ ), the maximin criterion yields lower estimates: the beginning of the operations should be shifted to  $T_s=12^{th}$  day and decrease its duration to  $durFR=4$  days. As the 'magnitude of optimism'

increases, the criterion reverts to the pessimism solution, suggesting decreasing the estimates by one day for a pessimistic farmer. Basing on the optimist's preferences it is recommended to start the operation on  $13^{th}$  day, i.e. one day later than for an 'extreme' pessimist. It is also suggested that the duration of agricultural operations is decreased one day compared to the 'extreme' pessimist's case.

From the contour graphs on Fig. 10b it can be noted that the maximum values of the criterion lie along the cone-shaped areas. Variations in 'magnitude of optimism' leads to shifts of local extreme located on the diagonal cone-sized areas in Fig. 10b. Such behaviour of the criterion surface causes the effect of non-monotonous sensitivity to the maximum values of criterion towards variations in the 'magnitude of optimism'.

Fig. 10a and Fig. 10b reflect the specific reaction of criterion towards realism with minimal reaction toward the decision-maker's optimism or pessimism.

### C. Optimal choice algorithm: Hurwicz criterion

The Hurwicz criterion is trying to arrive at a rational compromise between the two above mentioned criteria of extreme optimism and pessimism. Instead of taking either side, the Hurwicz criterion assigns certain weight to the optimism/pessimism expectations. The calculable expression reads as follows:

$$Hr(X_i) = a * (\text{maximum of "i" row}) + (1 - a) * (\text{minimum of "i" row}). \quad (3)$$

Introducing the weight of  $a$  in this context means that a complete maximax optimism of maximax ( $a=1$ ) is replaced by an altered optimism level decreased through the optimism coefficient  $a$ , with a belonging to the interval of  $0 \leq a \leq 1$ . Thus, for  $a=1$ , there is complete optimism prevailing (turning to maximax), whereas  $a=0$  would result in a risk-averse maximin. Under these notions, risk-averseness degree of the decision-maker would be expressed as  $(1 - a)$ . Now, the weighted average of  $H$  for each strategy  $X_i$  might be calculated. A critical analysis of the Hurwicz criterion reveals that the choice of  $a$  has no sufficient logic argument. Besides that, the optimal strategy is searched only across extreme elements of the **XS** matrix, and information contained in other cells of the **XS** matrix is not considered.

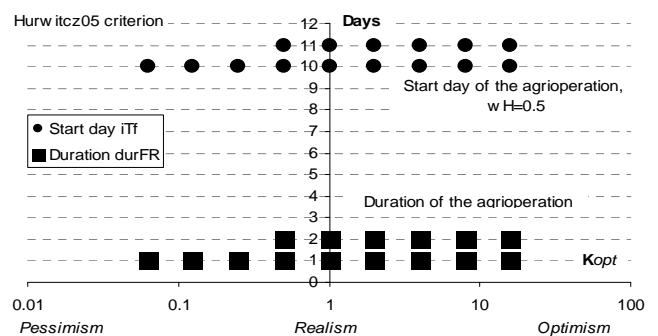


Fig. 11a. Fifteen strategies with 'magnitude of optimism' ranging from  $K_{min}=1/16$  to  $K_{max}=16$  considered optimal according to the Hurwicz criterion ( $a=0.5$ ).



Therefore, here we will present calculation results at  $a=0.5$ , whereas other values could be of interest and represent area for future research.

Both for pessimistic and optimistic decision-makers Hurwicz criterion ( $a=0.5$ ) recommends one single strategy of one day's duration starting on 10<sup>th</sup> day.

However, for optimists such a criterion adds one more optimal strategy with the parameters of  $T_s=11$ ,  $durFR=2$  whereas the pessimist has the choice of only one strategy of  $T_s=10$ ,  $durFR=1$ .

