

# Pantograph-Catenary System Modeling Using MATLAB-Simulink Algorithms

Aleksandrs Matvejevs, *Riga Technical University*, Andrejs Matvejevs, *Riga Technical University*

**Abstract** - Contacts between pantograph and catenary are the most critical parts in the transmission of electrical energy for modern high-speed trains. Contact wire oscillations change combined force between pantograph and catenary, and the contact may even get lost. Therefore special pantographs and catenaries have been developed and further constructive changes are under development. A design criterion includes the permanent contact of pantograph head and contact wire at high speed and the reduction of both aero acoustic noise and wear. Because of complicated dynamic behaviour and very high costs for prototypes, all modifications and new design concepts for the pantograph/catenary system are essentially based on dynamical simulation. Traditional approaches focus on the catenary, which is modelled as set of coupled strings and/or beams, whereas simplified lumped mass models are used to describe the pantograph. Nowadays increased computer power allows considering applications with more refined pantograph modes (e.g. the elasticity of the pantograph) and active control components in innovative pantograph concepts.

**Keywords** - interaction pantograph-catenary, mechanical multibody system, high performance computing in MATLAB

## I. INTRODUCTION

In the present paper are consider the combination of a beam model for the contact wire with a mechanical multibody system (MBS) model for the pantograph. Both substructures are modelled together resulting in partial difference equations of the catenary and differential-algebraic equations of the pantograph. Because of the geometrical contact between pantograph and contact wire the substructures are coupled. The efficient coupled dynamical simulation of pantograph and catenary is per se a challenging problem from the viewpoint of both mechanical engineering and numeric. In present paper there are consider mainly the modelling aspect of the problem. Most work was done in the design and implementation of numerical methods to handle the differential equations part. Theoretical investigations focussed on linear systems with constant coefficients. All works was done in the design of the numerical models and same dynamical characteristics of the intersection pantograph and catenary to use in MATLAB and Simulink.

The pantograph and catenary form an oscillating system that is coupled via the contact force between the pantograph head and the contact wire. Too large contact force variation can lead to loss of contact, arcing and wear as well as damage to the system. Thus, the dynamic behaviour plays a decisive role for high-speed trains from the power collecting point of view. It is therefore a need for enhanced possibilities to predict the dynamic behaviour of this type of system. To accomplish this, modelling, simulation and experiments have been used.

A simulation tool is developed to increase the actual use of computational support in the product development process.

We consider a linear model of the pantograph/catenary system, where the lower and upper pantograph frames, as well as the catenary, are modelled in terms of lumped masses, springs and dampers [1]. A schematic representation is depicted in Fig.1. The mechanical parameters of the upper and lower frame are constant and can be considered known with good accuracy. The catenary parameters, on the contrary, will be space-dependent functions of the current distance of the contact point from the closest adjacent towers and droppers (see Fig. 2).

Output signal of high speed train in the Catenary mathematical model is determined by the impact of the total input signal, which consists of:

- 1) the sum of  $N$  sinusoidal signals ( $N \geq 10$ ) with different amplitudes and frequencies,  $A_N$  and  $w_N$  ( $5 \leq w_N \leq 10$ );
- 2) the imposition of interference in the form of a random signal with uniform distribution on the total harmonic signal;
- 3) the disturbance in the form of step signal at certain time moments.

Pantograph mathematical model is considered in two versions – passive and active pantograph. Simultaneous investigation of both these models for different values of Catenary input signals confirms utility of adaptive algorithms in the pantograph structure.

## II. CATENARY MODEL ANALYSIS REVIEW AND DIFFERENTIAL EQUATIONS OF THE FORM

A system of  $n$  degrees of freedom has  $n$  natural modes. Associated with each mode is a natural frequency,  $w$ , and a natural mode shape,  $\varphi(x)$ . The mode shapes of a dynamic linear system are orthogonal and therefore system displacements can be expressed as a sum of the natural modes multiplied by appropriate, time-varying modal amplitudes, or modal response functions, a technique known as modal decomposition.

$$y(x, t) = \sum_{i=1}^n \varphi_i(x) z_i(t). \quad (1)$$

where

$y(x, t)$  – the time varying displacement of the system,

$\varphi_i(x)$  – the  $i$ th natural mode shape,

$z_i(t)$  – the modal amplitude of the  $i$ th mode,

$i$  – the mode number.