#### D. Optimal choice algorithm: Savage criterion

The Savage criterion yields results such as to minimize possible regrets arising from making a non-optimal decision. Strictly speaking, a *regret* is a subjective emotional condition, whose proper estimation is a rather problematic issue [10]. Nevertheless, it is possible to assume that regret is measurable for all the elements of the strategy-state of nature matrix.

assessing *regret*. Nevertheless, the Savage criterion is considered to be a better criterion compared to maximax, maximin or, probably, Hurwicz since the Savage criterion relies on more information among available on the problem. However, the Savage criterion is not able to take advantage of all the available information and it does not provide the ultimate solution to the problem of choosing the best farming strategy.

At  $K=16$  the surface of the criterion is very similar to the one of the maximax criterion. As the 'magnitude of optimism' diminishes, the area of maximum values increases and grows in the direction of the increasing values of agricultural operation duration  $durFR$ . For a pessimist with  $K=1/16$  the criterion allows a choice of strategies of operation durations of even 8 days and its start  $T_s$  from day 12 to day 14. Such criterion results allow additional strategies for the pessimists, which are, however, inefficient from the point of view of the 'realists'.

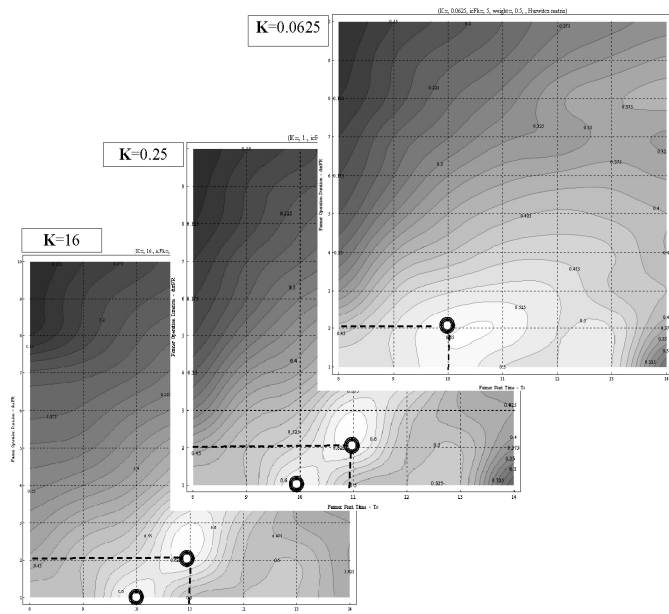


Fig. 11b. Hurwicz criterion contour graph ( $a=0.5$ ) contour graphs for three values of 'magnitude of optimism'  $K$ . The maximum criterion values are marked with circles.

The regret  $R_{ij}$  is defined as a possible loss of a part of a crop when  $X_i$  strategy is chosen at a  $S_j$  state of nature. Possible loss is a deviation between the highest crop attainable in a state of nature  $S_j$  and the actual result following from choosing the  $X_i$  strategy.

This can be formalised as follows:  $R_{ij} = MCF_{ij} - (\text{maximum } j\text{-th column of } XS \text{ matrix})$ , where  $R_{ij}$  - regret for row  $i$  and column  $j$  of regrets matrix  $R = \{R_{ij}\}$ .

It should be noted that losses of opportunities are defined as negative numbers. The best outcome would be zero (absence of regrets), and the higher the absolute value of  $R_{ij}$ , the higher is total regret.

A closer look at the Savage criterion reveals that regret linearly depends on the magnitude of the possible loss, whereas it is not the only one and not always a possible way of

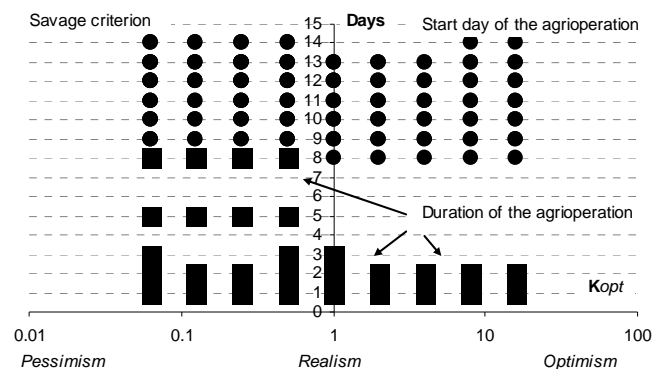


Fig. 12a. Fifteen strategies with 'magnitude of optimism' ranging from  $K_{min}=1/16$  to  $K_{max}=16$  considered optimal in accordance with the Savage criterion.

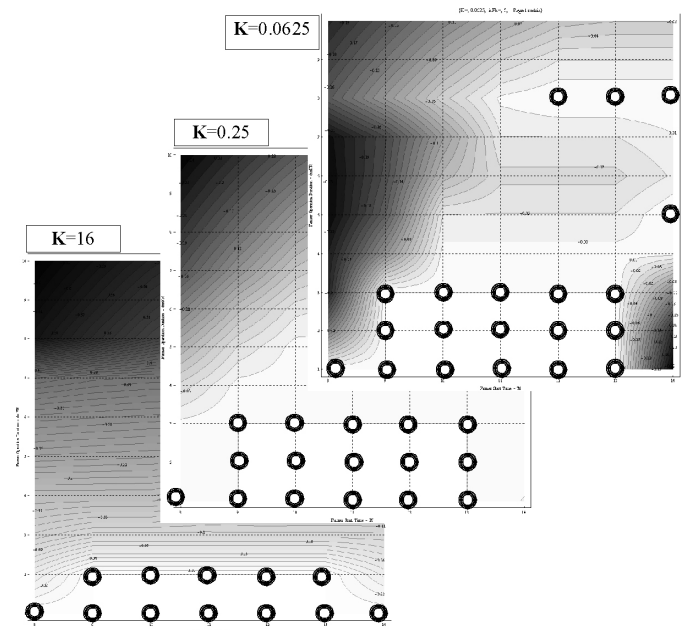


Fig. 12b. Savage criterion contour graphs for three values of 'magnitude of optimism'  $K$ . The maximum criterion values are marked with circles.



### E. Optimal choice algorithm: Hodges-Lehmann criterion

Under uncertainty, the Hodges-Lehmann criterion formally represents a combination of Laplace and Wald's maximin with the weighted  $h$  parameter. At  $h=0$  the Hodges-Lehmann criterion results in the solution according to Wald's rule and at  $h = 1$  the criterion switches to the Laplace's rule [11].

There will be presented results only for  $h=0.5$ , however other combinations might present interest for further research. The maximum value of the Hodges-Lehmann criterion corresponds to the farming strategies considered optimal as per the given criterion.

$$H(X_i) = h * \sum_j MCF_{ij} / N_s + (1-h) * \min_j MCF_{ij}, \quad (4)$$

where  $MCF_{ij}$  stands for the crop obtained through the application of  $i$ th strategy at  $j$ th state of nature.

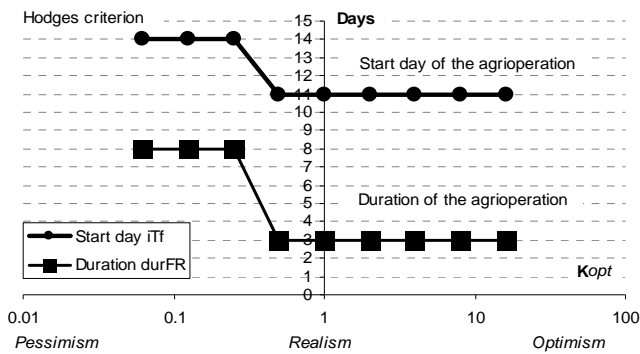


Fig. 13a. Fifteen strategies of 'magnitude of optimism' ( $K_{min}=1/16$ ,  $K_{max}=16$ ) considered optimal according to the Hodges-Lehmann criterion.