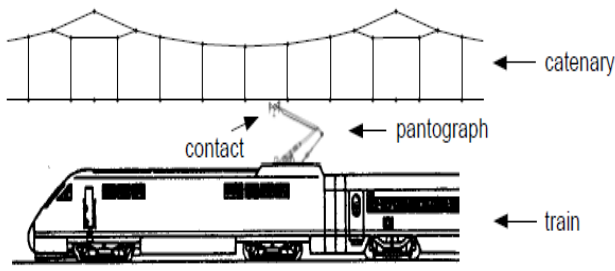


Fig. 1. Pantograph/catenary system

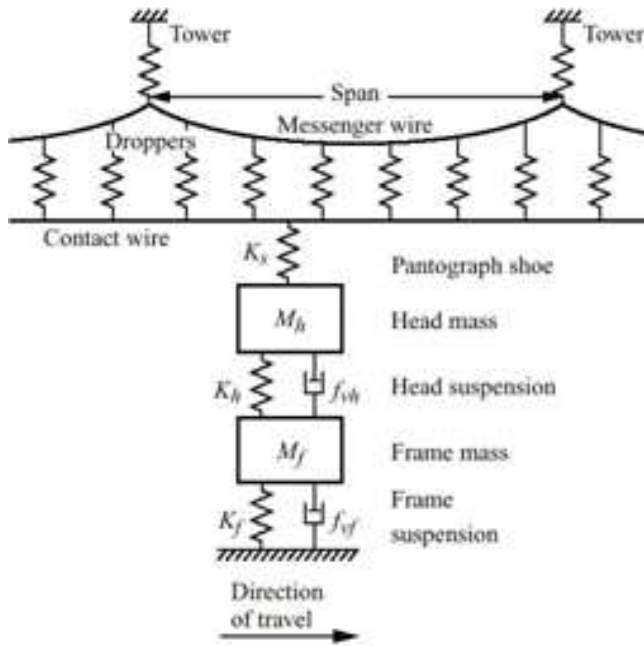


Fig. 2. Pantograph/catenary system blocks

The mode shape,  $\varphi$ , depends only upon position; and modal amplitude,  $z$ , depends only upon time. When a system is excited in a natural mode, the system and all system elements, maintain the same relative displacements to each other, and the mode shape describes this relation. Once the mode shapes are known, the dynamic of the system are determined by the amplitudes,  $z(t)$ . The benefit of separating the motion into modal components is the modes may be considered independently and the equations reduce to simple, linear, second order, differential equations of the form [2, 4]:

$$M_i \ddot{z}_i(t) + c_i \dot{z}_i(t) + K_i z_i(t) = Q_i, \quad (2)$$

where  $z_i$  – the  $i$  th modal amplitude,

$M_i$  – the modal mass of the  $i$  th mode,

$c_i$  – the modal damping of the  $i$  th mode,

$K_i$  – the modal stiffness of the  $i$  th mode,

$Q_i$  – the forcing function of the  $i$  th mode.

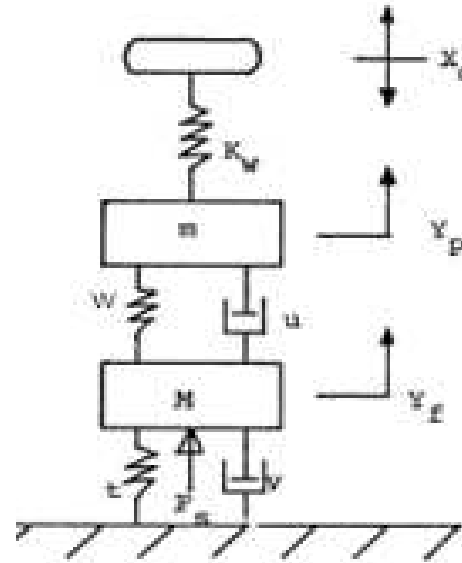


Fig. 3. Simplified block diagram of pantograph system for  $\frac{y_p}{x_o}, \frac{y_f}{x_o}$ .

The natural frequency of the system when vibrating in the  $i$  th mode is given by equation and follows from the natural frequency of a simple system as:

$$\omega_i = \sqrt{\frac{K_i}{M_i}}. \quad (3)$$

An efficient way to express equation (2) in terms of natural frequency, damping ratio, and damping and modal mass is the following:

$$M_i \ddot{z}_i(t) + 2M_i \varepsilon_i \omega_i \dot{z}_i(t) + M_i \omega_i^2 z_i(t) = Q_i, \quad (4)$$

Once the mode shapes and frequencies are known, the time response of each mode is determined by equation (4) and the total system response are determined by applying equation (1) and summing up the individual response.