Suggestions of the Hodges-Lehmann criterion resemble those of the Laplace criterion. For a pessimistic farmer the recommendations basing on either criterion are identical, whereas for the more optimistic farmer the Hodges-Lehmann criterion recommends higher values of the operation parameters than the Laplace criterion.

Thus, Hodges-Lehmann recommends duration of the operation of three days whereas the Laplace criterion under the same conditions supposes duration of one day.

The optimal start of the optimists' strategy the Hodges-Lehmann defines on the 11<sup>th</sup> day of the month whereas the Laplace considers optimal starting on the 10<sup>th</sup> day (see Fig. 13a and 13b).

### F. Optimal choice algorithm: maximed criterion

According to the **maximed** criterion the values of the median of each strategy for the states of nature are weighted with the coefficient  $m$  whereas the values of the deviation of the median are weighted with the coefficient of  $(1-m)$ . The strategies with the maximum values of this composite criterion are considered optimal:

$$\text{maximed}(X_i) = m * \text{median}_j(MCF_{ij}) + (1-m) * \text{medianDeviation}_j(MCF_{ij}) \quad (5)$$

In Fig. 14a and 14b we consider only results for  $m=0.5$ , whereas application of other values of the  $m$  parameter represents potential for future research.

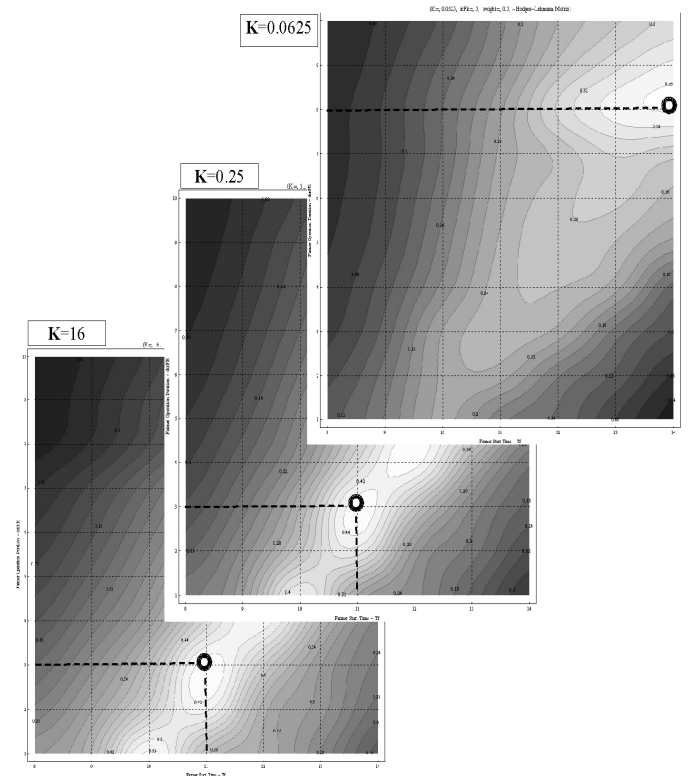


Fig. 13b. Hodges-Lehmann contour graphs for three values of 'magnitude of optimism'  $K$ . The maximum criterion values are marked with circles.

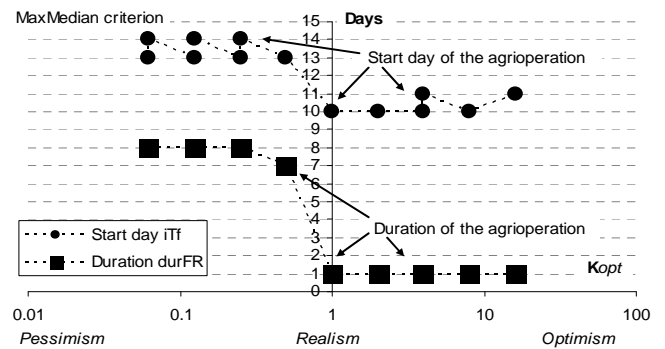


Fig. 14a. Fifteen strategies with 'magnitude of optimism' ranging from  $K_{min}=1/16$  to  $K_{max}=16$  considered optimal in accordance with the **maximed** criterion.