### III. CATENARY EQUATION DEVELOPMENT

For the catenary, let  $y(x, t)$  which is defined in (1) describe the displacements of the catenary wire, both as a function of position  $x$ , and of the time,  $t$ . The boundary conditions require displacements of the two ends ( $x = 0$  and  $x = L$ ). The two wires, the contact and the support wire, are written separately and as functions of sine terms only as[5]:

upper wire  
lower wire

$$y_A(x, t) = \sum_m A_m(t) \sin \left( \frac{m\pi x}{L} \right), \quad (5)$$

$$y_B(x, t) = \sum_m B_m(t) \sin \left( \frac{m\pi x}{L} \right), \quad (6)$$

where

– the displacement of the upper wire,

$y_B$  – the displacement of the lower wire,

$A_m$  – the amplitude of the  $m$  th sine term for the upper wire,

$B_m$  – the amplitude of the  $m$  th sine term for the lower wire,

$x$  – the distance along the catenary,

$L$  – the total length of the catenary,  
 $m$  – an integer, designates the harmonic number.

The shape of these wires is time varying, therefore the amplitudes  $A_m$  and  $B_m$  are time varying and can be written  $A_m(t)$  and  $B_m(t)$ . Since amplitudes describe the shape of the whole catenary at all times the amplitude can be used to write the equations of motion for the catenary, and obtain the natural modes of the catenary.

First, to describe the dynamic displacement of each wire from its equilibrium position, the displacement shape of each catenary wire is used by a sine-series expansion. A simple result of Fourier analysis of any shape with fixed ends can be mathematically described by superposing an infinite set of sine functions, each with any appropriate amplitude. A close approximation of the shape can be made with a finite number of sine terms. Considering a finite number of sine terms transforms the shape of the catenary wires from an infinite number of degrees of freedom into a finite number of degrees of freedom.

Therefore the displacements are described by a finite term sine-series expansion: one series representing the shape of the contact (lower) wire, a second series representing the shape of the messenger (upper) wire.

Then the system can be written in terms of time varying displacement as in (1). Using (4), (5) and (6) we find

$$F_i(x) = \sum_{m=1}^M A_{im} \sin\left(\frac{m\pi x_A}{L}\right) + \sum_{m=1}^M B_{im} \sin\left(\frac{m\pi x_B}{L}\right). \quad (7)$$

where  $M$  is the maximum number of sine terms in the sum.

#### IV. CATENARY EQUATIONS SIMULATION IN MATLAB

Parameters of the differential equation (4) are calculated based on equation (2) and are used to compute the total output of catenary signal (7). Some of these results were found using MATLAB. Ten harmonics of the function  $F_i(x)$  were used to compute it for a range of frequencies 0.1-0.5 Hz and 1.0-5.0 Hz.

The signals of these frequencies are combined into a single output catenary (Fig. 4). Coefficients  $M_i, C_i, K_i, Q_i$  of the catenary model are calculated in (2) according to [2]. MATLAB code is used to compute these coefficients. Then transfer functions are calculated for natural modes of vibration frequencies of 0.1-0.5 Hz and 1.0-5.0 Hz.

$$H_i(s) = \frac{Z_i(s)}{Q_i(s)} = \frac{1}{M_i s^2 + C_i s + K_i};$$

$$Y_1(s) = \frac{141.6180}{5.0139s^2 + 41.3247s + 69.0255} X_1(s)$$

$$Y_2(s) = \frac{245.1882}{15.0293s^2 + 123.8716s + 206.9052} X_2(s)$$

$$Y_3(s) = \frac{253.4573}{15.9337s^2 + 131.3254s + 219.3554} X_3(s) \quad (8)$$

$$Y_4(s) = \frac{193.666}{9.3766s^2 + 77.2822s + 129.086} X_4(s)$$

$$Y_5(s) = \frac{150.5337}{5.6651s^2 + 46.6917s + 77.9901} X_5(s)$$

The model in Simulink system, which corresponds to equation (4), shows vibration of sinusoid components using transfer functions (8). Catenary model for the calculated parameters in MATLAB is shown in Fig. 5. Analysis of the model leads to the following results.

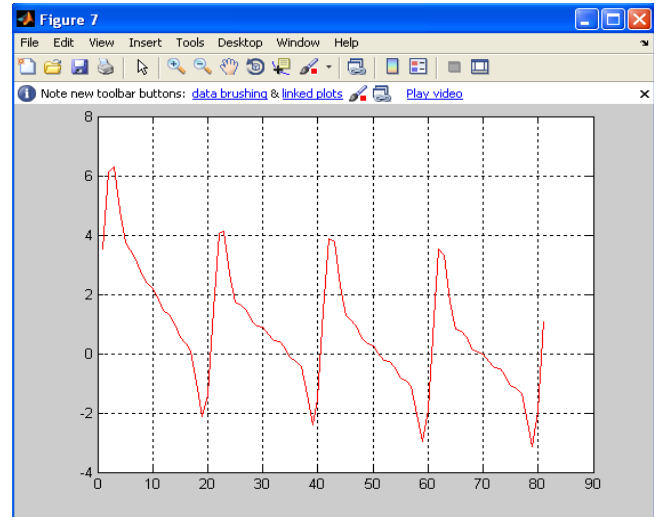


Fig. 4. Single output of catenary system

Input and output signals of the model are shown in Fig. 6, where

- the output signals of model catenary (every mode) are registered of the top schedule;
- the input catenary signal show of the bottom schedule (Uniform Random Numbers);
- the sums catenary output signal (step response plus Uniform Random Numbers) show of the average schedule.

#### V. A PHYSICAL AND MATHEMATICAL MODEL OF THE PANTOGRAPH.

The physical model pantograph is investigating of that section show on the figure 3 (passive pantograph) and figure 7 (ordinary active pantograph).

According to the descriptions of model [3] and shown to Fig. 7 in which the two mass models is represented, the definitions of symbols used in the analysis of mathematical model are:

$K_w$  – contact wire stiffness (kg/m);  $w$  – head spring rate (kg/m);  $u$  – head damping (kg · sec/m);  $t$  – frame stiffness (kg/m);  $v$  – frame damping (kg · sec/m);  $m$  – head mass (kg · sec<sup>2</sup> /m);  $M$  – frame mass (kg · sec<sup>2</sup> /m);  $F_s$  – constant lift force (kg).