The **maximed** criterion results resemble the results generated under the Laplace and Hodges-Lehmann criteria. For the pessimist, the recommended operation start  $T_{si}$ ,  $durFR_i$  falls on the 13<sup>th</sup>-14<sup>th</sup> day, i.e. the agricultural operation is delayed 3-4 days compared to the realist's estimation. The recommended duration of the operation extends to 8 days. The contour graphs in Fig. 14b depict decreasing solution stability when shifting from pessimism to realism (depicted by white areas).

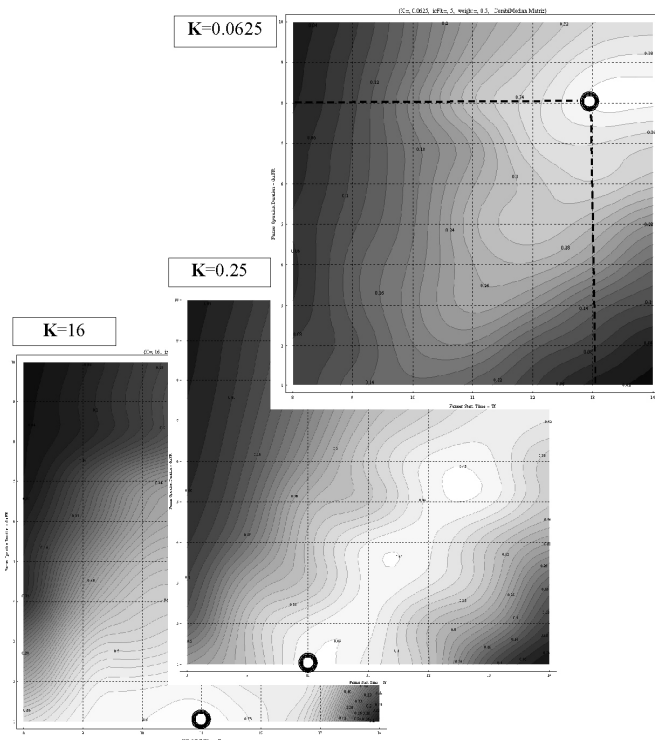


Fig. 14b. Maximized criterion contour graphs for the three values of 'magnitude of optimism'.

## VI. DISCUSSION AND CONCLUSIONS

The paper presents a simulation-based methodology of quantitative estimation and choice of optimal farming strategy on conditions of nature uncertainty. Uncertainty of nature states is modelled as alterations in values of respective parameters of the trapezoid agricultural function. In the context of the research, a farming strategy is defined by two major parameters, namely, the time to start performing agricultural operation and its duration. In the current paper, the average crop values are presented in the form of a farming strategies-state of nature matrix which had undergone application of several methods of decision-making theory.

Some sets of optimal farming strategies have been assessed using various optimality criteria. The resulting sets of optimal farming strategies based on the **maximax**, Laplace, Wald, Hurwicz, Savage, Hodges-Lehmann, and **maximed** criteria are compared in the present paper.

Influence of pessimism on the choice of strategy is various for different criteria. Relying on the examples above, one may also discuss the results of the criteria considered using the farmer's strategy under weather uncertainty. The most sensitive to pessimism are the Laplace, Hodges-Lehmann and **maximed** criteria followed by Wald and Savage criteria. Less sensitive to pessimism could be considered the **maximax** and Hurwicz criteria. Influence of pessimism results in notable discrepancies between the strategies compared to the choice of 'realists'.

Hurwicz criterion has only one strategy to offer equally suitable for the three groups of decision-makers ('optimists', 'pessimists', 'realists'), and one more strategy suitable for

'realists' and 'optimists'. Among the criteria considered it is the Hurwicz criterion which has the best indicators of indifference toward farmers' preferences and acts as the most objective and specific criterion in the research. The optimistic and realistic decision-makers for the majority of the considered criteria choose the same strategies and only Hurwicz criterion portrays a difference between the 'optimist's' and 'realist's' choice.