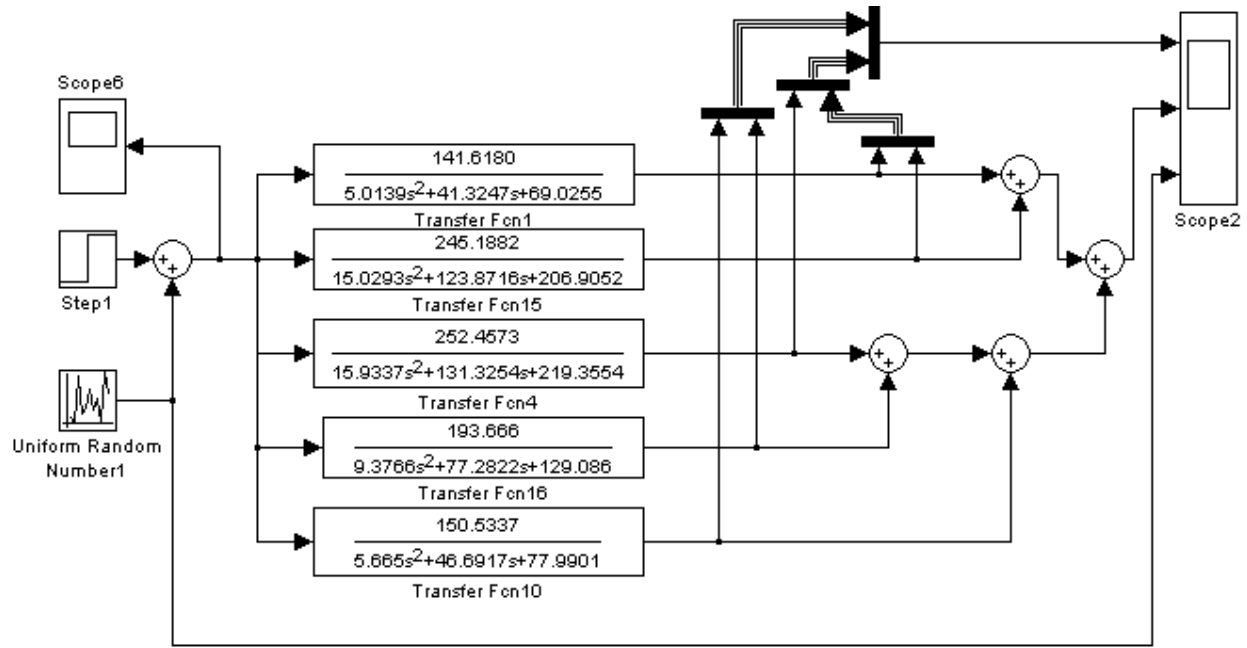


Fig. 5. Catenary model parameters

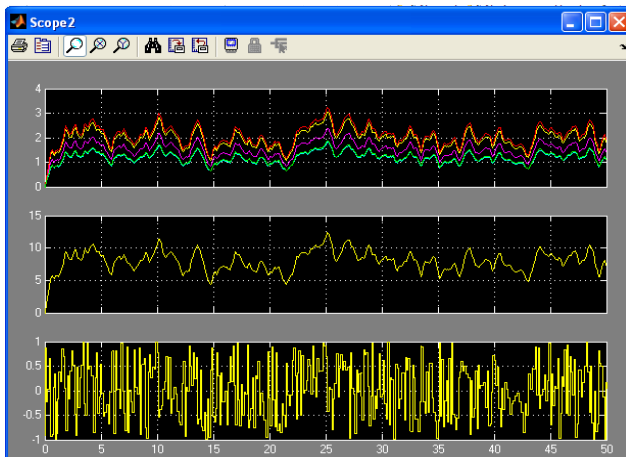


Fig. 6. Catenary model input and output signals

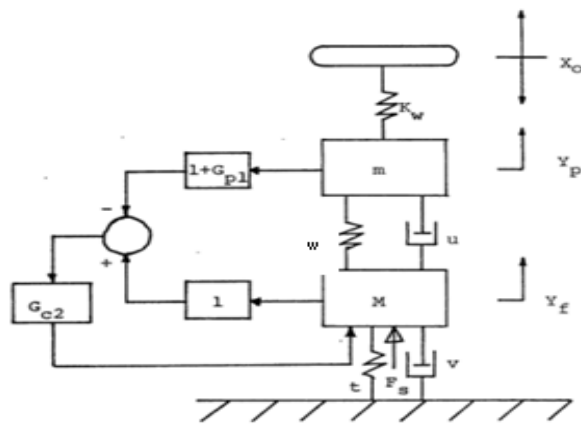


Fig. 7. Physical model of ordinary active pantograph

Passive pantograph parameters are the following(in system SI):

$K_w = 178,58 \text{ kg/m}$ ;  $w = 178.59 \text{ kg/m}$ ;  $u = 3.57159 \text{ kg} \cdot \text{sec/m}$ ;  $t = 34287.3 \text{ kg/m}$ ;  $v = 21.4295 \text{ kg} \cdot \text{sec/m}$ ;  $m = 1.1072 \text{ kg} \cdot \text{sec}^2/\text{m}$ ;  $M = 1.66079 \text{ kg} \cdot \text{sec}^2/\text{m}$ . (9)

The difference equations of this physical model are the following (for passive pantograph in Fig. 3):

$$m d^2 y_p = -F_c - w(y_p - y_f) - u(dy_p - dy_f)$$

$$M d^2 y_f = F_s + w(y_p - y_f) + u(dy_p - dy_f) - v(dy_f - dy_r) - t(y_f - y_r), (10)$$

where

$y_p$  – the displacement of the head,

$y_f$  – the displacement of the frame,

$y_r$  – the displacement of the vehicle roof,

$F_c$  – the contact force acting on the pantograph head,

$F_s$  – the static constant uplift force.

Hence, we take Laplace transform and assume zero initial conditions for Laplace transform of  $y_p$ ,  $y_f$  and  $F_c$ .

$$\begin{aligned} (ms^2 + us + w)Y_p + F_c &= (us + w)Y_f \\ (Ms^2 + (u + v)s + t)Y_f &= (us + w)Y_p \end{aligned} \quad (11)$$

Evaluating results of  $\bar{X}_0$  and  $\bar{X}_0$  we obtain

$$\frac{Y_p}{X_0} = \frac{K_w(Ms^2(u + v)s + w + t)}{\Delta}, \quad (12)$$

$$\frac{Y_f}{X_0} = \frac{K_w(us + w)}{\Delta}, \quad (13)$$

where

$$\dot{z} = Mms^4 + (Mu + m(u + v))s^3 + (M(w + Kw) + m(w + t) + u(u + v) - u^2)s^2 + ((u + v)(w + Kw) + u(w + t) - 2uw)s + ((w + t)(w + Kw) - w^2).$$

This model of a two-mass system is a system of two degree of freedom. Although the actual pantograph is very complicated in its structure, the model of two degree of freedom of which we have already built up is enough to represent the overall system of pantograph in initial studies. There is a system of one degree of freedom should be compute of the transfer function (12) with the parameters passive pantograph (9):

$$H_{px}(s) = \frac{Y_p(s)}{X_0(s)} = \frac{161.3s^2 + 2428s + 3347e05}{s^4 + 18.28s^3 + 20794s^2 + 66231s - 17341} \quad (14)$$

#### VI. A MATHEMATICAL MODEL OF THE PANTOGRAPH-CATENARY DYNAMIC INTERACTION WITH ONE CONTACT WIRE

##### a. Coupling between the models

The pantograph/catenary system model is shown on Fig. 9. Parameters of the catenary model computed for the range of frequencies 0.1 – 0.5Hz and 1.0 – 5.0 Hz.

Research of the main characteristics of model of a pantograph-catenary is spent by change of the entry signals operating on system catenary: casual signals, impulse and step signals on a model input catenary. Input and output signals of model are registered in the form of schedules.

The coupling between the pantograph and catenary comes exclusively through the contact force. When the pantograph is in contact with catenary in usual interaction, moving of the pantograph head and the lower catenary wire are identical, and the contact force has zero value. When the pantograph loses contact, the contact force becomes zero and the position of the pantograph and catenary are independent until the pantograph regains contact. Only during momentary losses of contact are the pantograph and catenary two separate systems. At all other times, they are directly coupled: they share the same position and they share the same force. The contact force of pantograph system is denoted by  $F_i(x)$  in equation (7).

##### b. Simulation technique coupling between the passive pantograph-catenary models

To simulate dynamic response of pantograph and catenary equations of motion were solved simultaneously for both using MATLAB code and Simulink blocks. (In previous papers it was done by integration technique using fourth Runge-Kutta [2,3,4]). The catenary equations ( $N$  equations, where  $N$  equals to the number of modes) are two pantograph equations (11). The response of each modal amplitude,  $z$ , and the response of the pantograph is calculated at each time step.

The position of the catenary wire at each instant is given by equation (1) and summing up the individual modes. The time is then incremented and the process repeated until the final time of the simulation is reached.

Two different pantograph-catenary models in Simulink are shown in Fig. 9. Model of passive pantograph contains transfer function Fcn6, and the active pantograph model contains transfer functions Fcn3 and Fcn6 as well as gain blocks Gain2 and Gain3.

Input and output signals of the models are shown in Fig. 8. There are:

- total output catenary signal (upper plot)
- transfer function of the system with passive pantograph (mid plot). It confirms that the system is unstable.
- transfer function of the system with active pantograph (lower plot). It confirms that the system is stable.

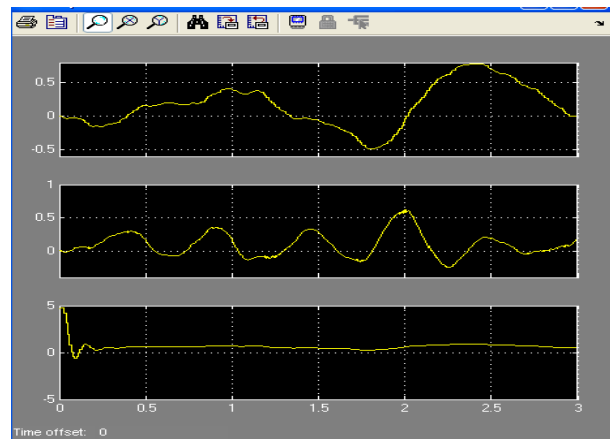


Fig. 8. Input and output signals of pantograph model

#### VII. FURTHER RESEARCH TO OPTIMIZE PANTOGRAPH-CATENARY SYSTEM

A linear model of pantograph-catenary system is considered, where the lower and upper pantograph frames, as well as catenary, are modelled in terms of lumped masses, springs and dampers. Mechanical parameters of the upper and lower pantograph frames are time dependable.

Similarly, catenary parameters are multivariable functions depend on distance between contact point and the neighbour adjacent tower and dropper (see Fig.3).

Model of active pantograph was introduced in the system to achieve optimal characteristics of the pantograph-catenary system with given conditions.

Additional requirements for the active pantograph model are the following:

- estimation of parameters time variations for the dynamic pantograph-catenary system;
- adaptive control algorithm elaboration, which makes up control signals based on system's parameters modifications.