Unfortunately, the results of the research are limited by the specificity of the model and the form of the agricultural functions as well as modelling uncertainty nature through random distribution of parameters.

Measuring the pessimism or optimism degree of the decision maker can also be approximated by an S-shaped probability function or linear approximation by a parallel line to the straight line featured in Fig. 1 [8, 9]. At the time of writing this work no research on decision-maker's subjective attitude influencing crop estimation was known to the authors. The present work aims at filling this gap. Let us note that the methodology illustrated above could possibly be applied in economic research for estimation of middle-term and long-term investment projects [12].

The present work illustrated the sensitivity of criteria to specific disturbance of the initial data. Interpretation of such disturbance as 'magnitude' of optimism of the decision-makers is conditioned by the specific crop model. Earlier, an analysis attempt of criteria sensitivity was undertaken in [13].

Concerning limitations of the present work, it may be argued if a farmer can numerically evaluate and compare a tremendous amount of histograms necessary for a complete crop analysis. The methodology presented above should be perceived as a conceptual approach to such type of problems. Should the optimism degree of the decision-maker remain unchanged while evaluating each histogram, it appears worthwhile to model such behaviour for a larger set of statistical data.

## ACKNOWLEDGMENTS

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## REFERENCES

- [1] Bardachenko, V., Merkuryev, Y., Solomennikov, A., Ruzha, A. (2009) Defining Optimal Farmer Strategy under Climate Uncertainty. Chapter 16. Cultural Landscape Across Disciplines. Ed. by Jozef Hernik. Oficyna Wydawnicza BRANTA, Bydgoszcz – Krakow. P. 323-340.
- [2] De Groot, M. (1971) Optimal Statistical Decisions. McGraw-Hill, New York, 512 pp.
- [3] Hodges, J.L., Lehmann, E.L. (1952). The use of previous experience in reaching statistical decision. Annals of Mathematical Statistics, 396-407.
- [4] Kilka, M. & Weber, M. (1998). "What Determines the Shape of the Probability Weighting Function under Uncertainty?" Management Science 47, 1712-1726.
- [5] Merkurjevs, J., Bardachenko, V., Arhipova, I., Rudusa, I. (2004) Agrofuctions Applying for Agricultural Risk Management. Proceedings of the Latvia University of Agriculture, Nr. 11 (306), 64.-69.

- [6] **Merkurjevs, J., Bardačenko, V., Solomenikova, I., Ruža, A.** (2005) Use of Simulation-Based Approach for Estimation of Agricultural Risks. Proceedings of the Latvia University of Agriculture, Nr. 15 (310), 81-86.
- [7] **Merkurjevs, J. Bardačenko, V. Ruža, A., Solomennikovs, A.** (2009) A. Simulation-Based Choice of Optimal Farming Strategies under Risks of Nature. Proceedings of the Latvia University of Agriculture. Nr. 22 (317). Jelgava. P. 53-64.
- [8] **Merkuryev Y., Bardachenko, V., Solomennikov, A.** (2005) Simulation model for evaluation of farmer's strategies under nature risks. Proceedings of the 2005 European Modelling and Simulation Symposium, EMSS 2005. October 20-22, 2005, Marseilles, France. Edited by Chiara Briano, Claudia Frydman, Antonio Guasch, Miquel Angel Piera, 59-64.
- [9] **Merkuryev, Y., Bardachenko, V., Solomennikov, A.** (2006) Simulation-based evaluation of agriculture strategies under uncertainty of weather forecast. International Mediterranean Modelling Multiconference. October, 4-6, 2006. Edited by Agostino G. Bruzzone, Antoni Guasch, Miquel Angel Piera, Jerzy Rozenblit. Barcelona, Spain, 131-138.
- [10] **Puri, M., Robinson, D. T.** "Optimism and Economic Choice," Journal of Financial Economics, vol. 86, pp. 71-99, 2007.
- [11] **Shaffer, M. J., Bartling, P. N. S., McMaster, G. S.** (2004) GPFARM modelling of corn yield and residual soil nitrate-N. Computers and Electronics in Agriculture, 43, 87-107.
- [12] **Tversky, A., Wakker, P.** (1995). "Risk Attitudes and Decision Weights".Econometrica 63, 1255-1280.
- [13] **Wakker, P.** (2001). "Testing and Characterizing Properties of Nonadditive Measuresthrough Violations of the Sure-Thing Principle". Econometrica 69, 1039-1059.