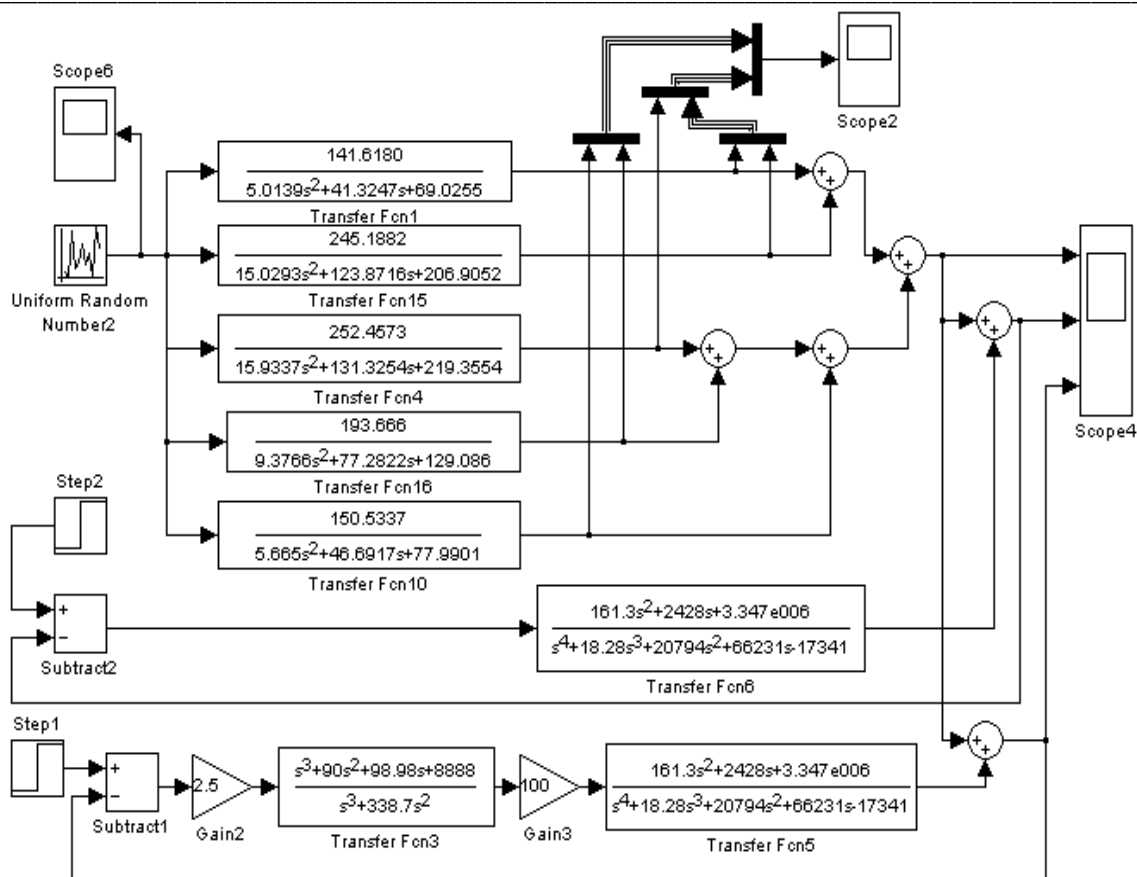


Fig. 9. Fcn6 – transfer function of passive pantograph; Fcn5 – transfer function of active pantograph

Realization of these requirements is possible based on the appropriate functions in the pantograph control contour:

- parametric identification algorithms in the system, that is, system pantograph-catenary parameters calculation in defined time interval;
- computing of gain matrix parameters of the active pantograph controller on measured interval of the system state vector;
- using of nano-accelerometers in the pantograph-catenary system.

### VIII. CONCLUSIONS

Some new models of pantograph-catenary system are considered to solve the following problems:

New model of the pantograph-catenary system leads to the following tasks which are considered:

- using output signals of the catenary system to form harmonic components of different frequencies;
- computing coefficients of catenary differential equations based on the output signals;
- computing of transfer functions for every component of the output catenary signal for modelling in Simulink;
- algorithms of active and passive pantograph development for modelling in Simulink.

Paper results confirm the effectiveness of the active pantograph on high speed trains. Pantograph basic parameters changing in time requires development algorithms of adaptive

control based on the algorithm of parametric identification in the pantograph-catenary system. Adaptive control system provides the desired reliability of the pantograph-catenary system on high speed trains.

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Andrejs Matvejevs is Doctor of Technical Sciences with speciality of information systems. Until 2009 he was a chief actuary in insurance company

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He is the author of about 40 scientific publications, 2 study books and numerous conference papers. In addition prof. Andrejs Matvejevs was a Editorial Board member of the Scientific Journal of Riga Technical University, section „Information Technology and Management Science”.