**Jurijs Merkurjevs, Vladimirs Bardačenko, Andrejs Solomenikovs. Pesimisma ietekmes uz lauksaimniecības stratēģijas izvēli imitācijas modelēšana nenoteiktības apstākļos**

Dabas riski var būtiski ietekmēt ražas apjomu atkarībā no agrārās operācijas veikšanas laika un intensitātes. Taču lēmuma pieņemšana nenoteiktības apstākļos atradās vēl vienā riska ietekmē, proti, pārmērīgi optimistiskās vai pesimistiskās sagaidāmās ražas vērtēšanas lēmuma ieņemšanas brīdī. Dotais raksts piedāvā agrooperācijas efektivitātes analīzes rezultātus pēc dažiem lēmumu pieņemšanas teorijā izmantojamiem optimalitātes kritērijiem, pamatojoties uz autoru izveidotā diskretā stohastiskā imitācijas modeļa izmantošanu. Modeļa veidošanas mērķis ir kvantitatīvs agrooperācijas efektivitātes novērtējums atkarībā no veicēja optimisma vai pesimisma pakāpes, ņemot vērā laika apstākļu svārstības. Analizējot lauksaimnieka pieņemto lēmumu optimalitāti, tiek lietoti Valda, Hurvica, Laplasa, maksimaksa, maksimēda, Sevidža un Hodžesa-Lēmāna kritēriji. Kā liecina veikto skaitlisko eksperimentu rezultāti, Laplasa, maksimēda un Hodžesa-Lēmāna kritēriji ir visvairāk jutīgi pret lauksaimnieka raksturu. Tiem seko Valda un Sevidža kritēriji. Savukārt, Hurvica un maksimaksa kritēriji ir vismazāk jutīgi pret lauksaimnieka raksturu. Pie tam tieši Hurvica kritērijs šajā gadījumā izskatās kā objektīvāks optimalitātes kritērijs.

**Юрий Меркурьев, Владимир Бардаченко, Андрей Соломенников. Имитационное моделирование влияния пессимизма фермера на выбор стратегии в условиях неопределенности**

Риски погоды могут оказывать существенное влияние на урожай в зависимости от времени и интенсивности проведения фермером необходимых агроопераций. Однако, принятие решений в условиях неопределенности так или иначе подвержено ещё одному риску - риску чрезмерного оптимизма или пессимизма фермера, принимающего решение. Данная статья представляет результаты анализа эффективности проведения агрооперации фермером с помощью ряда используемых в теории принятия решений критериев оптимальности, основываясь при этом на использовании разработанной авторами дискретной стохастической имитационной модели. Целью создания модели является получение количественной оценки эффективности проведения агроопераций оптимистичным и пессимистичным фермером при колебаниях погодных условий. В качестве рассматриваемых критериев оптимальности принимаемых решений используются критерии Вальда, Гурвица, Лапласа, максимакса, максимēда, Севиджа и Ходжеса-Лемана. Как показали результаты проведенных численных экспериментов, наиболее чувствительными к характеру фермера являются критерии Лапласа, максимēда и Ходжеса-Лемана, за которыми следуют критерии Вальда и Севиджа, а наименее чувствительными – критерии Гурвица и максимакса. При этом именно критерий Гурвица представляется в данном случае в качестве наиболее объективного критерия оптимальности.

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