#### **Aleksandrs Matvejevs, Andrejs Matvejevs. Sistēmas „Pantogrāfs-strāvas uztvērējs” modelēšana uz Matlab-Simulink algoritmu bāzes**

Kontakts starp pantogrāfu un kontaktu vadu ir vissvarīgākā daļa elektroenerģiju pārvadē modernu liela ātruma elektrovilcienos. Kontakta vada svārstības dēļ notiek mijiedarbības spēka izmaiņas starp pantogrāfu un vadu, kas nereti atved pie kontakta zaudējuma starp vadu un pantogrāfu.

Tādu situāciju izņēmumam pieprasa pastāvīgi pilnveidot šīs sistēmas komponentu mijiedarbību ar pastāvīga strāvas uztvērēja kontakta nodrošinājumu ar tīkla vadiem vilciena kustības augsta ātruma gadījumā, ievērojot šīs sistēmas elementu nodilumu un traucējumu signālu apslāpēšanas.

Šīs sistēmas dinamisku procesu sarežģītības dēļ, kuras notiek vilciena kustības reālos apstākļos, kā arī šīs sistēmas galveno elementu augstas izmaksas dēļ, visas izmaiņas sistēmā „pantogrāfs/strāvas uztvērējs” pamatojas dinamiskas modelēšanas posmā. Mūsdienīgu datoru programmas nodrošinājums ļauj pielietot sistēmas bloku funkcionēšanas režīmu modelēšanas vairāk modernu metodes un šīs sistēmas vadības metožu, ievērojot tās parametru un ārēju iedarbību izmaiņas.

Šajā darbā sistēmas „pantogrāfs/strāvas uztvērējs” matemātiskās modeļu priekšstatam bija izmantoti diferenciālvienādojumi, kas raksturo tās dinamiskas īpašības. Uz šo vienādojumu bāzes ar sistēmas Matlab/Simulink izmantošanu tika izveidotas vadu sistēmas atbilstošie modeļi, pasīva un aktīva pantogrāfu, kā arī pētāmās sistēmas kopējais modelis. Katram modelim tika saņemti atbilstošie dinamiskie raksturojumi (impulsu, pārveidošanas, biežumu), kas apstiprina to funkcionēšanas stabilitāti (nestabilitāti) ar parametru un ārēju iedarbību dažādām vērtībām. Uz izpildīto pētījumu rezultātu bāzes tiek piedāvāti rekomendācijas aktīva pantogrāfa tālākai algoritmu pilnveidošanai.

#### **Александр Матвеев, Андрей Матвеев. Моделирование системы «пantограф-токоприемник» с использованием алгоритмов системы Matlab**

Контакт между pantографом и контактным проводом является наиболее важным элементом в передаче электрической энергии в современных поездах, двигающихся с высокой скоростью. При таком движении в результате колебаний контактного провода происходит изменение силы взаимодействия между pantографом и проводом, что нередко приводит к потере контакта между проводом и pantографом.

Для исключения таких ситуаций требуется постоянно совершенствовать взаимодействие компонентов этой системы с целью обеспечения постоянного контакта токоприемника с проводами сети (pantограф/токоприемник) на высокой скорости движения поезда с учетом износа элементов этой системы и подавления сигналов помех.

В виду сложности динамических процессов, происходящих в этой системе при ее эксплуатации в реальных условиях движения поезда, а также учета высокой стоимости основных элементов этой системы, все изменения в системе pantограф/токоприемник обосновываются на этапе динамического моделирования. Программное обеспечение современных компьютеров позволяет применять более совершенные методы моделирования режимов функционирования основных блоков системы pantограф/токоприемники применения активных методов управления этой системой с учетом изменения ее параметров и внешних воздействий.

В данной статье для представления математических моделей системы «pantограф/токоприемник» были использовали дифференциальные уравнения, характеризующие их динамические свойства. На основе этих уравнений с использованием системы **Matlabi Simulink** были сформированы соответствующие модели системы проводов, пассивного и активного pantографов, а также общая модель исследуемой системы. Для каждой модели были получены соответствующие динамические характеристики (импульсные, переходные, частотные), подтверждающие устойчивость (неустойчивость) их функционирования при различных значениях параметров и внешних возмущений. На основании результатов выполненных исследований даны рекомендации для дальнейшего совершенствования алгоритмов активного pantографа.